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A Simulation Study Comparing Recent Approaches for the Estimation of Nonlinear Effects in SEM Under the Condition of Nonnormality

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In the past decade new approaches for the estimation of latent nonlinear interaction and quadratic effects in structural equation modeling have been proposed (Kelava & Brandt, 2009; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Marsh, Wen, & Hau, 2004; Mooijaart & Bentler, 2010; Wall & Amemiya, 2003). Most approaches have been developed for the analysis of normally distributed latent predictor variables. In this article, we investigate the performance of five recent approaches under the condition of nonnormally distributed data: the extended unconstrained approach (Kelava & Brandt, 2009), LMS (Klein & Moosbrugger, 2000), QML (Klein & Muthén, 2007), the 2SMM approach (Wall & Amemiya, 2003), and the method of moments approach by Mooijaart and Bentler (2010). Advantages and limitations of the approaches are discussed.

Keywords: estimators, interaction, nonlinear structural equation models, nonnormality, quadratic

Numerous theories have claimed an estimation of nonlinear effects (e.g., Ajzen, 1987; Cronbach & Snow, 1977; Karasek, 1979; Lusch & Brown, 1996; Snyder & Tanke, 1976). Within a latent variable framework, a structural equation that models one interaction and two quadratic effects (omitting a subject index) is given by

$$\eta = \alpha + \gamma_1\xi_1 + \gamma_2\xi_2 + \omega_{12}\xi_1\xi_2 + \omega_{11}\xi_1^2 + \omega_{22}\xi_2^2 + \zeta \quad (1)$$

where η is the latent criterion, α is the latent intercept, the γs are the linear effect parameters, the ωs are the nonlinear effect parameters for the latent predictors ξ_1 and ξ_2, and ζ is a latent disturbance term. Figure 1 shows a structural equation model with one latent interaction and two quadratic effects.

The analysis of interaction models is a common procedure in the social sciences (e.g., Aiken & West, 1991; Cohen, 1988). By including interaction effects, a model can account for differential effects occurring in hypotheses where the relationship between two variables might depend on a third variable; for example, where the relationship between intention and behavior might depend on the behavioral control that can be exerted by an individual (Ajzen, 1987). Quadratic effects model curvilinear associations between two variables that also occur in applied research, for instance, the quadratic relationship between arousal and performance (Yerkes & Dodson, 1908). In recent years, it has been proposed as a standard procedure to include quadratic effects when testing interaction effects to avoid spurious interactions (Ganzach, 1997; Klein, Schermelleh-Engel, Moosbrugger, & Kelava, 2009; MacCallum & Mar, 1995). Following this line of reasoning, it is recommended to use a baseline model with linear and quadratic effects when testing interactions, instead of a purely linear model (Klein et al., 2009). This suggests that estimation methods for analyzing interactions should be designed in a way that they can estimate interaction and quadratic effects simultaneously.

Several approaches for the analysis of nonlinear structural equation modeling (SEM) have been published within

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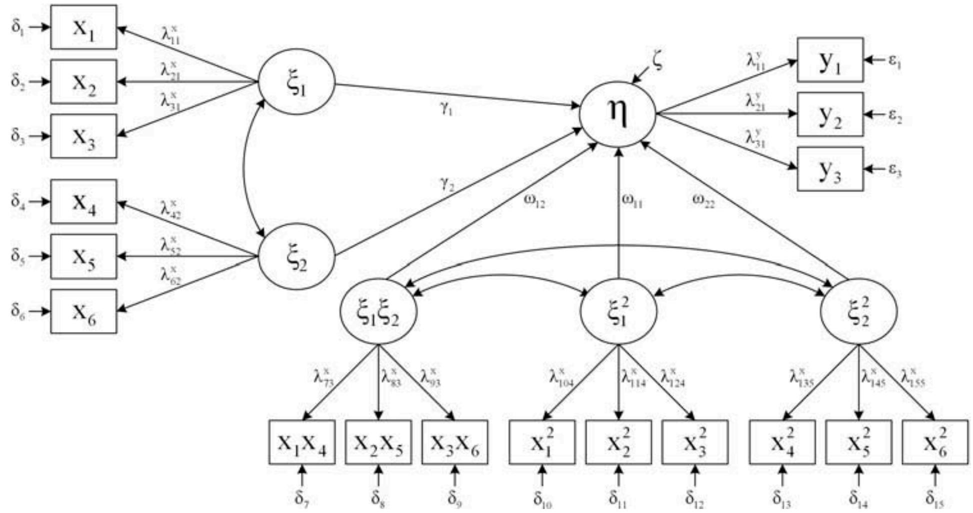

FIGURE 1 Nonlinear structural equation model with three latent nonlinear effects: one interaction effect and two quadratic effects. Note that the product indicators (e.g., x_1x_4, x_1^2, x_4^2, etc.) are only needed for product indicator approaches to implement a measurement model for the latent product terms ($\xi_1\xi_2, \xi_1^2$, and ξ_2^2).

the class of product indicator approaches (e.g., Bollen, 1995; Jaccard & Wan, 1995; Jöreskog & Yang, 1996; Kelava & Brandt, 2009; Little, Bovaird, & Widamam, 2006; Marsh et al., 2004; Marsh, Wen, & Hau, 2006; Ping, 1995, 1996; Wall & Amemiya, 2001). Within this class, products of indicators have been used to implement measurement models for latent product variables (e.g., $\xi_1\xi_2, \xi_1^2$). Multivariate normality for all indicators including the product indicators is assumed if these approaches are estimated using a maximum likelihood (ML) estimator (cf. Kelava et al., 2011; Schumacker & Marcoulides, 1998). Other approaches do not use product indicators, but also assume normality for all or some latent exogenous variables. For example, distribution analytic approaches (Klein & Moosbrugger, 2000; Klein & Muthén, 2007) assume normality for both latent predictors and residual variables (e.g., ξ_1, ξ_2, and $\delta_1, \ldots$), whereas the method of moments approach by Mooijaart and Bentler (2010) assumes normality only for predictors (e.g., ξ_1, ξ_2), and Wall and Amemiya's (2003) 2SMM approach assumes normality only for residual variables (e.g., $\delta_1, \ldots$).

In simulation studies, in particular, the product indicator approaches have been examined for normally distributed variables and, in some cases, for skewed latent predictors regarding their estimation properties (e.g., Lee, Song, & Poon, 2004; Marsh et al., 2004; Moulder & Algina, 2002; Schermelleh-Engel, Klein, & Moosbrugger, 1998). Other approaches have not been examined in close detail or only under very specific conditions of nonnormality (Klein & Muthén, 2007; Wall & Amemiya, 2003). Thus, most simulation studies have primarily been informative concerning the efficiency and unbiasedness of estimates in situations when distributional assumptions are met.

However, in the social and behavioral sciences many data sets can include nonnormally distributed (skewed) variables (Micceri, 1989). Different estimation methods might respond differently when nonnormally distributed variables are analyzed. Therefore, their estimation properties need to be examined when distributional assumptions are violated.

Until now, there have been only two important articles (Marsh et al., 2004; Moulder & Algina, 2002) that compare estimation methods for the analysis of latent interactions. Both only consider models without quadratic effects in the baseline model. Thus, we see a need for an updated comparison of current approaches for both interaction and quadratic effects when variables are nonnormally distributed. In this article, we compare five selected contemporary approaches that have been of interest in recent research: the extended unconstrained approach (ExUC; Kelava & Brandt, 2009), the latent moderated structural equations approach (LMS; Klein & Moosbrugger, 2000), the quasi-maximum likelihood approach (QML; Klein & Muthén, 2007), the two-stage method of moments approach (2SMM; Wall & Amemiya, 2000, 2003), and the method of moments approach by Mooijaart and Bentler (2010). This comparison includes empirically relevant settings for normally and nonnormally distributed predictors, and a simultaneous estimation of multiple nonlinear effects.

The article is structured as follows: In the next section, we present different contemporary approaches for the estimation of latent nonlinear interaction and quadratic effects in structural equation models. We report on the status quo of their performance in the face of normally and nonnormally distributed variables, and we discuss their theoretical strengths and weaknesses. We then report on a simulation study that compares these approaches in empirically common settings with nonnormally distributed data when nonlinear models with interaction and quadratic effects are estimated. In the last section, we discuss the results of the simulation study, and we give guidelines for the application of the approaches.

APPROACHES FOR THE ESTIMATION OF LATENT NONLINEAR EFFECTS

In the past 10 years some approaches for the estimation of latent interaction and quadratic effects have received attention (Jöreskog & Yang, 1996; Kelava & Brandt, 2009; Kelava et al., 2011; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Little et al., 2006; Marsh et al., 2004; 2006; Mooijaart & Bentler, 2010; Moulder & Algina, 2002; Schumacker & Marcoulides, 1998; Wall & Amemiya, 2000, 2003, among others). Most of them can be divided into three major classes: (a) product indicator approaches, (b) distribution analytic approaches, and (c) method of moments approaches. In the following, we briefly present the main approaches of these classes and their underlying assumptions. Then, we summarize the results of simulation studies that have been conducted up to the present.

Product Indicator Approaches

The largest class of approaches, the class of product indicator approaches, relates to Kenny and Judd's (1984) idea to use product indicators to identify interaction effects. Kenny and Judd proposed to specify a measurement model for the latent nonlinear product variables (e.g., for $\xi_1\xi_2$). Products of the indicators of the linear measurement model (e.g., x_1x_4; see Figure 1) are used as indicators for the nonlinear latent product variables. Whereas Kenny and Judd (1984) proposed to constrain the parameters for the nonlinear measurement model part of the structural equation model, Marsh et al. (2004, 2006) showed with their unconstrained approach that some constraints can be relaxed, and thereby, estimation properties can be improved.

With the ExUC approach, Kelava and Brandt (2009) presented an extension of the unconstrained approach to models with quadratic and interaction effects. They showed that the inclusion of multiple product indicators of the same indicator (e.g., x_2^2 and x_1x_2) entails residual covariances between product indicators that have to be taken into account to avoid spurious nonlinear effects (Kelava & Brandt, 2009; Kelava, Moosbrugger, Dimitruk, & Schermelleh-Engel, 2008).

For the constrained approach, normality of latent (linear) predictor variables (e.g., ξ_1, ξ_2) and residual variables (e.g., $\delta_1, \ldots, \varepsilon_1, \ldots, \zeta$; see Figure 1) is assumed for the specification of parameter constraints (Kenny & Judd, 1984; Wall & Amemiya, 2001). The relaxation of constraints in the unconstrained approach results in more robust estimates (Marsh et al., 2004, 2006). Still, for the unconstrained approach and its extended version, the ExUC approach, the normality assumption is used for assuming covariances to be zero between the measurement model of the (linear) predictors and the measurement model of the product terms (for a relaxation of this assumption and its potential drawbacks, see Kelava & Brandt, 2009).

In addition to the normality assumption for the constraint specification, multivariate normality is implicitly assumed for both the (linear) indicators and the product indicators when using the standard ML estimation algorithm. This assumption is always violated because even if the indicators themselves are normally distributed, their products are nonnormal in general (Aroian, 1944). As a consequence, standard errors and fit indices might be severely biased, even if the indicators of the linear measurement model are normally distributed (for general consequences using the ML estimator for nonnormally distributed variables in linear models see Bollen, 1989; Boomsma, 1983; West, Finch, & Curran, 1995). If linear indicators are nonnormal as well, additional estimation bias—also for parameter estimates—might be induced due to the fact that the constraints concerning the covariances between the measurement models of the linear predictors and the product terms are invalid.

Recently, more robust estimators for standard errors (Satorra & Bentler, 1994; White, 1982; Yuan & Bentler, 2000) have become available in standard software packages, for instance, the sandwich estimator in M*plus* (Muthén & Muthén, 1998–2010) or in the R-package lavaan (R Development Core Team, 2011). The sandwich estimator approximates the variance of the parameter estimates using the observed Fisher information instead of the expected Fisher information. It is a quasi ML estimator that leads to consistent results even if distributional assumptions are violated (White, 1982) and thus should improve standard error estimation for nonnormally distributed data (Yuan & Bentler, 2000). It has not yet been investigated if the sandwich estimator might enhance the standard error estimation for the unconstrained or the ExUC approach.

Distribution Analytic Approaches

Within the class of distribution analytic approaches, mainly two methods have drawn attention (cf. Kelava et al., 2011): the LMS approach (Klein & Moosbrugger, 2000) and the QML approach (Klein & Muthén 2007). For LMS a likelihood function for a nonnormal distribution is derived, which is approximated by numerical methods, and maximized using the expectation maximization algorithm (EM; Dempster, Laird & Rubin, 1977). QML is a quasi ML estimation procedure. It approximates the multivariate nonnormal distribution of the indicator variables by a product of a conditionally normal and an unconditionally normal distribution. Neither of the two approaches uses product indicators.

LMS and QML both assume multivariate normality for all latent exogenous variables, that is, for the latent predictors (e.g., ξ_1, ξ_2) and residual variables (e.g., $\delta_1, \ldots, \varepsilon_1, \ldots, \zeta$).

The advantage of QML over LMS is that QML is theoretically more robust than LMS with respect to nonnormal data and that the computational burden is lower. The possible advantage of LMS over QML lies in the fact that LMS produces true ML estimates that are theoretically more efficient when the underlying assumptions are met (Kelava et al., 2011).

Method of Moments Approaches

Within the class of method of moments based approaches, two methods have addressed the issue of estimating nonlinear effects. The 2SMM approach by Wall and Amemiya (2000, 2003) and the method of moments (MM) approach by Mooijaart and Bentler (2010). Both approaches do not use product indicators and they relax the assumption of multivariate normality for all variables.

The 2SMM approach by Wall and Amemiya (2003) is a two-stage procedure. In a first step, the measurement model part is estimated using standard SEM procedures. Bartlett factor scores and their variances are obtained. In a second step, the factor scores and variances are used to estimate the parameters of the structural model. The method is based on an errors-in-variable regression that takes into account that the factor scores obtained in the first step are estimates and therefore have a variance that has to be modeled in the second step. A method of moment estimator is applied to estimate the structural model parameters. The 2SMM approach has been specified for models including either a single interaction or a single quadratic effect (for a general polynomial model, see Wall & Amemiya, 2000) and has been implemented for those models in the SAS software (Wall & Amemiya, 2003).

The relevant distributional assumption for the second step, the parameter estimation in the structural model, is the normality of residual variables (e.g., $\delta_1, \ldots, \varepsilon_1, \ldots$) which is needed for the specification of the estimating equations.[1] This assumption is also used for the derivation of the standard errors. When an ML estimator is used for the Bartlett factor scores, multivariate normality for all indicators is implicitly assumed in the first step. If indicators are nonnormal, it can be expected that the factor scores are unbiased, because they only depend on the parameter estimates of the measurement model (see earlier; Bollen, 1989; Boomsma, 1983; West et al., 1995). If indicator variables are nonnormal, however, the variances of the parameter estimates for the measurement models might be underestimated, and as a consequence, standard errors of the structural model could be underestimated, too.

Recently, Mooijaart and Bentler (2010) proposed a moment-based approach that takes higher order moments of the variables into account. In their approach, the assumption of normally distributed measurement error variables is relaxed. The measurement error variables can be nonnormally distributed. By applying an MM estimator, the discrepancy of observed and model-implied higher order moments is minimized to fit the nonlinear model. The MM approach assumes normality of the latent predictor variables (e.g., ξ_1, ξ_2) for the consistency of its estimates. The nonnormality due to the interaction effect is taken into account for the calculation of the higher order moments.

The MM approach might show two potential drawbacks regarding its estimation properties. First, the estimation of higher order moments might be unstable in small samples. Second, if additional nonnormality is present in the data that is not due to nonlinear effects but to nonnormally distributed constructs, higher order moments could contain two sources of nonnormality and thus induce biased estimates.

Comparison of the Approaches

In the previous subsections we presented different approaches for the estimation of nonlinear effects. Table 1 summarizes the properties of these approaches.

All approaches make distributional assumptions, but it is not straightforward to evaluate their robustness regarding parameter and standard error estimation in situations when these assumptions are violated. Especially the estimation properties for finite sample sizes have to be examined. Simulation studies have been published primarily for models with single interaction effects (Marsh et al., 2004, 2006; Moulder & Algina, 2002), although in empirical settings the full nonlinear model is needed to avoid spurious interaction effects (Klein et al., 2009). Only sparse results are available for models with more than one nonlinear effect (Kelava et al., 2008; Kelava et al., 2011; Klein & Muthén, 2007; Moosbrugger, Dimitruk Schermelleh-Engel, Kelava, & Klein, 2009). Here, we briefly summarize and compare the results of previous simulation studies.

For models with a single interaction effect and normally distributed variables, LMS and QML have been shown to provide unbiased estimates and to be more efficient than the product indicator approaches (Klein & Muthén 2007; Marsh et al., 2004). For nonnormally distributed latent predictors, the unconstrained approach has provided rather unbiased parameter estimates, but has underestimated the standard errors (Marsh et al., 2004). The originally found bias of the standard error estimation for QML (with a standard error estimation using the expected Fisher information) under the condition of nonnormal predictors (Marsh, et al., 2004) could be removed in a later QML version (using the observed Fisher information for standard error estimation; Klein &

[1] Wall and Amemiya (2003) used a set of equations including higher order moments. These higher order moments can either be estimated or assumed to be zero if normality of the residuals is assumed.

TABLE 1
Summary of the Properties of the Five Different Approaches

Approach	*Distributional Assumptions*	*Situations in Which Assumptions Are Violated*	*Consequences*
ExUC approach (and unconstrained approach)	Multivariate normally distributed indicators and product indicators (equivalent to multivariate normally distributed errors and latent variables)	Never fulfilled due to product indicators; more severely violated if indicators are nonnormal	Slightly biased parameter estimates, underestimated standard errors (SE); if predictors are nonnormal, bias of the SEs might increase
LMS	Normally distributed exogenous variables (latent predictors and residuals)	Not violated per se; only if indicators are nonnormal	Parameter estimates might be biased if assumptions are violated
QML	Normally distributed exogenous variables (latent predictors and residuals); (approximately) conditionally normal latent criterion (given the *x* variables)	Not violated per se; only if indicators are nonnormal	Parameter estimates might be less biased than for LMS if assumptions are violated
2SMM approach	Normally distributed residual variables	Violated if residual variables are nonnormal	Should be robust against nonnormality of indicators, especially if indicators have a high reliability
MM approach	Normally distributed latent predictors	Violated if predictors are nonnormal	Should not be robust to additional nonlinear effects; biased estimates if predictors are nonnormal

Note. ExUC = extended unconstrained approach; LMS = latent moterated structural equations approach; QML = quasi-maximum likelihood approach; 2SMM = two-stage method of moments; MM = method of moments approach.

Muthén 2007). Wall and Amemiya (2000, 2003) showed in two simulation studies that the 2SMM approach is (asymptotically) unbiased analyzing either a single interaction or a single quadratic effect in the presence of nonnormally distributed data. In their main article, Mooijaart and Bentler (2010) showed that the MM approach for an interaction model leads to results comparable to LMS under the condition of normally distributed variables. There are no systematic simulation studies comparing the efficiency of the moment-based approaches to either distribution analytic or product indicator approaches.

For models containing interaction and quadratic effects, it was shown that LMS and QML produced very similar results when variables were normally distributed (Kelava et al., 2011). A comparison of the distribution analytic approaches with the ExUC approach could show that LMS and QML were more efficient than the ExUC approach under the condition of normally distributed latent predictors (Kelava et al., 2011; Moosbrugger et al., 2009). A comparison of the ExUC approach, LMS, and QML under the condition of nonnormally distributed variables has not yet been conducted. Additionally, no simulation studies for the moment-based approaches for models containing more than one nonlinear effect have been conducted.

Hence, the applied researcher is confronted with several approaches for the estimation of nonlinear SEM that are difficult to compare regarding their potential strengths and weaknesses. This lack of comparability is at least partly due to the fact that some of the approaches are not feasible and readily available, and therefore, simulation studies comparing these approaches have not been conducted. With this article, we are trying to fill this gap.

SIMULATION STUDY

In this section, we examine the properties of the approaches already presented with a simulation study, in which the degree of nonnormality of the latent predictors is varied. We selected the MM approach by Mooijaart and Bentler (2010) and the 2SMM approach by Wall and Amemiya (2003) for the class of method of moments-based approaches, LMS (Klein & Moosbrugger, 2000) and QML (Klein & Muthén, 2007) for the class of distribution analytic approaches, and the ExUC approach (Kelava & Brandt, 2009) for the class of product indicator approaches. We examine the relative bias of parameter and standard error estimates in the structural model as well as the Type I error rates and power for population models with and without nonlinear effects. Special emphasis is placed on the inspection of the bias of the parameters.

Design of the Simulation Study

Data were generated according to three structural models. We restricted our simulation study to models with symmetric effects of the predictors. The first model for data generation,

M_{lin}, included no nonlinear effects and was used to examine the Type I error rate:

$$\eta = .316\xi_1 + .316\xi_2 + \zeta. \tag{2}$$

The second model, M_{int}, included one interaction effect:

$$\eta = -.255 + .316\xi_1 + .316\xi_2 + .139\xi_1\xi_2 + \zeta. \tag{3}$$

The third model, M_{full}, additionally included two quadratic effects, such that each nonlinear effect was accounting for approximately 2.2% of the latent criterion's variance, which is a realistic size of a nonlinear effect in the social sciences (see Chaplin, 1991; 2007):

$$\begin{aligned}\eta =& -.255 + .316\xi_1 + .316\xi_2 + .139\xi_1\xi_2 + .101\xi_1^2 \\ &+ .101\xi_2^2 + \zeta.\end{aligned} \tag{4}$$

In all three population models, M_{lin}, M_{int}, and M_{full}, the bivariate distribution of the latent predictors ξ_1 and ξ_2 was specified with means and variances $E(\xi_1) = E(\xi_2) = 0$, $Var(\xi_1) = Var(\xi_2) = 1$, and $Cov(\xi_1, \xi_2) = .375$. The disturbance variable ζ was normally distributed with a variance specified such that $Var(\eta) = 1$ under the condition of normally distributed predictors.

Two conditions were selected for the latent predictors' distribution (in line with the values used by Curran, West, & Finch, 1996): (a) normality with skewness 0 and kurtosis 0, and (b) nonnormality with skewness 2 and kurtosis 7. Nonnormality of the latent predictors was induced using the Fleishman (1978) transformation implemented in EQS (Bentler, 1995).

For each of the predictor variables ξ_1, ξ_2 and for the latent outcome variable η three indicator variables were specified. The reliability of the indicators was set to .80. The indicators were generated as unidimensional measures with normally distributed residual variables. All observed deviations of the indicator variables' distribution from normality were due to the nonnormal latent predictor variables and the latent nonlinear effects.

The sample size was set to $N = 400$. Raw data for a total of 500 replications were generated and analyzed for each of the resulting conditions. A solution was considered proper when there were no negative estimates for the variances or standard errors. Outliers were identified by analyzing box plots and z-scores (cf. Paxton, Curran, Bollen, Kirby, & Chen, 2001).

For the analysis two types of models were specified: an interaction model A_{int} (for the analysis of M_{lin} and M_{int}) and a full model with interaction and quadratic effects A_{full} (for the analysis of models M_{lin} and M_{full}).

Approaches and Software Implementations

In the simulation study, we compared five contemporary approaches for the estimation of latent nonlinear effects. We implemented the MM approach (Mooijaart & Bentler, 2010) in MATLAB (MATLAB, 2010). The 2SMM approach (Wall & Amemiya, 2003) and the ExUC approach[2] (Kelava, 2008; Kelava & Brandt, 2009) were implemented in R (R Development Core Team, 2011). For LMS (Klein & Moosbrugger, 2000) we used the implementation in M*plus* v6.11 (Muthén & Muthén 1998–2010), for QML (Klein & Muthén, 2007) we used a stand-alone software (QML v3.11) that can be obtained from the original author. Syntax for LMS and QML can be found in Kelava et al., (2011). Syntax for the ExUC approach can be found in Kelava and Brandt (2009). Syntax for the 2SMM and the MM approach can be obtained from the authors.

To conduct a fair comparison among the five approaches, the standard error estimates were based on a robust estimation using the observed Fisher information (sandwich estimator; White, 1982; Yuan & Bentler, 2000). The sandwich estimator is already implemented for QML in QML v3.11 as well as for LMS in M*plus*; for the ExUC and the 2SMM approach the package lavaan in R was used (ML estimation with observed Fisher information). Beyond this, default settings (e.g., default start values, default number of iterations, and default numerical algorithms) supplied by the programs were used.

Results of the Simulation Study

In the first subsection, we present the results that were obtained when an interaction model was specified as analysis model (A_{int}). In the second subsection, we present the results that were obtained when a model with both interaction and quadratic effects was specified as analysis model (A_{full}). For each analysis model, we show results for Type I error and power conditions for normally and nonnormally distributed latent predictor variables. The percentage of significant t-values for the parameter estimates was calculated for each linear and nonlinear effect of the structural model. These percentages are interpreted as power for detecting the corresponding parameter when the population parameter is different from zero, or as Type I error when the population parameter is equal to zero.

We report the relative bias for the linear and the nonlinear effects. Across models we calculated the ratio SE/SD between the average estimated standard error ($\widehat{\overline{SE}}$) and the standard deviation of the parameter estimates (Monte-Carlo SD). A relative bias of the parameter estimates above 5% was interpreted as slightly biased, and above 10% as biased.

[2]For the interaction model actually the term *unconstrained approach* (Marsh et al., 2004, 2006) would be more adequate, but for simplicity we refer to the approach as ExUC approach throughout the rest of the article, which is the more general approach.

Accordingly, a ratio SE/SD below .90 or above 1.10 was interpreted as biased (cf. Hoogland & Boomsma, 1998).

Analysis Model: Interaction Model (A_{int})

Results for the analysis model A_{int} specified with a single interaction effect are presented in Tables 2 to 5.

Population model M_{lin}. The analysis of the linear population model M_{lin} for normally distributed predictors (Type I error condition; Table 2) showed unbiased parameter estimates across all five approaches. The mean estimates for the interaction parameter lay between .003 (for QML) and .006 (for the MM approach). The MM approach showed somewhat inflated standard error estimates (SE/SD

TABLE 2
Simulation Results for the Linear Population Model (M_{lin}) Under the Condition of Normally Distributed Data (Analysis Model A_{int})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.314	−0.61%	.051	.050	.973	100.00%
γ_2	.316	.317	0.34%	.049	.050	1.010	100.00%
ω_{12}	.000	.006	n. def.	.072	.082	1.150	3.20%
2SMM approach							
γ_1	.316	.315	−0.50%	.055	.052	.946	100.00%
γ_2	.316	.318	0.62%	.052	.051	.992	100.00%
ω_{12}	.000	.004	n. def.	.047	.046	.982	5.40%
LMS							
γ_1	.316	.315	−0.53%	.055	.053	.967	100.00%
γ_2	.316	.318	0.56%	.052	.053	1.013	100.00%
ω_{12}	.000	.004	n. def.	.046	.046	.989	6.00%
QML							
γ_1	.316	.314	−0.78%	.055	.053	.950	100.00%
γ_2	.316	.318	0.60%	.054	.052	.975	100.00%
ω_{12}	.000	.003	n. def.	.046	.046	.996	5.71%
ExUC approach							
γ_1	.316	.315	−0.54%	.055	.053	.966	100.00%
γ_2	.316	.318	0.55%	.052	.052	1.012	100.00%
ω_{12}	.000	.004	n. def.	.047	.046	.978	5.60%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

TABLE 3
Simulation Results for the Linear Population Model (M_{lin}) Under the Condition of Nonnormally Distributed Data (Analysis Model A_{int})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.314	−0.70%	.051	.050	.975	100.00%
γ_2	.316	.318	0.63%	.051	.050	.986	100.00%
ω_{12}	.000	.309	n. def.	.129	.118	.913	77.80%
2SMM approach							
γ_1	.316	.313	−1.13%	.056	.054	.963	99.60%
γ_2	.316	.318	0.48%	.056	.054	.972	100.00%
ω_{12}	.000	.005	n. def.	.043	.039	.909	8.00%
LMS							
γ_1	.316	.308	−2.66%	.056	.055	.978	99.60%
γ_2	.316	.313	−1.13%	.055	.055	.997	100.00%
ω_{12}	.000	.018	n. def.	.043	.039	.914	10.60%
QML							
γ_1	.316	.308	−2.70%	.058	.055	.951	99.59%
γ_2	.316	.313	−0.95%	.057	.055	.965	100.00%
ω_{12}	.000	.017	n. def.	.043	.039	.904	10.52%
ExUC approach							
γ_1	.316	.309	−2.42%	.056	.054	.973	99.60%
γ_2	.316	.314	−0.84%	.055	.055	.989	100.00%
ω_{12}	.000	.017	n. def.	.042	.038	.902	11.40%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

TABLE 4
Simulation Results for the Population Model With Interaction Effect (M_{int}) Under the Condition of Normally Distributed Data (Analysis Model A_{int})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.315	−0.49%	.053	.052	.973	100.00%
γ_2	.316	.318	0.44%	.051	.051	1.010	100.00%
ω_{12}	.139	.145	4.76%	.075	.084	1.125	37.80%
2SMM approach							
γ_1	.316	.315	−0.53%	.052	.048	.924	100.00%
γ_2	.316	.318	0.46%	.049	.048	.985	100.00%
ω_{12}	.139	.143	3.05%	.045	.043	.961	89.80%
LMS							
γ_1	.316	.315	−0.50%	.052	.050	.958	100.00%
γ_2	.316	.318	0.43%	.049	.050	1.019	100.00%
ω_{12}	.139	.143	2.92%	.044	.043	.975	90.00%
QML							
γ_1	.316	.315	−0.47%	.053	.049	.938	100.00%
γ_2	.316	.317	0.39%	.050	.049	.979	100.00%
ω_{12}	.139	.142	2.40%	.045	.043	.963	89.92%
ExUC approach							
γ_1	.316	.315	−0.54%	.052	.049	.950	100.00%
γ_2	.316	.318	0.44%	.049	.049	1.009	100.00%
ω_{12}	.139	.143	2.96%	.046	.044	.970	88.80%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

TABLE 5
Simulation Results for the Population Model With Interaction Effect (M_{int}) Under the Condition of Nonnormally Distributed Data (Analysis Model A_{int})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.363	14.95%	.057	.057	.997	100.00%
γ_2	.316	.368	16.49%	.055	.057	1.039	100.00%
ω_{12}	.139	.516	271.55%	.155	.177	1.146	91.53%
2SMM approach							
γ_1	.316	.313	−1.03%	.054	.051	.940	100.00%
γ_2	.316	.317	0.18%	.052	.051	.977	100.00%
ω_{12}	.139	.145	4.32%	.042	.038	.896	93.40%
LMS							
γ_1	.316	.308	−2.45%	.054	.053	.972	100.00%
γ_2	.316	.312	−1.22%	.052	.053	1.020	100.00%
ω_{12}	.139	.161	15.88%	.041	.038	.924	96.80%
QML							
γ_1	.316	.309	−2.42%	.055	.052	.947	100.00%
γ_2	.316	.312	−1.26%	.053	.052	.990	100.00%
ω_{12}	.139	.160	15.40%	.041	.038	.920	96.58%
ExUC approach							
γ_1	.316	.315	−0.53%	.054	.052	.957	100.00%
γ_2	.316	.318	0.68%	.052	.052	1.000	100.00%
ω_{12}	.139	.154	10.67%	.043	.038	.881	95.80%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

= 1.150). The Type I error rates for the latent interaction parameter were close to the nominal 5% rate (ranging from 3.20% for the MM approach to 6.00% for LMS).

The analysis of the linear population model M_{lin} for nonnormally distributed predictors (Table 3) led to an overestimated average interaction effect parameter (.309) for the MM approach. The average interaction effect parameters for the other four approaches lay between .005 (for the 2SMM approach) and .018 (for LMS). Standard error estimates were fairly unbiased for all approaches with a SE/SD ratio between .902 (for the ExUC approach) and .914 (for LMS). The Type I error rates were inflated for the MM approach (77.80%), for LMS (10.60%), for QML (10.52%), and for the ExUC approach (11.40%). The 2SMM approach showed a Type I error rate of 8.00%.

Population model M_{int}. Analyzing the nonlinear population model M_{int} for normally distributed predictors (power condition; Table 4) showed unbiased parameter estimates for all five approaches. The relative bias lay between 2.40% (for QML) and 4.76% (for the MM approach). The mean standard error was overestimated for the MM approach ($SE/SD = 1.125$) and unbiased for the other four approaches (with SE/SD between .961 for the 2SMM approach and .975 for LMS). The MM approach showed inefficient estimates for the interaction effect with a power of 37.80% in comparison to 89.80% for the 2SMM approach, 90.00% for LMS, 89.92% for QML, and 88.80% for the ExUC approach.

The results for the nonlinear population model M_{int} for nonnormally distributed predictors (Table 5) showed biased parameter estimates for the interaction effect for the MM approach with a relative bias of 271.55% (.516 instead of .139) as well as for LMS (15.88%), QML (15.40%), and the ExUC approach (10.67%). For the the 2SMM approach the interaction effect parameter was unbiased with a relative bias of 4.32%. The average standard errors were biased for the MM, the 2SMM, and the ExUC approach ($SE/SD = 1.146$, .896, and .881, respectively). The average standard error of the interaction effect for the MM approach was larger than for the other approaches (.177 vs. .038 for each of the other approaches). The power for the interaction effect lay between 91.53% (for the MM approach) and 96.80% (for LMS).

Specified Model: Nonlinear Model With Interaction and Quadratic Effects (A_{full})

Results for the analysis model A_{full}, specified with interaction and quadratic effects are presented in Tables 6 to 9.

TABLE 6
Simulation Results for the Linear Population Model (M_{lin}) Under the Condition of Normally Distributed Data (Analysis Model A_{full})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\widehat{\overline{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.314	−0.63%	.055	.053	.971	99.80%
γ_2	.316	.317	0.37%	.053	.053	1.012	100.00%
ω_{12}	.000	−.064	n. def.	.174	.075	.431	36.80%
ω_{11}	.000	.056	n. def.	.207	.025	.120	74.80%
ω_{22}	.000	.056	n. def.	.205	.025	.121	72.00%
2SMM approach							
γ_1	.316	.315	−0.44%	.055	.052	.936	99.80%
γ_2	.316	.318	0.70%	.052	.052	.995	99.80%
ω_{12}	.000	.006	n. def.	.067	.064	.949	6.40%
ω_{11}	.000	−.002	n. def.	.046	.042	.905	6.60%
ω_{22}	.000	−.001	n. def.	.046	.041	.911	7.80%
LMS							
γ_1	.316	.315	−0.53%	.055	.053	.961	99.80%
γ_2	.316	.318	0.61%	.052	.053	1.021	100.00%
ω_{12}	.000	.006	n. def.	.066	.063	.956	6.20%
ω_{11}	.000	−.002	n. def.	.046	.042	.914	6.60%
ω_{22}	.000	−.001	n. def.	.045	.041	.918	7.60%
QML							
γ_1	.316	.314	−0.76%	.056	.053	.934	100.00%
γ_2	.316	.318	0.63%	.052	.052	.999	100.00%
ω_{12}	.000	.005	n. def.	.066	.063	.954	7.04%
ω_{11}	.000	−.002	n. def.	.045	.041	.910	7.45%
ω_{22}	.000	−.001	n. def.	.045	.041	.906	8.28%
ExUC approach							
γ_1	.316	.315	−0.53%	.055	.053	.957	99.80%
γ_2	.316	.318	0.58%	.052	.052	1.015	99.80%
ω_{12}	.000	.007	n. def.	.069	.065	.940	6.80%
ω_{11}	.000	−.002	n. def.	.047	.042	.900	6.80%
ω_{22}	.000	−.001	n. def.	.046	.042	.917	8.20%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

TABLE 7
Simulation Results for the Linear Population Model (M_{lin}) Under the Condition of Nonnormally Distributed Data (Analysis Model A_{full})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.314	−0.70%	.055	.053	.973	100.00%
γ_2	.316	.318	0.69%	.054	.053	.987	100.00%
ω_{12}	.000	.208	n. def.	.198	.110	.555	52.71%
ω_{11}	.000	.084	n. def.	.192	.035	.181	68.14%
ω_{22}	.000	.075	n. def.	.184	.033	.181	67.74%
2SMM approach							
γ_1	.316	.315	−0.51%	.081	.077	.953	98.20%
γ_2	.316	.319	0.96%	.076	.077	1.016	98.40%
ω_{12}	.000	.009	n. def.	.058	.054	.935	9.00%
ω_{11}	.000	−.002	n. def.	.034	.030	.882	8.60%
ω_{22}	.000	−.002	n. def.	.033	.030	.909	8.60%
LMS							
γ_1	.316	.298	−5.72%	.071	.069	.971	99.00%
γ_2	.316	.303	−4.11%	.066	.068	1.040	99.40%
ω_{12}	.000	.002	n. def.	.058	.055	.942	7.80%
ω_{11}	.000	.011	n. def.	.033	.030	.903	10.40%
ω_{22}	.000	.011	n. def.	.033	.030	.912	11.00%
QML							
γ_1	.316	.298	−5.77%	.073	.068	.943	99.38%
γ_2	.316	.303	−4.06%	.067	.068	1.022	99.79%
ω_{12}	.000	.001	n. def.	.059	.055	.931	8.47%
ω_{11}	.000	.011	n. def.	.034	.030	.888	10.12%
ω_{22}	.000	.011	n. def.	.033	.030	.908	11.16%
ExUC approach							
γ_1	.316	.298	−5.64%	.071	.068	.967	99.20%
γ_2	.316	.303	−4.20%	.065	.068	1.040	99.20%
ω_{12}	.000	.004	n. def.	.060	.055	.913	8.00%
ω_{11}	.000	.009	n. def.	.030	.027	.883	11.20%
ω_{22}	.000	.009	n. def.	.030	.027	.906	11.80%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

Population model M_{lin}. The analysis of data of the linear population model M_{lin} for normally distributed predictors (Type I error condition; Table 6) showed that the mean parameter estimates for the MM approach were close to the population parameters (–.064 for the interaction effect and .056 for both quadratic effects). The estimates for the 2SMM approach, for LMS, QML, and for the ExUC approach were unbiased with mean parameter estimates between −.001 (for the quadratic effect ω_{11} for all four approaches) and .007 (for the interaction effect ω_{12} for the ExUC approach). The standard error estimates for the MM approach were severely underestimated (SE/SD between .120 and .431). This ratio was not inflated due to outliers; a graphical inspection showed that the parameter estimates followed a normal distribution. For all other approaches the standard errors were close to unbiased (SE/SD between .900 for the ExUC approach and .956 for LMS). The MM approach produced 36.80% spurious interaction and 72.00% to 74.80% spurious quadratic effects. The Type I error rates for the other four approaches were acceptable (ranging from 6.20% for LMS to 8.28% for QML).

Analyzing data of the linear population model M_{lin} for nonnormally distributed predictors (Table 7) for the MM approach showed a biased interaction effect estimate of .208 and biased quadratic effect estimates of .084 and .075. For the 2SMM and the ExUC approach, parameter estimates were unbiased (between −.002 and .009). For LMS and QML the quadratic effects were slightly biased, with an average estimate of .011. The standard errors of the MM approach were severely underestimated (SE/SD ranging from .181 for the quadratic effects to .555 for the interaction effect). Biased standard error estimates were also found for the first quadratic effect for the 2SMM approach, for QML, and for the ExUC approach (SE/SD between .882 and .888). The Monte Carlo SD for the MM approach was about four to five times larger than the Monte Carlo SDs for the other approaches. The Type I error rate of the MM approach was severely inflated (52.71% to 68.14%). LMS, QML, and the ExUC approach showed minimally higher Type I error rates (between 7.80% and 11.80%) than the 2SMM approach (between 8.60% and 9.00%).

TABLE 8
Simulation Results for the Nonlinear Population Model (M_{full}) Under the Condition of Normally Distributed Data (Analysis Model A_{full})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.316	−0.11%	.058	.057	.975	99.80%
γ_2	.316	.317	0.24%	.055	.057	1.025	100.00%
ω_{12}	.139	.151	8.98%	.178	.083	.463	56.40%
ω_{11}	.101	.113	11.50%	.207	.027	.130	74.80%
ω_{22}	.101	.082	−19.38%	.202	.027	.134	74.40%
2SMM approach							
γ_1	.316	.314	−0.60%	.055	.049	.888	100.00%
γ_2	.316	.317	0.36%	.050	.049	.977	100.00%
ω_{12}	.139	.146	5.28%	.064	.061	.948	67.20%
ω_{11}	.101	.099	−1.93%	.044	.040	.907	69.40%
ω_{22}	.101	.100	−1.40%	.043	.039	.922	71.20%
LMS							
γ_1	.316	.315	−0.48%	.055	.052	.952	100.00%
γ_2	.316	.317	0.34%	.049	.052	1.045	100.00%
ω_{12}	.139	.145	4.18%	.063	.060	.955	66.80%
ω_{11}	.101	.100	−1.33%	.043	.040	.928	70.40%
ω_{22}	.101	.100	−0.92%	.042	.039	.937	71.20%
QML							
γ_1	.316	.315	−0.46%	.055	.050	.902	100.00%
γ_2	.316	.317	0.20%	.050	.050	.982	100.00%
ω_{12}	.139	.145	4.26%	.063	.060	.953	67.20%
ω_{11}	.101	.099	−1.82%	.043	.040	.925	69.42%
ω_{22}	.101	.100	−1.06%	.042	.039	.934	71.63%
ExUC approach							
γ_1	.316	.314	−0.56%	.054	.050	.914	100.00%
γ_2	.316	.317	0.29%	.049	.049	1.003	100.00%
ω_{12}	.139	.146	5.01%	.067	.062	.933	64.60%
ω_{11}	.101	.100	−1.27%	.045	.041	.901	66.80%
ω_{22}	.101	.100	−0.98%	.044	.040	.929	69.40%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

Population model M_{full}. The results for the data of the nonlinear population model M_{full} for normally distributed predictors (power condition; Table 8) showed that the mean parameter estimates for the MM approach were biased (11.50% and –19.38% for the quadratic effects, and 8.98% for the interaction effect). The 2SMM approach, LMS, QML, and the ExUC approach showed unbiased (or only slightly biased) parameter estimates with a relative bias between –0.92% (for LMS) and 5.28% (for the 2SMM approach). The standard error estimates for the MM approach were strongly biased with SE/SD between .130 and .463. The standard errors for the other four approaches were unbiased with a ratio SE/SD between .901 (for the ExUC approach) and .955 (for LMS). The power for the nonlinear effects was slightly larger for the 2SMM approach and for QML (between 67.20% and 71.63%) compared to the ExUC approach (64.60%–69.40%).

The results for the data of the nonlinear population model M_{full} for nonnormally distributed predictors (Table 9) showed that the MM approach produced biased quadratic effects (–30.95% and –29.40%) and a biased interaction effect (207.71%). The 2SMM and the ExUC approach showed unbiased (or only slightly biased) parameter estimates (between –2.12% and 9.22%). LMS and QML showed overestimated quadratic effects with a bias of about 19%. The standard error estimates for the MM approach were underestimated (SE/SD ranging from .221 and .224 for the quadratic effects to .714 for the interaction effect). For the 2SMM approach the standard error estimate for the first quadratic effect was underestimated ($SE/SD = .880$). The standard error estimates for LMS and QML were unbiased. For the ExUC approach standard errors were underestimated for all nonlinear effects (SE/SD between .857 and .898). For the MM approach the power for the quadratic effects was smaller than the estimated Type I error rate (which was due to the biased estimates and the larger standard error estimates in the power condition). The power for the 2SMM approach, LMS, QML, and the ExUC approach was high on average: For the interaction effect the power was above 65.00% and for the quadratic effects it was above 80.96%.

TABLE 9
Simulation Results for the Nonlinear Population Model (M_{full}) Under the Condition of Nonnormally Distributed Data (Analysis Model A_{full})

	θ	$\bar{\hat{\theta}}$	$bias(\bar{\hat{\theta}})\%$	SD	$\overline{\widehat{SE}}$	SE/SD	% significance
MM approach							
γ_1	.316	.363	14.93%	.058	.057	.994	100.00%
γ_2	.316	.368	16.38%	.054	.057	1.045	100.00%
ω_{12}	.139	.427	207.71%	.229	.163	.714	72.47%
ω_{11}	.101	.070	−30.95%	.148	.033	.224	47.77%
ω_{22}	.101	.071	−29.40%	.151	.033	.221	45.14%
2SMM approach							
γ_1	.316	.315	−0.42%	.078	.073	.942	98.40%
γ_2	.316	.320	1.05%	.072	.074	1.029	98.80%
ω_{12}	.139	.152	9.22%	.060	.058	.962	73.75%
ω_{11}	.101	.099	−2.12%	.034	.030	.880	84.17%
ω_{22}	.101	.098	−3.15%	.033	.031	.946	80.96%
LMS							
γ_1	.316	.315	−0.35%	.068	.066	.969	99.80%
γ_2	.316	.319	0.82%	.063	.066	1.045	99.60%
ω_{12}	.139	.137	−1.69%	.060	.057	.947	65.00%
ω_{11}	.101	.121	19.51%	.034	.032	.927	92.60%
ω_{22}	.101	.120	18.99%	.033	.032	.944	91.80%
QML							
γ_1	.316	.314	−0.74%	.069	.065	.940	100.00%
γ_2	.316	.317	0.28%	.063	.064	1.022	99.80%
ω_{12}	.139	.137	−1.53%	.060	.057	.949	65.66%
ω_{11}	.101	.120	18.96%	.034	.032	.935	92.57%
ω_{22}	.101	.120	18.77%	.033	.032	.944	91.97%
ExUC approach							
γ_1	.316	.316	−0.16%	.068	.064	.943	99.80%
γ_2	.316	.319	0.85%	.063	.064	1.017	99.60%
ω_{12}	.139	.149	7.38%	.063	.056	.898	72.40%
ω_{11}	.101	.106	4.34%	.032	.027	.857	92.00%
ω_{22}	.101	.105	3.55%	.031	.027	.898	91.80%

Note. MM = method of moments approach; 2SMM = two-stage method of moments; LMS = latent moderated structural equations approach; QML = quasi-maximum likelihood approach; ExUC = extended unconstrained approach.

DISCUSSION

For the analysis of latent nonlinear effects in empirical settings it is necessary to rely on procedures that result in estimates with an acceptably small bias and correct inferences that include a nominal Type I error rate and a high power. Although most approaches rely on certain distributional assumptions that include the normality of latent and manifest variables, these assumptions are often not met in applied empirical settings. In this article, we examined five different approaches in a simulation study regarding their properties in two different settings: First, we tested their performance with regard to the analysis of single interaction effects versus the analysis of multiple nonlinear effects. Second, we tested their performance in the presence of either normally or nonnormally distributed latent predictors.

Discussion of the Results of the Simulation Study

The results of the simulation study showed that the parameter estimates for the ExUC approach led only to a small bias for all nonlinear effects under the condition of normal or nonnormal latent predictors. Although a robust estimator for the standard errors was used, the standard errors were underestimated under the condition of nonnormal predictors. This resulted in an inflated Type I error rate that is in line with other simulation studies reported previously (e.g.,Marsh et al., 2004). Although the sandwich estimator is asymptotically unbiased (White, 1982), its properties in smaller samples might not always lead to unbiased standard error estimates.

The advantage of the two distribution analytic approaches, LMS and QML, lies in a high efficiency for the estimation of multiple nonlinear effects in settings with normal latent predictors. Under the condition of nonnormal predictors, however, parameter estimates might be moderately biased, whereas standard error estimates remained unbiased. The two methods showed only small differences in their estimation results, which is in line with the findings by Kelava et al. (2011).

The 2SMM approach showed unbiased and efficient parameter and standard error estimates under the condition of normal predictors. The standard errors were slightly underestimated under the conditions of nonnormal predictors. The results for the 2SMM approach supported the claimed robustness against nonnormal predictors (Wall & Amemiya, 2003).

The MM approach led to unbiased parameter estimates for the interaction model under the condition of normal latent predictors. Under the condition of nonnormal predictors or multiple nonlinear effects, parameter and standard error estimates were severely biased in the presented simulation study.

Guidelines for Applications

For different degrees of normality of the indicators of the predictors and for different numbers of nonlinear effects we give four recommendations for the selection of the approaches. A nonnormality of the indicators of the dependent variable is not taken into account here, because their distribution should be nonnormal in general if nonlinear effects are present in the data (cf. Klein & Moosbrugger, 2000). The guidelines should be used with caution and under the limitations concerning latent and manifest distributions presented next.

First, when indicators of the latent predictors are normally distributed and single interaction effects are estimated, each one of the approaches can be used, for instance, the MM approach or LMS, although power for detecting an interaction using the MM approach might be low. Second, when indicators of the latent predictors are normally distributed and multiple nonlinear effects are estimated, the distribution analytic approaches, LMS and QML, lead to the most efficient estimates with the highest power and should be used due to their theoretical and empirical properties. Third, when indicators of the latent predictors are nonnormally distributed and it is not expected that residuals are strongly nonnormal, especially the 2SMM approach leads to unbiased results and can be used if single interaction effects are estimated. If its implementation is difficult the ExUC approach is a useful alternative. Fourth, when indicators of the latent predictors are nonnormally distributed and multiple nonlinear effects are estimated, the 2SMM and the ExUC approach lead to unbiased estimates, but the 2SMM covers the Type I error rate slightly better. Therefore, the 2SMM approach should be preferred.

Limitations

There are also two limitations that are of relevance to discuss. The first limitation concerns the validation of the distributional assumptions. In the simulation study presented in this article, we assumed that the nonnormality of the indicators was caused by nonnormally distributed latent predictor variables. In practice, however, the source of nonnormality cannot be precisely attributed to either the predictors or the measurement errors. It can only be inferred by plausible assumptions, but not statistically (Molenaar, Dolan, & Verhelst, 2010). These assumptions might include findings from previous studies concerning the distribution of the construct of interest, or specific situational aspects that influence the distribution of the measurement errors, for example, ceiling effects that occurred in the measurement of the constructs. However, in most practical situations one would only implement an SEM model when the indicator variables are reliable (MacCallum & Austin, 2000). In this case, the indicators have a large amount of "true" variance that amounts to the latent predictor. When indicators are skewed, it is plausible to assume that the skewness is caused by the latent predictor. In the case of unreliable indicators, this conclusion appears not to be valid. The results of the approaches discussed in this article should then be interpreted with caution.

The second limitation concerns the question of what a realistic sample size is. The sample size in the simulation study was set to $N = 400$ to reflect a typical medium to large sample size for a structural equation model in the social sciences (Jaccard & Wan, 1995). We did not examine smaller or larger sample sizes. For smaller sample sizes it can be expected that the method of moments-based approaches and the ExUC approach might become less stable because the estimation of higher order moments needs a sufficient sample size, and because the ExUC approach has a large number of free parameters. With a larger sample size, at least the estimates of the method of moments-based approaches can be expected to improve. It was beyond the scope of this article, however, to find the critical sample size when estimates become acceptable in the context of nonnormally distributed data.

Final Considerations and Future Directions

The decision on whether a distributional assumption poses a restriction on the application of a method depends on at least three aspects: the plausibility of the assumption in empirically relevant settings, the robustness of the method against a violation of the assumption, and the testability of the assumption. For the approaches within the nonlinear SEM framework we discussed the plausibility of the distributional assumptions and we then tested their robustness. The answer to the third aspect, the testability of the assumption, is—as was pointed out earlier—only possible to a limited degree. Our results indicate that the assumption of normal latent predictors is a serious assumption for some approaches, and that researchers need to attend to this assumption when making a decision about what approach to use.

Future research should focus on the refinement of existing approaches in two ways. First, the unbiased and efficient estimation of multiple effects should be explored in ways where distributional assumptions are violated in other empirically relevant settings, for example, when ordinal or censored data are used. There, the application of robust versions of the estimators for the different approaches might improve estimation properties; for example, bootstrap standard errors for the ExUC approach might improve the Type I error rate in certain settings (Brandt, 2009). Second, there is still a lack of implementation and straightforward extension in existing SEM software for some of the approaches, especially the 2SMM approach.

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