Structural Equation Modeling for Ordinal Data

Chong Xing[1], Meghan Sullivan[1], Paul Johnson[1] & Ben Kite[2]

[1]Center for Research Methods and Data Analysis
[2]H&R Block

2018

Goals of this Session

During this session:

- Understand the non-normality issues in SEM (or data analysis in general)
- Understand the estimations specifically developed for treating non-normality issues in SEM
- Obtain working knowledge on running SEM models with ordinal data
- Reflect on own analysis project(s) and the decisions made

After the workshop:

- Make better statistical judgment
- Produce quality research
- Educate and help others

Outline

Outline ...

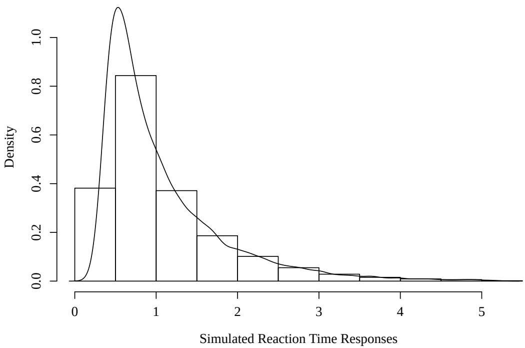
Outline

Non-Normal Continuous Data - Reaction Time (RT)

"Non-normal data" is often used as a general (sometimes vague) expression referring to sample distribution patterns that *don't look normal*.

- Responses collected using **continuous measures** (interval or ratio) can exhibit substantial departure from normality. For instance, observations are often skewed in reaction time (RT) studies.

Non-Normal Continuous Data - Reaction Time (RT) ...



- E.g., Correll et al. (2002) conducted experimental studies on the reaction time for shooting decisions made under 2 (Black or White target) $\times$ 2 (Unarmed or armed) conditions.

Non-Normal Continuous Data - Reaction Time (RT) ...

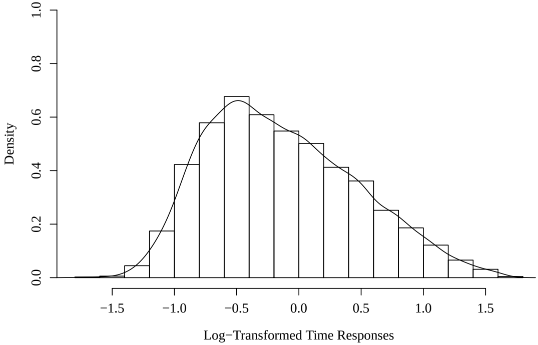
Results and Discussion

To analyze the resulting reaction times, we excluded all trials on which the participant had either timed-out (i.e., failed to make a decision in the allotted 850-ms window) or made an incorrect response (e.g., shooting a target holding a non-gun). This resulted in the exclusion of data from 7% of the trials across participants, with a maximum of 20% of the trials for any one participant. Response latencies on the remaining trials were log-transformed and then averaged within subject across trials occurring in the same cell of the 2×2 within-subject research design. An analysis of variance (ANOVA) of the resulting mean latencies was then conducted, treating Target Ethnicity (White vs. African American) and Object Type (gun versus no gun) as within-subject factors.

- The non-normal continuous RT data, following a transformation, may appear normal. Hence, researchers may feel more comfortable applying linear modeling techniques to the transformed data.

KU

Non-Normal Continuous Data - Reaction Time (RT) ...



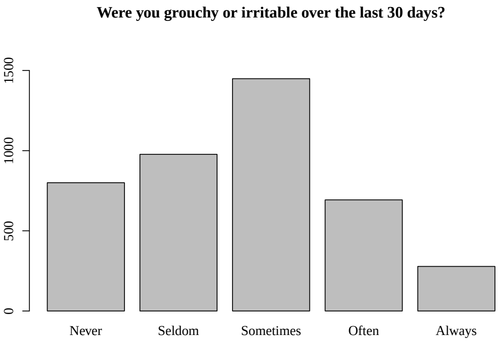
- This log-transformation approach (and its variations) has traditionally been used for non-normal continuous data, but is now **outdated**. <- cite/evidence - still used today

Categorical Data

- Responses collected using **categorical measures** (nominal or ordinal) contain information separated in distinct and mutually exclusive categories.

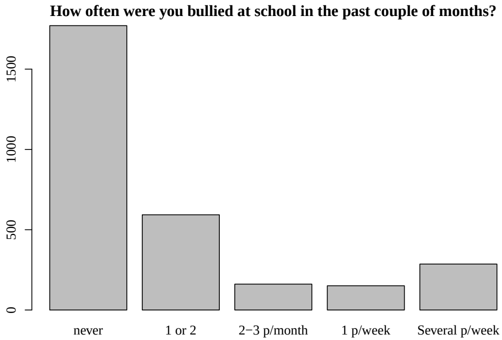
Categorical Data ...

- Sometimes, ordinal responses may exhibit a unimodal-symmetric distribution that “looks normal,”



Categorical Data ...

- but fundamentally ordinal measures with ordered-categorical responses rarely meet the assumption of equal spacing between response levels (O'Brien, 1985).



Categorical Data ...

- The terms “non-normal data” and “non-normality” could mean non-normal continuous data or categorical data (or something else). Nevertheless, different estimation approaches have been developed targeting specific non-normality issues in SEM framework.
- For analysts, it is important to understand the nature of our data prior to executing any formal analyses. For instance, being able to differentiate **non-normal continuous data** from **categorical data** is vitally important when decisions are to be made about choosing appropriate estimation techniques (or to rule out the wrong choices) in SEM analysis.

Outline

Some Published Work on the Big Five Personality

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	2	3	4	5

- A variable with numeric codes 1-2-3-4-5 cannot be "normal" or even approximately so. A normal variable is a floating point number. Any variable measured 1-2-3-4-5 cannot, by definition, be drawn from a normal distribution.

Measures.

Big Five Inventory. As described above, the original Big Five Inventory (BFI; John, Donahue, & Kentle, 1991; see John et al., 2008) was developed to measure the prototypical features of each Big Five domain. Its 44 items are short, descriptive phrases that respondents rate on a 5-point scale ranging from *disagree strongly* to *agree strongly*. A total of 641 ESCS members completed the BFI; in this sample, the domain scales' alpha reliabilities were .86 for Extraversion, .82 for Agreeableness, .83 for Conscientiousness, .85 for Negative Emotionality, and .84 for Open-Mindedness.

Some Published Work on the Big Five Personality ...

Soto and John (2017a) on Big Five 2

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	2	3	4	5

- However, for many years after the invention of SEM, it was standard to treat "Likert-type" variables, scored 1-2-3-4-5, as if they were normally distributed. Even now,

To examine the multidimensional structure of the BFI-2-S and BFI-2-XS, we submitted each short form's items to a random intercept EFA that extracted and varimax-rotated five factors, using maximum likelihood estimation. Random intercept EFA (Aichholzer, 2014) is a procedure for examining the multidimensional structure of an item set while using a method factor to model individual differences in acquiescent responding, thereby minimizing the negative structural effects of acquiescence variance (Rammstedt & Farmer, 2013; Soto et al., 2008). For both short

Soto and John (2017b) on Big Five 2-S and ES

Some Published Work on the Big Five Personality ...

- Sometimes, “non-normality” issues with five-point Likert-type scale were addressed (as if the data were continuous) and techniques developed for continuous data would be used to analyze the truly ordinal responses.

> Prior to analysis, all items were checked for non-normality. Skewness and kurtosis of the data were −1.18 to 0.25 (kurtosis) and −0.87 to 0.95 (skeweness), showing that items were mainly close to normal. Because population weights were used to draw highly precise conclusions about the population, a restricted maximum likelihood (MLR) estimator was applied (Muthén & Muthén, 1998–2013). Factor models were separately fitted for each of the Big Five traits.

Another Big Five study (Göllner et al., 2017)

- Under what conditions is it reasonable to treat an ordinal variable as if it were a numeric?

The Long-Lasting "Interval-Ordinal Debate"

If the situation arises that ordinal data could form a unimodal-symetric distribution pattern that looks normal, two claims could be made:

- Claim 1: Ordered-categorical data should be analyzed only using methods developed for categorical data (e.g., WLSMV).
- Claim 2: Ordered-categorical data can be analyzed using methods developed for continuous data (e.g., ML or MLR), **if...**

Support for Claim 1:

- a. More appropriate model specifications
- b. Less biased (more accurate) estimates
- c. Inferences are made from valid models that align with data

Reservations of Claim 2:

- a. **If** there are more than five response options
- b. **If** the sample size is small (e.g., $N < 50$)
- c. **If** there are missing data

Why (or when) ordinal data should not be treated as if it were numeric

Support 1a. - Treating ordinal data as numeric can lead to model misspecifications (Muthén & Kaplan, 1985).

- "*With five-category Likert scales, the [indicator] variables are often scored by consecutive integers, say 0, 1, 2, 3, 4. The analysis then proceeds as if these [indicator] variables are interval scaled .... Strictly speaking,* **the model cannot be correct unless there are infinitely many categories for each [indicator variable]** *...*" (p. 172).
- "*[For indicator variables based on Likert scales] with extreme skews it should further be noted that the ordinary measures of association, covariances and Pearson product moment correlations, my not be suitable.* **Not only the [indicator variables] discrete, but they are also limited in range with strong censoring. In such cases, a linear model for [ordinal data] may be unrealistic**, *and a non-linear model ... may be more appropriate*" (p. 172).

Why (or when) ordinal data should not be treated as if it were numeric ...

Support 1b. - Applying modeling techniques developed for numeric data to ordinal data can lead to incorrect parameter and global model fit estimates (Beauducel & Herzberg, 2006; Flora & Curran, 2004).

- Spuriously inflated model χ^2values (i.e., over-rejection of solutions)
- Modest underestimation of fit indices (e.g., TLI, CFI)
- moderate to severe underestimation of the standard errors of the parameter estimates (inflated Type I error)
- These issues are exacerbated when the sample size is small.

Why (or when) ordinal data should not be treated as if it were numeric ...

Second, this study replicated previous results that factor loadings are typically underestimated by MLR but are essentially unbiased with WLSMV (Beauducel & Herzberg, 2006; DiStefano, 2002; Flora & Curran, 2004). Interestingly, a clear superiority of WLSMV over MLR in factor loading estimates was found in this study, irrespective of the number of categories. This study also revealed that the factor loadings obtained by WLSMV were more precise and accurate than those obtained by MLR when the latent normality assumption was moderately violated. Generally speaking, WLSMV was preferable to MLR across most of the conditions observed in this study, given its properties of being less biased and having small sampling variation in estimating factor loadings.

WLSMV vs. MLR (Li, 2016)

Why (or when) ordinal data can be treated as if is numeric

Reservation 1 - If there are more categories (Bollen & Barb, 1981, p. 234)

AMERICAN SOCIOLOGICAL REVIEW

Table 1. Comparison of Mean Correlation Coefficients for Original and Collapsed Variables

Number of Categories	Mean Collapsed Correlation				
2	0.117	0.264	0.414	0.579	0.719
3	0.149	0.302	0.445	0.617	0.727
4	0.165	0.336	0.504	0.669	0.774
5	0.182	0.359	0.533	0.712	0.806
6	0.180	0.371	0.554	0.733	0.833
7	0.192	0.380	0.563	0.750	0.850
8	0.193	0.384	0.571	0.760	0.860
9	0.194	0.389	0.577	0.771	0.870
10	0.197	0.391	0.581	0.772	0.875
Mean Original Correlation	0.203	0.404	0.599	0.796	0.901

Based on 50 samples of 500 observations each.

Why (or when) ordinal data can be treated as if is numeric ...

- Reservation 1's reservation - information of **continuous concepts** are coerced into equally-spaced ordered categories.

Reservation 2 - If the sample size is small, treating ordinal data as numeric may facilitate model convergence.

WLSM and WLSMV also reported sensitivity to extreme levels of nonnormality, but generally, with more categories (five, seven) and smaller sample sizes (200, 400). Under these conditions, WLSMV reported greater numbers of replications that did not converge than with the two- or three-category conditions. With many categories and item-level nonnormality, a contingency table between two observed variables could result in empty cells. The *Mplus* Web site noted the sensitivity of categorical estimators to empty cells and the software has implemented procedures to assist with categorical estimation beginning with *Mplus* version 4.0 (http://www.statmodel.com); however, some sensitivity to these conditions remains.

Why (or when) ordinal data can be treated as if is numeric ...

DiStefano and Morgan (2014, p. 430); also see Flora and Curran (2004)

Reservation 3 - If there are missing values in ordinal data, treating them as numeric may facilitate information recovery.

- The commonly used weighted least squares estimators (e.g., DWLS, WLSMV) do not have missing data information as a part of their model specifications; least-wise deletion is the only option.
- Full information maximum likelihood (FIML; Arbuckle,1996), along with multiple imputation (MI; Allison, 2003), are two popular missing data methods for continuous data. These two methods can be used if ordinal data are treated as numeric.
- One SEM book claims "[a] full-information version of ML (FIML) estimation for noncontinuous indicators is becoming increasingly available in SEM computer tools, including LISERL, Mplus, Stata, and others" (Kline,2016, p. 331). Yet, published studies evaluating such implementations are scarce (e.g., van der Palm, van der Ark, & Vermunt, 2016).

Outline

Mathmatical definition of Maximum Likelihood

(McArdle & Nesselroade, 2014)

> When estimating structural equation models, the choice of which estimation technique to use is an important decision. The fit function is responsible for producing the model parameters, standard errors of parameters, and fit indices to evaluate the proposed model. Researchers who select an estimator without attending to its requirements and assumptions might inadvertently choose a technique that adversely impacts results (Finney & DiStefano, 2006).

DiStefano and Morgan (2014, p. 425)

- When analyzing non-normal continuous data, the appropriate choice of estimator should be a robust/corrected version of ML; estimators from the weighted least squares family (developed for categorical data) are not recommended for handling this type of non-normality.

Mathmatical definition of Maximum Likelihood ...

- When the observations were made using nominal or ordinal measures, WLS family estimators are recommended. However, decisions have to be made in terms of which WLS to use (e.g., WLS vs. DWLS vs. WLSM vs. WLSMV).
- This section reviews ML first, then extends the discussion to the robust versions of ML and WLS family estimators.

The previous section has already alluded to the idea that analysis decisions must be made based on a clear understanding of the sample data and the capabilities of the specific statistical technique. This section focuses on commonly used estimators in SEM and situtations in which one estimator may be preferred over another. It is important to understand the characteristics (e.g., assumptions) of individual estimators to ensure that they align with the characteristics of the data. If the assumption that the data is continuous does not hold, generalized models should be used instead.

> When estimating structural equation models, the choice of which estimation technique to use is an important decision. The fit function is responsible for producing the model parameters, standard errors of parameters, and fit indices to evaluate the proposed model. Researchers who select an estimator without attending to its requirements and assumptions might inadvertently choose a technique that adversely impacts results (Finney & DiStefano, 2006).

DiStefano and Morgan (2014, p. 425)

- When analyzing non-normal continuous data, the appropriate choice of estimator should be a robust/corrected version of ML; estimators from the weighted least squares family (developed for categorical data) are not recommended for handling this type of non-normality.
- When the observations were made using nominal or ordinal measures, WLS family estimators are recommended. However, decisions have to be made in terms of which WLS to use (e.g., WLS vs. DWLS vs. WLSM vs. WLSMV).
- This section reviews ML first, then extends the discussion to the robust versions of ML and WLS family estimators.

How ML works?

Maximum likelihood

Properties of ML

- Asymptotic Efficiency: an estimator is said to be asymptotically efficient if it is consistent and asymptotically normally distributed and has an asymoptotic covariance matrix that is not larger than the asymptotic covariance matrix of any other consistent, asymptotically normally distributed estimator.
- Regularity Conditions

Test procedures

- Likelihood ratio tests
- Wald tests are based on the unrestricted model estimates.
- Lagrange multiplier tests are tests based on the restricted model instead of an unrestricted model. These tests supposes that if the restrictions on parameters are valid, then imposing them would not significantly affect the value of the likelihood function.

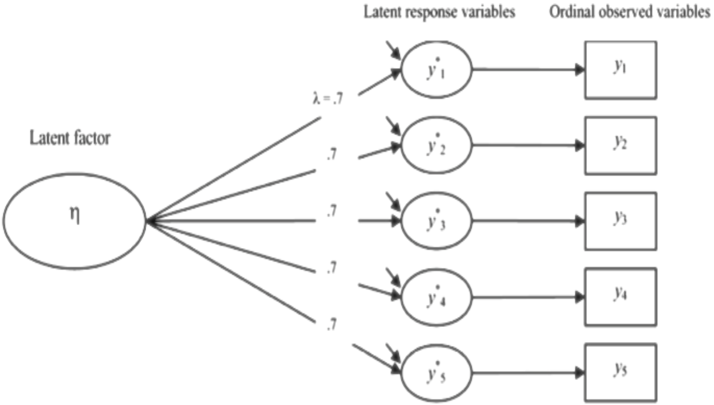
How ML works? ...

Robust ML still assumes multivariate normal data, however corrects for kurtosis (i.e., a measure of the shape of the distribution) with a scaling factor that rescales the loglikelihood under the likelihood ratio tests and adjusts standard errors for all parameter estimates.

AIC and BIC are used to compare nonnested models to test the validity of parameters in the model.

1. MLM
    1. MLR (Yuan & Bentler, 2000) Robust ML still assumes multivariate normal data, however corrects for kurtosis (i.e., a measure of the shape of the distribution) with a scaling factor that rescales the loglikelihood under the likelihood ratio tests and adjusts standard errors for all parameter estimates.
    2. FIML

Categorical SEM: The key conceptual ideas



Flora and Curran (2004, p. 473)

Categorical SEM: The key conceptual ideas ...

1. A latent response variable is a normally distributed latent variable with threshold estimates (τ; tau)
2. A threshold estimate indicates the level of a factor that a person needs to possess to move from one category to the next category (e.g., from responding "agree" to "strongly agree" to an ordinal survey item)
3. The number of threshold estimates equals to the number of categories of an ordinal item -1

Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	2	3	4	5

$5 - 1 = 4$ thresholds

WLS estimators for categorical data

1. WLS - weighted least squares
2.
3. Diagonally weighted least squares (developed to facilitate model estimation)
    1. WLSM - weighted least squares-mean adjusted
    2. WLSMV - weighted least squares mean and variance adjusted
    3. DWLS - diagonal weighted least squares

Outline

The nature of your data

- A categorical variable has a measurement scale consisting of a set of categories Agresti (2007).
- Analysts need to be aware of the data type of the variables (indicators) when using R for SEM. Problems, such as uninterpretable/incorrect results, will arise if the variable types are not specified correctly.

```
## The demonstration data - a subset of HBSC
## Health Behaviour in School-Aged Children
names(hbsc)
```

The nature of your data ...

```
 [1] "stud_id"  "schl_id"  "Gender"   "Age"      "Grade"    "body1_o"
      "body2_o"  "body3_o"
 [9] "body4_o"  "body5_o"  "phys1_o"  "phys2_o"  "phys3_o"  "phys4_o"
      "phys5_o"  "phys6_o"
[17] "phys7_o"  "phys8_o"  "depre1_o" "depre2_o" "depre3_o" "depre4_o"
     "depre5_o" "depre6_o"
[25] "gotBu1_o" "gotBu2_o" "gotBu3_o" "gotBu4_o" "gotBu5_o" "gotBu6_o"
     "gotBu7_o" "gotBu8_o"
[33] "gotBu9_o" "buOth1_o" "buOth2_o" "buOth3_o" "buOth4_o" "buOth5_o"
     "buOth6_o" "buOth7_o"
[41] "buOth8_o" "buOth9_o" "alc1_o"   "alc2_o"   "alc3_o"   "alc4_o"
     "alc5_o"   "body1_i"
[49] "body2_i"  "body3_i"  "body4_i"  "body5_i"  "phys1_i"  "phys2_i"
     "phys3_i"  "phys4_i"
[57] "phys5_i"  "phys6_i"  "phys7_i"  "phys8_i"  "depre1_i" "depre2_i"
     "depre3_i" "depre4_i"
[65] "depre5_i" "depre6_i" "gotBu1_i" "gotBu2_i" "gotBu3_i" "gotBu4_i"
     "gotBu5_i" "gotBu6_i"
[73] "gotBu7_i" "gotBu8_i" "gotBu9_i" "buOth1_i" "buOth2_i" "buOth3_i"
     "buOth4_i" "buOth5_i"
[81] "buOth6_i" "buOth7_i" "buOth8_i" "buOth9_i" "alc1_i"   "alc2_i"
     "alc3_i"   "alc4_i"
[89] "alc5_i"
```

The nature of your data ...

- R can find out the class of a variable by `class()` and show the levels of a factor variable by `levels()`

```
class(hbsc$Age)
```

```
[1] "integer"
```

```
class(hbsc$Gender)
```

```
[1] "factor"
```

```
levels(hbsc$Gender)
```

```
[1] "male"   "female"
```

The nature of your data

- A common mistake may occur when reading in unordered-factor variables

```
## An ordered-factor avariable in R should look
   like
class(hbsc$depre1_o)
```

```
[1] "ordered" "factor"
```

```
levels(hbsc$depre1_o)
```

```
[1] "Never"      "Seldom"     "Sometimes" "Often"      "Always"
```

```
str(hbsc$depre1_o)
```

```
Ord.factor w/ 5 levels "Never"<"Seldom"<..: 2 2 3 3 2 3 1 3 1 1 ...
```

KU

A one-factor categorical CFA model

```
## Specifying the model-structure object
cfa.01.v.02 <- '
 depress =~ NA*depre1_o + depre2_o + depre3_o +
               depre4_o + depre5_o + depre6_o
 depress ~~ 1*depress '
## Estimating the model
cfa.01.v.02.fit <-
 cfa(model = cfa.01.v.02, data = hbsc,
     mimic = "Mplus", estimator = "WLSMV",
     ordered = c("depre1_o", "depre2_o",
                 "depre3_o", "depre4_o",
                 "depre5_o", "depre6_o"))
## Requesting an estimation summary
summary(cfa.01.v.02.fit, fit.measures = TRUE,
        standardized = TRUE)
```

A one-factor categorical CFA model ...

```
lavaan (0.5-23.1097) converged normally after  12 iterations

                                                  Used       Total
  Number of observations                          4103        4284

  Estimator                                       DWLS      Robust
  Minimum Function Test Statistic              170.031     315.344
  Degrees of freedom                                 9           9
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.540
  Shift parameter                                            0.231
    for simple second-order correction (WLSMV)

Model test baseline model:

  Minimum Function Test Statistic            16019.445   10984.581
  Degrees of freedom                                15          15
  P-value                                        0.000       0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.990       0.972
  Tucker-Lewis Index (TLI)                       0.983       0.953

  Robust Comparative Fit Index (CFI)                            NA
```

A one-factor categorical CFA model ...

```
  Robust Tucker-Lewis Index (TLI)                                  NA

Root Mean Square Error of Approximation:

  RMSEA                                         0.066        0.091
  90 Percent Confidence Interval        0.058   0.075        0.083
       0.100
  P-value RMSEA <= 0.05                         0.001        0.000

  Robust RMSEA                                                 NA
  90 Percent Confidence Interval                               NA
       NA

Standardized Root Mean Square Residual:

  SRMR                                          0.034        0.034

Weighted Root Mean Square Residual:

  WRMR                                          2.088        2.088

Parameter Estimates:

  Information                                Expected
  Standard Errors                          Robust.sem
```

A one-factor categorical CFA model ...

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  depress =~
    depre1_o          0.710    0.010   69.620    0.000    0.710    0.710
    depre2_o          0.655    0.011   59.565    0.000    0.655    0.655
    depre3_o          0.744    0.011   66.214    0.000    0.744    0.744
    depre4_o          0.715    0.011   66.132    0.000    0.715    0.715
    depre5_o          0.603    0.013   47.857    0.000    0.603    0.603
    depre6_o          0.589    0.013   45.792    0.000    0.589    0.589

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .depre1_o          0.000                               0.000    0.000
   .depre2_o          0.000                               0.000    0.000
   .depre3_o          0.000                               0.000    0.000
   .depre4_o          0.000                               0.000    0.000
   .depre5_o          0.000                               0.000    0.000
   .depre6_o          0.000                               0.000    0.000
    depress           0.000                               0.000    0.000

Thresholds:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    depre1_o|t1      -0.564    0.021  -27.190    0.000   -0.564   -0.564
    depre1_o|t2       0.133    0.020    6.757    0.000    0.133    0.133
    depre1_o|t3       1.095    0.024   44.714    0.000    1.095    1.095
    depre1_o|t4       1.786    0.036   49.027    0.000    1.786    1.786
```

A one-factor categorical CFA model ...

```
    depre2_o|t1      -0.870    0.023  -38.655    0.000   -0.870   -0.870
    depre2_o|t2      -0.193    0.020   -9.813    0.000   -0.193   -0.193
    depre2_o|t3       0.737    0.022   34.076    0.000    0.737    0.737
    depre2_o|t4       1.510    0.030   49.863    0.000    1.510    1.510
    depre3_o|t1       0.289    0.020   14.544    0.000    0.289    0.289
    depre3_o|t2       0.668    0.021   31.455    0.000    0.668    0.668
    depre3_o|t3       1.177    0.025   46.355    0.000    1.177    1.177
    depre3_o|t4       1.624    0.033   49.905    0.000    1.624    1.624
    depre4_o|t1      -0.107    0.020   -5.447    0.000   -0.107   -0.107
    depre4_o|t2       0.255    0.020   12.896    0.000    0.255    0.255
    depre4_o|t3       0.888    0.023   39.219    0.000    0.888    0.888
    depre4_o|t4       1.436    0.029   49.526    0.000    1.436    1.436
    depre5_o|t1      -0.370    0.020  -18.423    0.000   -0.370   -0.370
    depre5_o|t2       0.072    0.020    3.668    0.000    0.072    0.072
    depre5_o|t3       0.669    0.021   31.485    0.000    0.669    0.669
    depre5_o|t4       1.232    0.026   47.298    0.000    1.232    1.232
    depre6_o|t1      -0.428    0.020  -21.175    0.000   -0.428   -0.428
    depre6_o|t2       0.085    0.020    4.323    0.000    0.085    0.085
    depre6_o|t3       0.741    0.022   34.223    0.000    0.741    0.741
    depre6_o|t4       1.255    0.026   47.639    0.000    1.255    1.255

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    depress           1.000                                1.000    1.000
   .depre1_o          0.496                                0.496    0.496
   .depre2_o          0.571                                0.571    0.571
```

A one-factor categorical CFA model ...

```
   .depre3_o        0.446                                      0.446    0.446
   .depre4_o        0.489                                      0.489    0.489
   .depre5_o        0.637                                      0.637    0.637
   .depre6_o        0.653                                      0.653    0.653

Scales y*:
                 Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    depre1_o        1.000                                      1.000    1.000
    depre2_o        1.000                                      1.000    1.000
    depre3_o        1.000                                      1.000    1.000
    depre4_o        1.000                                      1.000    1.000
    depre5_o        1.000                                      1.000    1.000
    depre6_o        1.000                                      1.000    1.000
```

A three-factor categorical CFA model

```
## Specifying the model-structure object
cfa.04.v.02 <- '
 gotBully =~ NA*gotBu1_o + gotBu2_o + gotBu3_o +
                gotBu4_o + gotBu5_o + gotBu6_o +
                gotBu7_o + gotBu8_o + gotBu9_o
 gotBully ~~ 1*gotBully

 depress =~ NA*depre1_o + depre2_o + depre3_o +
               depre4_o + depre5_o + depre6_o
 depress ~~ 1*depress

 alcohol =~ NA*alc1_o + alc2_o + alc3_o +
               alc4_o + alc5_o
 alcohol ~~ 1*alcohol '
## Estimating the model
cfa.04.v.02.fit <-
 cfa(model = cfa.04.v.02, data = hbsc,
```

A three-factor categorical CFA model ...

```
      mimic = "Mplus", estimator = "WLSMV")
## Requesting an estimation summery
summary(cfa.04.v.02.fit, fit.measures = TRUE,
         standardized = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  23 iterations

                                                  Used       Total
  Number of observations                          2684        4284

  Estimator                                       DWLS      Robust
  Minimum Function Test Statistic              870.308    1046.718
  Degrees of freedom                               167         167
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.876
  Shift parameter                                           52.908
    for simple second-order correction (WLSMV)

Model test baseline model:

  Minimum Function Test Statistic            75123.246   30297.601
  Degrees of freedom                               190         190
  P-value                                        0.000       0.000
```

A three-factor categorical CFA model ...

```
User model versus baseline model:

  Comparative Fit Index (CFI)                        0.991        0.971
  Tucker-Lewis Index (TLI)                           0.989        0.967

  Robust Comparative Fit Index (CFI)                                 NA
  Robust Tucker-Lewis Index (TLI)                                    NA

Root Mean Square Error of Approximation:

  RMSEA                                              0.040        0.044
  90 Percent Confidence Interval              0.037  0.042        0.042
      0.047
  P-value RMSEA <= 0.05                              1.000        1.000

  Robust RMSEA                                                       NA
  90 Percent Confidence Interval                                     NA
      NA

Standardized Root Mean Square Residual:

  SRMR                                               0.052        0.052

Weighted Root Mean Square Residual:

  WRMR                                               1.795        1.795
```

A three-factor categorical CFA model ...

```
Parameter Estimates:

  Information                                Expected
  Standard Errors                          Robust.sem

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  gotBully =~
    gotBu1_o          0.752    0.013   56.031    0.000    0.752    0.752
    gotBu2_o          0.780    0.013   59.272    0.000    0.780    0.780
    gotBu3_o          0.778    0.015   53.300    0.000    0.778    0.778
    gotBu4_o          0.807    0.011   70.933    0.000    0.807    0.807
    gotBu5_o          0.840    0.012   67.431    0.000    0.840    0.840
    gotBu6_o          0.866    0.013   65.092    0.000    0.866    0.866
    gotBu7_o          0.770    0.015   52.315    0.000    0.770    0.770
    gotBu8_o          0.861    0.014   61.471    0.000    0.861    0.861
    gotBu9_o          0.891    0.016   57.304    0.000    0.891    0.891
  depress =~
    depre1_o          0.692    0.014   49.508    0.000    0.692    0.692
    depre2_o          0.640    0.015   43.866    0.000    0.640    0.640
    depre3_o          0.753    0.015   48.967    0.000    0.753    0.753
    depre4_o          0.710    0.015   47.978    0.000    0.710    0.710
    depre5_o          0.597    0.017   35.108    0.000    0.597    0.597
    depre6_o          0.628    0.016   38.116    0.000    0.628    0.628
  alcohol =~
```

A three-factor categorical CFA model ...

```
    alc1_o          0.859    0.013   67.435    0.000    0.859    0.859
    alc2_o          0.774    0.016   47.132    0.000    0.774    0.774
    alc3_o          0.951    0.009  106.921    0.000    0.951    0.951
    alc4_o          0.882    0.010   84.810    0.000    0.882    0.882
    alc5_o          0.907    0.009  100.960    0.000    0.907    0.907

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  gotBully ~~
    depress         0.438    0.024   18.442    0.000    0.438    0.438
    alcohol         0.262    0.031    8.362    0.000    0.262    0.262
  depress ~~
    alcohol         0.354    0.028   12.469    0.000    0.354    0.354

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .gotBu1_o        0.000                               0.000    0.000
   .gotBu2_o        0.000                               0.000    0.000
   .gotBu3_o        0.000                               0.000    0.000
   .gotBu4_o        0.000                               0.000    0.000
   .gotBu5_o        0.000                               0.000    0.000
   .gotBu6_o        0.000                               0.000    0.000
   .gotBu7_o        0.000                               0.000    0.000
   .gotBu8_o        0.000                               0.000    0.000
   .gotBu9_o        0.000                               0.000    0.000
   .depre1_o        0.000                               0.000    0.000
```

A three-factor categorical CFA model ...

```
    .depre2_o         0.000                                         0.000    0.000
    .depre3_o         0.000                                         0.000    0.000
    .depre4_o         0.000                                         0.000    0.000
    .depre5_o         0.000                                         0.000    0.000
    .depre6_o         0.000                                         0.000    0.000
    .alc1_o           0.000                                         0.000    0.000
    .alc2_o           0.000                                         0.000    0.000
    .alc3_o           0.000                                         0.000    0.000
    .alc4_o           0.000                                         0.000    0.000
    .alc5_o           0.000                                         0.000    0.000
     gotBully         0.000                                         0.000    0.000
     depress          0.000                                         0.000    0.000
     alcohol          0.000                                         0.000    0.000

Thresholds:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
     gotBu1_o|t1      0.266    0.025   10.872    0.000    0.266    0.266
     gotBu1_o|t2      0.852    0.028   30.787    0.000    0.852    0.852
     gotBu1_o|t3      1.070    0.030   35.702    0.000    1.070    1.070
     gotBu1_o|t4      1.319    0.034   39.215    0.000    1.319    1.319
     gotBu2_o|t1      0.470    0.025   18.643    0.000    0.470    0.470
     gotBu2_o|t2      1.028    0.029   34.882    0.000    1.028    1.028
     gotBu2_o|t3      1.241    0.032   38.363    0.000    1.241    1.241
     gotBu2_o|t4      1.539    0.038   40.380    0.000    1.539    1.539
     gotBu3_o|t1      0.906    0.028   32.147    0.000    0.906    0.906
     gotBu3_o|t2      1.331    0.034   39.319    0.000    1.331    1.331
```

A three-factor categorical CFA model ...

```
    gotBu3_o|t3      1.546    0.038   40.387    0.000    1.546    1.546
    gotBu3_o|t4      1.802    0.046   39.535    0.000    1.802    1.802
    gotBu4_o|t1      0.304    0.025   12.371    0.000    0.304    0.304
    gotBu4_o|t2      0.917    0.028   32.422    0.000    0.917    0.917
    gotBu4_o|t3      1.204    0.032   37.874    0.000    1.204    1.204
    gotBu4_o|t4      1.459    0.036   40.162    0.000    1.459    1.459
    gotBu5_o|t1      1.012    0.029   34.560    0.000    1.012    1.012
    gotBu5_o|t2      1.375    0.035   39.679    0.000    1.375    1.375
    gotBu5_o|t3      1.555    0.038   40.396    0.000    1.555    1.555
    gotBu5_o|t4      1.831    0.047   39.305    0.000    1.831    1.831
    gotBu6_o|t1      1.251    0.033   38.486    0.000    1.251    1.251
    gotBu6_o|t2      1.580    0.039   40.402    0.000    1.580    1.580
    gotBu6_o|t3      1.788    0.045   39.637    0.000    1.788    1.788
    gotBu6_o|t4      2.029    0.055   37.133    0.000    2.029    2.029
    gotBu7_o|t1      0.649    0.026   24.823    0.000    0.649    0.649
    gotBu7_o|t2      1.077    0.030   35.826    0.000    1.077    1.077
    gotBu7_o|t3      1.291    0.033   38.933    0.000    1.291    1.291
    gotBu7_o|t4      1.564    0.039   40.401    0.000    1.564    1.564
    gotBu8_o|t1      1.361    0.034   39.571    0.000    1.361    1.361
    gotBu8_o|t2      1.695    0.042   40.157    0.000    1.695    1.695
    gotBu8_o|t3      1.889    0.049   38.776    0.000    1.889    1.889
    gotBu8_o|t4      2.083    0.057   36.372    0.000    2.083    2.083
    gotBu9_o|t1      1.492    0.037   40.281    0.000    1.492    1.492
    gotBu9_o|t2      1.744    0.044   39.918    0.000    1.744    1.744
    gotBu9_o|t3      1.889    0.049   38.776    0.000    1.889    1.889
    gotBu9_o|t4      2.075    0.057   36.490    0.000    2.075    2.075
```

KU

A three-factor categorical CFA model ...

```
    depre1_o|t1      -0.553    0.026  -21.599    0.000   -0.553   -0.553
    depre1_o|t2       0.163    0.024    6.713    0.000    0.163    0.163
    depre1_o|t3       1.095    0.030   36.162    0.000    1.095    1.095
    depre1_o|t4       1.812    0.046   39.463    0.000    1.812    1.812
    depre2_o|t1      -0.906    0.028  -32.147    0.000   -0.906   -0.906
    depre2_o|t2      -0.175    0.024   -7.176    0.000   -0.175   -0.175
    depre2_o|t3       0.749    0.027   27.917    0.000    0.749    0.749
    depre2_o|t4       1.533    0.038   40.372    0.000    1.533    1.533
    depre3_o|t1       0.293    0.025   11.910    0.000    0.293    0.293
    depre3_o|t2       0.687    0.026   26.046    0.000    0.687    0.687
    depre3_o|t3       1.168    0.031   37.356    0.000    1.168    1.168
    depre3_o|t4       1.642    0.041   40.325    0.000    1.642    1.642
    depre4_o|t1      -0.152    0.024   -6.251    0.000   -0.152   -0.152
    depre4_o|t2       0.247    0.024   10.103    0.000    0.247    0.247
    depre4_o|t3       0.897    0.028   31.940    0.000    0.897    0.897
    depre4_o|t4       1.478    0.037   40.236    0.000    1.478    1.478
    depre5_o|t1      -0.410    0.025  -16.432    0.000   -0.410   -0.410
    depre5_o|t2       0.053    0.024    2.200    0.028    0.053    0.053
    depre5_o|t3       0.656    0.026   25.046    0.000    0.656    0.656
    depre5_o|t4       1.237    0.032   38.313    0.000    1.237    1.237
    depre6_o|t1      -0.454    0.025  -18.073    0.000   -0.454   -0.454
    depre6_o|t2       0.092    0.024    3.782    0.000    0.092    0.092
    depre6_o|t3       0.733    0.027   27.443    0.000    0.733    0.733
    depre6_o|t4       1.261    0.033   38.607    0.000    1.261    1.261
    alc1_o|t1         1.060    0.030   35.515    0.000    1.060    1.060
    alc1_o|t2         1.836    0.047   39.263    0.000    1.836    1.836
```

A three-factor categorical CFA model ...

```
    alc1_o|t3         2.193    0.063   34.644    0.000    2.193    2.193
    alc1_o|t4         2.435    0.081   30.196    0.000    2.435    2.435
    alc2_o|t1         0.927    0.028   32.661    0.000    0.927    0.927
    alc2_o|t2         1.816    0.046   39.425    0.000    1.816    1.816
    alc2_o|t3         2.117    0.059   35.865    0.000    2.117    2.117
    alc2_o|t4         2.473    0.084   29.444    0.000    2.473    2.473
    alc3_o|t1         1.317    0.034   39.194    0.000    1.317    1.317
    alc3_o|t2         1.816    0.046   39.425    0.000    1.816    1.816
    alc3_o|t3         2.075    0.057   36.490    0.000    2.075    2.075
    alc3_o|t4         2.435    0.081   30.196    0.000    2.435    2.435
    alc4_o|t1         0.983    0.029   33.938    0.000    0.983    0.983
    alc4_o|t2         1.604    0.040   40.388    0.000    1.604    1.604
    alc4_o|t3         2.000    0.053   37.506    0.000    2.000    2.000
    alc4_o|t4         2.353    0.074   31.779    0.000    2.353    2.353
    alc5_o|t1         1.067    0.030   35.640    0.000    1.067    1.067
    alc5_o|t2         1.719    0.043   40.050    0.000    1.719    1.719
    alc5_o|t3         2.036    0.055   37.033    0.000    2.036    2.036
    alc5_o|t4         2.353    0.074   31.779    0.000    2.353    2.353

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    gotBully          1.000                               1.000    1.000
    depress           1.000                               1.000    1.000
    alcohol           1.000                               1.000    1.000
   .gotBu1_o          0.435                               0.435    0.435
   .gotBu2_o          0.391                               0.391    0.391
```

A three-factor categorical CFA model ...

```
    .gotBu3_o          0.395                                       0.395    0.395
    .gotBu4_o          0.348                                       0.348    0.348
    .gotBu5_o          0.295                                       0.295    0.295
    .gotBu6_o          0.249                                       0.249    0.249
    .gotBu7_o          0.407                                       0.407    0.407
    .gotBu8_o          0.259                                       0.259    0.259
    .gotBu9_o          0.206                                       0.206    0.206
    .depre1_o          0.521                                       0.521    0.521
    .depre2_o          0.591                                       0.591    0.591
    .depre3_o          0.432                                       0.432    0.432
    .depre4_o          0.495                                       0.495    0.495
    .depre5_o          0.643                                       0.643    0.643
    .depre6_o          0.606                                       0.606    0.606
    .alc1_o            0.262                                       0.262    0.262
    .alc2_o            0.401                                       0.401    0.401
    .alc3_o            0.095                                       0.095    0.095
    .alc4_o            0.221                                       0.221    0.221
    .alc5_o            0.178                                       0.178    0.178

Scales y*:
                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
     gotBu1_o          1.000                                       1.000    1.000
     gotBu2_o          1.000                                       1.000    1.000
     gotBu3_o          1.000                                       1.000    1.000
     gotBu4_o          1.000                                       1.000    1.000
     gotBu5_o          1.000                                       1.000    1.000
```

A three-factor categorical CFA model ...

```
    gotBu6_o        1.000                                    1.000     1.000
    gotBu7_o        1.000                                    1.000     1.000
    gotBu8_o        1.000                                    1.000     1.000
    gotBu9_o        1.000                                    1.000     1.000
    depre1_o        1.000                                    1.000     1.000
    depre2_o        1.000                                    1.000     1.000
    depre3_o        1.000                                    1.000     1.000
    depre4_o        1.000                                    1.000     1.000
    depre5_o        1.000                                    1.000     1.000
    depre6_o        1.000                                    1.000     1.000
    alc1_o          1.000                                    1.000     1.000
    alc2_o          1.000                                    1.000     1.000
    alc3_o          1.000                                    1.000     1.000
    alc4_o          1.000                                    1.000     1.000
    alc5_o          1.000                                    1.000     1.000
```

Outline

Outline

The Chi Square Calculation/Correction

References

Agresti, A. (2007). *An introduction to categorical data analysis* (2nd ed.). Hoboken, NJ: John Wiley and Sons.

Allison, P. D. (2003). Missing data techniques for structural equation modeling. *Journal of Abnormal Psychology*, *112*, 54. doi:10.1037/0021-843x.112.4.545

Arbuckle, J. L. (1996). Advanced structural equation modeling: Issues and techniques. In G. A. Marcoulides & R. E. Schumacker (Eds.), (p. 243-277). Mahwah, NJ: Lawrence Erlbaum. doi:10.4324/9781315827414

Beauducel, A., & Herzberg, P. Y. (2006). On the performance of maximum likelihood versus means and variance adjusted weighted least squares estimation in CFA. *Structural Equation Modeling*, *13*, 186-203. doi:10.1207/s15328007sem1302_2

References ...

Bollen, K. A., & Barb, K. H. (1981). Pearson's R and coarsely categorized measures. *American Sociological Review*, *46*, 232-239. doi:10.2307/2094981

Correll, J., Park, B., Judd, C. M., & Wittenbrink, B. (2002). The police officer's dilemma: Using ethnicity to disambiguate potentially threatening individuals. *Journal of Personality and Social Psychology: Attitudes and Social Cognition*, *83*, 1314-1329. doi:10.1037//0022-3514.83.6.1314

DiStefano, C., & Morgan, G. B. (2014). A comparison of diagonal weighted least squares robust estimation techniques for ordinal data. *Structural Equation Modeling*, *21*, 425-438. doi:10.1080/10705511.2014.915373

Flora, D. B., & Curran, P. J. (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. *Psychological Methods*, *9*, 466-491. doi:10.1037/1082-989x.9.4.466

References ...

Göllner, R., Roberts, B. W., Damian, R. I., Lüdtke, O., Jonkmann, K., & Trautwein, U. (2017). Whose "strom and stress" is it? Parent and child reports of personality development in the transition to early adolescence. *Journal of Personalily*, *85*, 376-387. doi:10.1111/jopy.12246

Kline, R. B. (2016). *Principles and practice of structural equation modeling (4th ed.)*. New York: Guilford.

Li, C. -H. (2016). Confirmatory factor analysis with ordinal data: Comparing robust maximum likelihood and diagonally weighted least squares. *Behavior Research Methods*, *48*, 936-949. doi:10.3758/s13428-015-0619-7

McArdle, J. J., & Nesselroade, J. R. (2014). *Longitudinal data analysis using structural equation models*. Washington, DC: American Psychological Association.

References ...

Muthén, B., & Kaplan, D. (1985). A comparison of some methodologies for the factor analysis of non-normal likert variables. *British Journal of Mathematical and Statistical Psycholgoy*, *38*, 171-189. doi:10.1111/j.2044-8317.1985.tb00832.x

O'Brien, R. M. (1985). The relationship between ordinal measures and their underlying values: Why all the disagreement? *Quality and Quantity*, *19*, 265-277. doi:10.1007/bf00170998

Soto, C. J., & John, O. P. (2017a). The next Big Five Inventory (BFI-2): Developing and assessing a hierarchical model with 15 facets to enhence bandwidth, fidelity, and predictive power. *Journal of Personality and Social Psychology*, *113*, 117-143. doi:10.1037/pspp0000096

Soto, C. J., & John, O. P. (2017b). Short and extra-short forms of the Big Five Inventory-2: The BFI-2-S and BFI-2-XS. *Journal of Research in Personality*, *68*, 69-81. doi:10.1016/j.jrp.2017.02.004

References ...

van der Palm, D. W., van der Ark, L. A., & Vermunt, J. K. (2016). A comparison of incomplete-data methods for categorical data. *Statistical Methods in Medical Research*, *25*, 754-774. doi:10.1177/0962280212465502

Yuan, K. -H., & Bentler, P. M. (2000). Three likelihood-based methods for mean and covariance structure analysis with nonnormal missing data. *Sociological Methodology*, *30*, 165-200. doi:10.1111/0081-1750.00078

Session

```
sessionInfo()
```

```
R version 3.4.4 (2018-03-15)
Platform: x86_64-apple-darwin15.6.0 (64-bit)
Running under: macOS High Sierra 10.13.4

Matrix products: default
BLAS:
    /Library/Frameworks/R.framework/Versions/3.4/Resources/lib/libRblas.0.dyli
LAPACK:
    /Library/Frameworks/R.framework/Versions/3.4/Resources/lib/libRlapack.dyli

locale:
[1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8

attached base packages:
[1] stats     graphics  grDevices utils     datasets  base

other attached packages:
[1] plyr_1.8.4          lavaan_0.5-23.1097 foreign_0.8-70
    kutils_1.40         crmda_0.70
[6] stationery_0.79

loaded via a namespace (and not attached):
```

Session ...

```
 [1] Rcpp_0.12.16    quadprog_1.5-5  rprojroot_1.3-2 digest_0.6.15
     backports_1.1.2 xtable_1.8-2
 [7] magrittr_1.5    stats4_3.4.4    evaluate_0.10.1 stringi_1.2.2
     pbivnorm_0.6.0  openxlsx_4.0.17
[13] rmarkdown_1.9   tools_3.4.4     stringr_1.3.0   compiler_3.4.4
    mnormt_1.5-5    htmltools_0.3.6
[19] knitr_1.20      methods_3.4.4
```