SEM mediation models

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KU CENTER FOR RESEARCH METHODS & DATA ANALYSIS

College of Liberal Arts & Sciences

Outline

KU

For examples: we will use the affect data

The affect data is available in the SEM workshop `data` folder

```
ddir <- "../../data"
fn <- "affect-2.csv"
dat <- read.csv(file.path(ddir, fn))
```

```
head(dat)
```

```
  Agency1 Agency2 Agency3 Intrin1 Intrin2 Intrin3 Extrin1 Extrin2 Extrin3 PosAFF1 PosAFF2 PosAFF3
1  3.5000  4.0000  4.0000  4.0000     4.0       4  1.0000  1.0000  1.5000  4.0000     4.0     4.0
2  2.5000  3.1667  3.0000  3.2123     2.0       3  1.8333  2.6667  1.8333  3.0000     3.5     2.5
3  1.8333  2.0000  1.5000  3.0000     3.0       2  1.0000  1.0000  1.0000  3.0184     2.5     3.0
4  2.7714  3.0602  2.3639  3.1337     4.0       3  1.0774  1.1667  1.0000  3.0000     2.5     3.0
5  3.1667  3.3333  2.8333  3.5000     4.0       4  1.8333  2.0000  1.8333  3.7804     3.5     3.0
6  2.3333  2.8333  2.3333  3.0000     2.5       3  3.0588  2.4125  2.6667  4.0000     3.0     3.0
  NegAFF1 NegAFF2 NegAFF3 Sex gender ethnicity     race
1     1.0  1.0000     1.0   1   male  Hispanic Nonwhite
2     1.5  1.6858     1.5   1   male     White    White
3     1.0  1.0000     1.0   1   male     White    White
4     2.5  2.5000     1.5   1   male     White    White
5     2.5  2.0000     3.0   1   male     White    White
6     2.0  1.5000     2.0   1   male     White    White
```

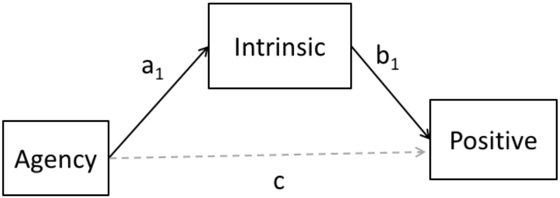
For examples: we will use the affect data ...

The primary R (R Core Team, 2017) package we use is `lavaan`, but also will illustrate `semPlot`.

```
library(lavaan)
library(semPlot)
```

A model with observed variables

This is a "usual" kind of regression story



$$Positive_i = c \cdot Agency_i + b_1 Intrinsic_i + \varepsilon 1_i \quad (1)$$

$$Intrinsic_i = a_1 Agency_i + \varepsilon 2_i \quad (2)$$

- c is the *direct* effect of Agency on Positive affect
- The product "$a_1 \cdot b_1$" is the *indirect effect* of Agency on Positive affect

A model with observed variables ...

- See why "$a_1 \cdot b_1$" is the *indirect effect*?
  - Insert equation (2) into (1):

$$\begin{aligned} Positive &= c \cdot Agency_i + b_1\{a_1 Agency_i + \varepsilon 2_i\} + \varepsilon 1_i \qquad (3) \\ &= c \cdot Agency_i + \{b_1 \cdot a_1\} Agency_i + \{\epsilon 1_i + b_1 \epsilon 2_i\} \end{aligned}$$

The coefficients in that one-liner cannot be estimated, they are not identifiable.

$$\{b_1 \cdot a_1 + c\} Agency_i + \{\epsilon 1_i + b_1 \epsilon 2_i\} \qquad (4)$$

A model with observed variables ...

- The primary research question is whether the "direct" path c is more/less important than the indirect path $a_1 b_1$
- To estimate this with ordinary regression analysis, we would need to calculate 2 regression models:

$$Positive_i = c \cdot Agency_i + b_1 Intrinsic_i + \varepsilon 1_i \tag{5}$$

$$Intrinsic_i = a_1 Agency_i + \varepsilon 2_i \tag{6}$$

- From the output, take the individual estimates, $\hat{a}_1$ and $\hat{b}_1$, and multiply to create $\hat{a}_1 \hat{b}_1$ and interpret that as an estimate of $a_1 b_1$.
  - (We just asserted that $\hat{a}_1 \hat{b}_1$ is a consistent/unbiased estimator of $\widehat{a_1 b_1}$, and I'm not entirely sure how that is justified. But assume it is OK)
- But...

A model with observed variables ...

More problems present themselves:

1. Are the error terms $\varepsilon 1_i$ and $\varepsilon 2_i$ uncorrelated with each other? If not, a system-wide estimation procedure is needed
    1. "seemingly unrelated regression" (3SLS), so-called "limited information maximum likelihood"
    2. system-wide maximum Likelihood (ML), often called "full information maximum likelihood"

A model with observed variables ...

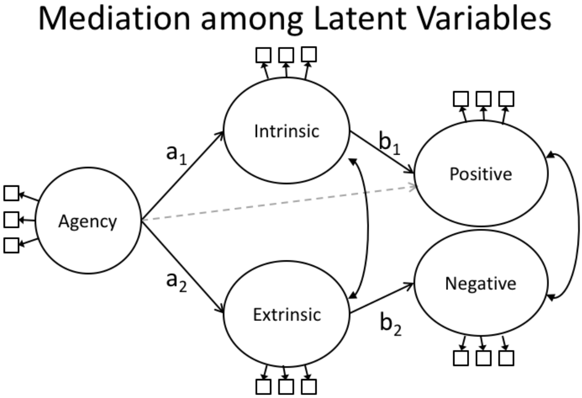
2. How can we conduct a hypothesis test for the product $\hat{a}_1 \cdot \hat{b}_1$? Does anybody know if we can do a t-test of a product of 2 variables? What would we plug in for the standard error, $s.e.(\widehat{a_1 b_1})$ in a formula like

$$\hat{t} = \frac{\widehat{a_1 b_1}}{s.e.(\widehat{a_1 b_1})} = \frac{\hat{a}_1 \hat{b}_1}{?} \tag{7}$$

   1. Think about what the 'nil' hypo means:
      1. $H_0 : a_1 \cdot b_1 = 0$
      2. Means that either $H_0 : a_1 = 0$ and/or $b_1 = 0$
   2. Tough to visualize, no known statistical distribution can be used for a direct test. Various methods have been proposed.

Latent Variable Model with Mediation

The circles are unmeasured variables.



Latent Variable Model with Mediation

- The SEM integrates
  - "measurement error" analysis (correlation of error terms for Positive and Negative terms, with
  - analysis of the underlying relationships.
- We've spiced this example with correlations between
  - latent variables Intrinsic and Extrinsic
  - error terms for Positive and Negative affect outcomes

Recall SEM assumptions

1. observed indicators reflect unmeasured latent variables.
2. the latent variables are multivariate normal distributions (with inter-correlations we'd like to estimate)

In the "big picture", there are 2 chores.

1. Find a way to test the indirect effect's importance, and
2. Decide if the path model we sketched above is "good enough". We end up doing a χ^2 "diff test" to contrast the "full" CFA and a simpler SEM.

   CFA: correlations among latent traits exist, allow those correlations to be freely estimated.

   - uses "most" parameters because all links are estimated

   SEM: structural equation model: Limit estimation of linkages to a theoretically inspired set of connections.

Outline

The CFA model procedure

```
cfa.mod <- '
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 '
cfa3 <- cfa(cfa.mod, data = dat, std.lv = TRUE)
```

The CFA model procedure ...

```
summary(cfa3)
```

```
lavaan 0.6-3 ended normally after 58 iterations

  Optimization method                           NLMINB
  Number of free parameters                         40

  Number of observations                           380

  Estimator                                         ML
  Model Fit Test Statistic                     106.847
  Degrees of freedom                                80
  P-value (Chi-square)                           0.024

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Agency =~
    Agency1           0.466    0.021   22.560    0.000
```

The CFA model procedure ...

```
    Agency2             0.492    0.021    23.774    0.000
    Agency3             0.497    0.022    22.815    0.000
  Intrinsic =~
    Intrin1             0.541    0.039    13.960    0.000
    Intrin2             0.581    0.043    13.489    0.000
    Intrin3             0.615    0.038    16.201    0.000
  Extrinsic =~
    Extrin1             0.388    0.022    17.606    0.000
    Extrin2             0.456    0.027    17.168    0.000
    Extrin3             0.471    0.026    18.100    0.000
  Positive =~
    PosAFF1             0.569    0.028    20.132    0.000
    PosAFF2             0.603    0.029    21.150    0.000
    PosAFF3             0.632    0.029    21.648    0.000
  Negative =~
    NegAFF1             0.634    0.029    21.670    0.000
    NegAFF2             0.585    0.027    21.457    0.000
    NegAFF3             0.598    0.026    22.805    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  Agency ~~
    Intrinsic           0.507    0.047    10.782    0.000
    Extrinsic           0.270    0.054     5.028    0.000
    Positive            0.302    0.051     5.950    0.000
    Negative            0.017    0.055     0.312    0.755
```

The CFA model procedure ...

```
  Intrinsic ~~
    Extrinsic        -0.028    0.063   -0.441    0.659
    Positive          0.414    0.052    7.918    0.000
    Negative          0.021    0.060    0.341    0.733
  Extrinsic ~~
    Positive         -0.029    0.058   -0.503    0.615
    Negative          0.209    0.056    3.764    0.000
  Positive ~~
    Negative         -0.071    0.056   -1.274    0.203

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Agency1           0.048    0.005    9.649    0.000
   .Agency2           0.036    0.005    7.604    0.000
   .Agency3           0.051    0.005    9.278    0.000
   .Intrin1           0.298    0.029   10.204    0.000
   .Intrin2           0.386    0.036   10.620    0.000
   .Intrin3           0.214    0.029    7.474    0.000
   .Extrin1           0.080    0.009    8.924    0.000
   .Extrin2           0.123    0.013    9.465    0.000
   .Extrin3           0.103    0.012    8.264    0.000
   .PosAFF1           0.121    0.012    9.921    0.000
   .PosAFF2           0.104    0.012    8.615    0.000
   .PosAFF3           0.099    0.013    7.869    0.000
   .NegAFF1           0.107    0.012    9.202    0.000
   .NegAFF2           0.096    0.010    9.498    0.000
```

The CFA model procedure ...

```
    .NegAFF3           0.067    0.009    7.369    0.000
     Agency            1.000
     Intrinsic         1.000
     Extrinsic         1.000
     Positive          1.000
     Negative          1.000
```

Make a nice table with semTable

Start by creating a label vector `vl`

```
vl <- c("Intrin1" = "Intrinsic1", "Intrin2" =
    "Intrinsic2",  "Intrin3" = "Intrinsic3",
    "Extrin1" = "Extrinsic1", "Extrin2" =
    "Extrinsic2",  "Extrin3" = "Extrinsic3",
    "PosAFF1" = "Positive1", "PosAFF2" =
    "Positive2", "PosAFF3" = "Positive3",
    "NegAFF1" = "Negative1", "NegAFF2" =
    "Negative2", "NegAFF3" = "Negative3")
```

Not much customization needed:

```
library(semTable)
cfa3.tab <- semTable(list("Confirmatory Factor
    Analysis" = cfa3), columns = c("estsestars"),
    varLabels = vl, longtable = TRUE,
    print.results = FALSE)
```

And results are

```
cat(cfa3.tab)
```

		Confirmatory Factor Analysis
		Estimate(Std.Err.)
		Factor Loadings
Agency		
	Agency1	0.47(0.02)***
	Agency2	0.49(0.02)***
	Agency3	0.50(0.02)***
Intrinsic		
	Intrinsic1	0.54(0.04)***
	Intrinsic2	0.58(0.04)***
	Intrinsic3	0.62(0.04)***
Extrinsic		
	Extrinsic1	0.39(0.02)***
	Extrinsic2	0.46(0.03)***

And results are ...

	Extrinsic3	0.47(0.03)***
Positive		
	Positive1	0.57(0.03)***
	Positive2	0.60(0.03)***
	Positive3	0.63(0.03)***
Negative		
	Negative1	0.63(0.03)***
	Negative2	0.59(0.03)***
	Negative3	0.60(0.03)***
		Residual Variances
	Agency1	0.05(0.00)***
	Agency2	0.04(0.00)***
	Agency3	0.05(0.01)***
	Intrinsic1	0.30(0.03)***
	Intrinsic2	0.39(0.04)***
	Intrinsic3	0.21(0.03)***
	Extrinsic1	0.08(0.01)***

And results are ...

Extrinsic2	0.12(0.01)***
Extrinsic3	0.10(0.01)***
Positive1	0.12(0.01)***
Positive2	0.10(0.01)***
Positive3	0.10(0.01)***
Negative1	0.11(0.01)***
Negative2	0.10(0.01)***
Negative3	0.07(0.01)***
	Latent Variances
Agency	1.00^{+}
Intrinsic	1.00^{+}
Extrinsic	1.00^{+}
Positive	1.00^{+}
Negative	1.00^{+}
	Latent Covariances
Agency w/Intrinsic	0.51(0.05)***
Agency w/Extrinsic	0.27(0.05)***

And results are ...

Agency w/Positive	0.30(0.05)***
Agency w/Negative	0.02(0.05)
Intrinsic w/Extrinsic	−0.03(0.06)
Intrinsic w/Positive	0.41(0.05)***
Intrinsic w/Negative	0.02(0.06)
Extrinsic w/Positive	−0.03(0.06)
Extrinsic w/Negative	0.21(0.06)***
Positive w/Negative	−0.07(0.06)
	Fit Indices
χ^2(df)	106.85(80)*
CFI	0.99
TLI	0.99
RMSEA	0.03

+Fixed parameter

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Outline

1. R warmups
2. Path Model of Mediation
3. Mediation among Latent Variables
   - CFA Review
   - SEM with mediation
4. Supplementary Hypothesis Test
5. Conclusion

Equations we need to estimate

Going by the path diagram, write out the equations:

$$Positive_i = c \cdot Agency_i + b_1 Intrinsic_i + \varepsilon 1_i \tag{8}$$
$$Intrinsic_i = a_1 Agency_i + \varepsilon 2_i \tag{9}$$
$$Negative_i = b_2 Extrinsic_i + \varepsilon 3_i \tag{10}$$
$$Extrinsic_i = a_2 Agency + \varepsilon 4_i \tag{11}$$

- Agency affects Intrinsic motivation (equation 9)
- Positive affect is predicted by Intrinsic motivation and Agency (equation 8)
- Negative affect is driven by Agency *only indirectly*
- And we inserted also correlations that should be estimated,
    - $Cov(\varepsilon 1, \varepsilon 3) \neq 0$
    - $Cov(Intrinsic, Extrinsic) \neq 0$

Equations we need to estimate ...

- But we assert there is no direct link from
  - Extrinsic to Positive
  - Intrinsic to Negative
  - Agency to Negative

Estimate mediation with lavaan

The distinctive feature is that our `lavaan` code introduces 4 named parameters, `a1`, `a2`, `b1`, `b2`.

We will create two "constructed" variables, `ind1` and `ind2`, to represent the indirect path estimates.

Estimate mediation with lavaan ...

```
mediat.mod.2 <- '
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
 Intrinsic ~ a1*Agency
 Extrinsic ~ a2*Agency
 Intrinsic ~~ Extrinsic
 ## define mediation parameters (indirect effects)
 ind1 := a1 * b1
 ind2 := a2 * b2
 '
```

Estimate mediation with lavaan ...

```
mediat.mod.2 <- '
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
 Intrinsic ~ a1*Agency
 Extrinsic ~ a2*Agency
 Intrinsic ~~ Extrinsic
 ## define mediation parameters (indirect effects)
 ind1 := a1 * b1
 ind2 := a2 * b2
 '
```

1. Named parameters for indirect effect $ind1 = a_1 \cdot b_1$

```
Positive ~ b1*Intrinsic + Agency
Negative ~ b2*Extrinsic
Intrinsic ~ a1*Agency
Extrinsic ~ a2*Agency
## define mediation parameters (indirect effects)
ind1 := a1 * b1
ind2 := a2 * b2
```

- The product $a_1 \cdot b_1$ can be tested because we represent it by `ind1`

2. Bootstrap the estimate of $a_1 b_1$

- The hull hypothesis we would like to test,

$$H_0 : a_1 b_1 = 0$$

- At the current time, no analytical "formula" exists for testing that. Instead, it is necessary to use 'bootstrapped standard errors'.
- The disadvantage of the bootstrap method is that the model must be calculated 100s or 1000s of times (slow!)

Structural Regressions among Latent Constructs

We will set the number of bootstrap resampling exercises as `Nboot`

```
Nboot <- 500
```

```
sem(mediat.mod.2, data = dat, std.lv = TRUE, se =
    "boot", bootstrap = Nboot)
```

- The arguments in `sem()` that concern the bootstrap are
  - `se="boot"` , and
  - `bootstrap = Nboot`.
- Rather than analytically approximate standard errors, the standard deviations of the bootstrapped estimates are used as standard errors.

Make a nice table

```
semcfa.tab <- semTable(list("SEM" = mediat.out.2,
    "CFA" = cfa3), columns = c("estsestars"),
    varLabels = vl, longtable = TRUE,
    print.results = FALSE)
```

And results are

```
cat(semcfa.tab)
```

		SEM	CFA
		Estimate(Std.Err.)	Estimate(Std.Err.)
		Factor Loadings	
Agency			
	Agency1	0.47(0.02)***	0.47(0.02)***
	Agency2	0.49(0.02)***	0.49(0.02)***
	Agency3	0.50(0.02)***	0.50(0.02)***
Intrinsic			
	Intrinsic1	0.47(0.04)***	0.54(0.04)***
	Intrinsic2	0.50(0.04)***	0.58(0.04)***
	Intrinsic3	0.53(0.03)***	0.62(0.04)***
Extrinsic			
	Extrinsic1	0.37(0.02)***	0.39(0.02)***
	Extrinsic2	0.44(0.03)***	0.46(0.03)***

And results are ...

	Extrinsic3	0.45(0.03)***	0.47(0.03)***
Positive			
	Positive1	0.51(0.03)***	0.57(0.03)***
	Positive2	0.54(0.03)***	0.60(0.03)***
	Positive3	0.57(0.03)***	0.63(0.03)***
Negative			
	Negative1	0.62(0.04)***	0.63(0.03)***
	Negative2	0.57(0.04)***	0.59(0.03)***
	Negative3	0.59(0.03)***	0.60(0.03)***
		Regression Slopes	
Positive			
	Intrinsic	0.34(0.08)***	
	Agency	0.13(0.07)	
Negative			
	Extrinsic	0.20(0.07)**	
Intrinsic			
	Agency	0.59(0.08)***	

And results are ...

Extrinsic		
Agency	0.28(0.06)***	
	Residual Variances	
Agency1	0.05(0.01)***	0.05(0.00)***
Agency2	0.04(0.00)***	0.04(0.00)***
Agency3	0.05(0.01)***	0.05(0.01)***
Intrinsic1	0.30(0.03)***	0.30(0.03)***
Intrinsic2	0.39(0.04)***	0.39(0.04)***
Intrinsic3	0.21(0.04)***	0.21(0.03)***
Extrinsic1	0.08(0.01)***	0.08(0.01)***
Extrinsic2	0.12(0.02)***	0.12(0.01)***
Extrinsic3	0.10(0.02)***	0.10(0.01)***
Positive1	0.12(0.02)***	0.12(0.01)***
Positive2	0.10(0.02)***	0.10(0.01)***
Positive3	0.10(0.01)***	0.10(0.01)***
Negative1	0.11(0.02)***	0.11(0.01)***
Negative2	0.10(0.01)***	0.10(0.01)***

And results are ...

Negative3	$0.07(0.01)^{***}$	$0.07(0.01)^{***}$
	Latent Variances	
Agency	1.00^{+}	1.00^{+}
Intrinsic	1.00^{+}	1.00^{+}
Extrinsic	1.00^{+}	1.00^{+}
Positive	1.00^{+}	1.00^{+}
Negative	1.00^{+}	1.00^{+}
	Latent Covariances	
Intrinsic w/Extrinsic	$-0.20(0.06)^{**}$	$-0.03(0.06)$
Positive w/Negative	$-0.08(0.07)$	$-0.07(0.06)$
Agency w/Intrinsic		$0.51(0.05)^{***}$
Agency w/Extrinsic		$0.27(0.05)^{***}$
Agency w/Positive		$0.30(0.05)^{***}$
Agency w/Negative		$0.02(0.05)$
Intrinsic w/Positive		$0.41(0.05)^{***}$
Intrinsic w/Negative		$0.02(0.06)$
Extrinsic w/Positive		$-0.03(0.06)$

And results are ...

Extrinsic w/Negative		$0.21(0.06)^{***}$
	Constructed	
ind1	$0.20(0.05)^{***}$	
ind2	$0.06(0.02)^{*}$	
	Fit Indices	
$\chi^2(\mathrm{df})$	$109.15(83)^{*}$	$106.85(80)^{*}$
CFI	0.99	0.99
TLI	0.99	0.99
RMSEA	0.03	0.03

$^{+}$Fixed parameter

* p<0.05, ** p<0.01, ***p<0.001

Use lavaan's anova function

- The mediation model with latent variables estimates fewer parameters than the CFA, but it fits just as well as the CFA.

```
anova(cfa3, mediat.out.2)
```

```
Chi Square Difference Test

             Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
cfa3         80 7478.2 7635.8 106.85
mediat.out.2 83 7474.5 7620.3 109.15     2.3051       3     0.5116
```


```
anova(cfa3, mediat.out.2)
```

```
Chi Square Difference Test

             Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
cfa3         80 7478.2 7635.8 106.85
mediat.out.2 83 7474.5 7620.3 109.15     2.3051       3     0.5116
```

Make a pleasant table with compareLavaan (semTable)

```
# if nesting is not specified, the diff is not
   reported
compareLavaan(list("CFA" = cfa3, "SEM" =
   mediat.out.2), nesting = "CFA > SEM", fitmeas
   = c("chisq", "df", "pvalue", "rmsea", "cfi",
   "bic"),  type="latex")
```

	χ^2	df	*p*-value	rmsea	cfi	bic	$\Delta\chi^2$	Δdf	*p*
CFA	106.85	80.00	0.02	0.03	0.99	7635.83	-	-	-
SEM	109.15	83.00	0.03	0.03	0.99	7620.31	2.305a	3	0.5
a = SEM vs CFA									

Make a pleasant table with compareLavaan (semTable) ...

```
mediat.mod.3 <- '
 ## measurement model
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 ## structural model
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
 Intrinsic ~ a1*Agency
 Extrinsic ~ a2*Agency
 Intrinsic ~~ Extrinsic
 ## define mediation parameters (indirect effects)
 c1 := a1 * b1
 c2 := a2 * b2
 c1 == c2
```

Make a pleasant table with compareLavaan (semTable) ...

```
,
```

Compare Standardized factor loadings

- We have standardized latent variables, but we did not standardize the observed variables.
- Various methods are available

  Method 1: Refit from scratch:.

```
mediat.out.2.std.A <- sem(mediat.mod.2, data
    = dat, std.lv = TRUE, std.ov=TRUE, se =
    "boot", bootstrap = Nboot)
summary(mediat.out.2.std.A, fit = TRUE)
```

Compare Standardized factor loadings ...

```
lavaan 0.6-3 ended normally after 31 iterations

  Optimization method                           NLMINB
  Number of free parameters                         37

  Number of observations                           380

  Estimator                                         ML
  Model Fit Test Statistic                     109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.991

Loglikelihood and Information Criteria:
```

Compare Standardized factor loadings ...

```
  Loglikelihood user model (H0)               -6260.310
  Loglikelihood unrestricted model (H1)       -6205.734

  Number of free parameters                          37
  Akaike (AIC)                                12594.621
  Bayesian (BIC)                              12740.407
  Sample-size adjusted Bayesian (BIC)         12623.013

Root Mean Square Error of Approximation:

  RMSEA                                           0.029
  90 Percent Confidence Interval          0.010   0.043
  P-value RMSEA <= 0.05                           0.996

Standardized Root Mean Square Residual:

  SRMR                                            0.034

Parameter Estimates:

  Standard Errors                             Bootstrap
  Number of requested bootstrap draws               500
  Number of successful bootstrap draws              500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
```

Compare Standardized factor loadings ...

```
  Agency =~
    Agency1          0.904    0.042   21.531    0.000
    Agency2          0.932    0.042   22.325    0.000
    Agency3          0.910    0.044   20.914    0.000
  Intrinsic =~
    Intrin1          0.605    0.052   11.579    0.000
    Intrin2          0.587    0.045   13.046    0.000
    Intrin3          0.689    0.041   16.750    0.000
  Extrinsic =~
    Extrin1          0.777    0.048   16.079    0.000
    Extrin2          0.763    0.049   15.634    0.000
    Extrin3          0.795    0.053   14.918    0.000
  Positive =~
    PosAFF1          0.770    0.037   20.728    0.000
    PosAFF2          0.796    0.041   19.302    0.000
    PosAFF3          0.807    0.039   20.575    0.000
  Negative =~
    NegAFF1          0.869    0.051   16.875    0.000
    NegAFF2          0.864    0.057   15.044    0.000
    NegAFF3          0.897    0.051   17.756    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Positive ~
    Intrinsic (b1)   0.343    0.073    4.684    0.000
    Agency           0.129    0.074    1.739    0.082
```

Compare Standardized factor loadings ...

```
  Negative ~
    Extrinsic (b2)    0.199    0.068    2.939    0.003
  Intrinsic ~
    Agency    (a1)    0.588    0.087    6.788    0.000
  Extrinsic ~
    Agency    (a2)    0.278    0.062    4.450    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .Intrinsic ~~
   .Extrinsic        -0.202    0.060   -3.350    0.001
 .Positive ~~
   .Negative         -0.076    0.073   -1.052    0.293

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Agency1           0.181    0.022    8.400    0.000
   .Agency2           0.129    0.017    7.696    0.000
   .Agency3           0.169    0.018    9.134    0.000
   .Intrin1           0.505    0.056    8.949    0.000
   .Intrin2           0.534    0.056    9.600    0.000
   .Intrin3           0.358    0.062    5.793    0.000
   .Extrin1           0.347    0.047    7.445    0.000
   .Extrin2           0.371    0.045    8.278    0.000
   .Extrin3           0.316    0.051    6.232    0.000
   .PosAFF1           0.271    0.034    7.846    0.000
```

Compare Standardized factor loadings ...

```
   .PosAFF2           0.221    0.034    6.437    0.000
   .PosAFF3           0.199    0.028    6.978    0.000
   .NegAFF1           0.210    0.030    6.972    0.000
   .NegAFF2           0.219    0.030    7.261    0.000
   .NegAFF3           0.157    0.025    6.405    0.000
    Agency            1.000
   .Intrinsic         1.000
   .Extrinsic         1.000
   .Positive          1.000
   .Negative          1.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ind1              0.202    0.045    4.515    0.000
    ind2              0.055    0.023    2.445    0.014
```

Method 2: R update function (re-specify standardization)

```
mediat.out.2.std.B <- update(mediat.out.2,
    std.lv = TRUE, std.ov = TRUE)
summary(mediat.out.2.std.B, fit = TRUE)
```

Compare Standardized factor loadings ...

```
lavaan 0.6-3 ended normally after 31 iterations

  Optimization method                           NLMINB
  Number of free parameters                         37

  Number of observations                           380

  Estimator                                         ML
  Model Fit Test Statistic                     109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.991

Loglikelihood and Information Criteria:

```

Compare Standardized factor loadings ...

```
  Loglikelihood user model (H0)                -6260.310
  Loglikelihood unrestricted model (H1)        -6205.734

  Number of free parameters                           37
  Akaike (AIC)                                 12594.621
  Bayesian (BIC)                               12740.407
  Sample-size adjusted Bayesian (BIC)          12623.013

Root Mean Square Error of Approximation:

  RMSEA                                            0.029
  90 Percent Confidence Interval          0.010    0.043
  P-value RMSEA <= 0.05                            0.996

Standardized Root Mean Square Residual:

  SRMR                                             0.034

Parameter Estimates:

  Standard Errors                              Bootstrap
  Number of requested bootstrap draws                500
  Number of successful bootstrap draws               500

Latent Variables:
                 Estimate  Std.Err  z-value  P(>|z|)
```

Compare Standardized factor loadings ...

```
  Agency =~
    Agency1          0.904    0.039   23.380    0.000
    Agency2          0.932    0.039   23.715    0.000
    Agency3          0.910    0.040   22.895    0.000
  Intrinsic =~
    Intrin1          0.605    0.050   12.100    0.000
    Intrin2          0.587    0.041   14.319    0.000
    Intrin3          0.689    0.042   16.423    0.000
  Extrinsic =~
    Extrin1          0.777    0.046   16.848    0.000
    Extrin2          0.763    0.046   16.560    0.000
    Extrin3          0.795    0.053   15.124    0.000
  Positive =~
    PosAFF1          0.770    0.038   20.246    0.000
    PosAFF2          0.796    0.043   18.540    0.000
    PosAFF3          0.807    0.040   20.015    0.000
  Negative =~
    NegAFF1          0.869    0.055   15.779    0.000
    NegAFF2          0.864    0.057   15.247    0.000
    NegAFF3          0.897    0.051   17.522    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Positive ~
    Intrinsic (b1)   0.343    0.075    4.582    0.000
    Agency           0.129    0.072    1.777    0.076
```

Compare Standardized factor loadings ...

```
  Negative ~
    Extrinsic (b2)    0.199    0.070    2.862    0.004
  Intrinsic ~
    Agency    (a1)    0.588    0.085    6.916    0.000
  Extrinsic ~
    Agency    (a2)    0.278    0.064    4.354    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .Intrinsic ~~
   .Extrinsic        -0.202    0.065   -3.106    0.002
 .Positive ~~
   .Negative         -0.076    0.070   -1.092    0.275

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .Agency1           0.181    0.021    8.428    0.000
   .Agency2           0.129    0.017    7.707    0.000
   .Agency3           0.169    0.019    9.060    0.000
   .Intrin1           0.505    0.055    9.131    0.000
   .Intrin2           0.534    0.052   10.253    0.000
   .Intrin3           0.358    0.065    5.514    0.000
   .Extrin1           0.347    0.046    7.482    0.000
   .Extrin2           0.371    0.047    7.863    0.000
   .Extrin3           0.316    0.051    6.158    0.000
   .PosAFF1           0.271    0.036    7.458    0.000
```

Compare Standardized factor loadings ...

```
    .PosAFF2          0.221    0.033    6.719    0.000
    .PosAFF3          0.199    0.028    6.981    0.000
    .NegAFF1          0.210    0.031    6.799    0.000
    .NegAFF2          0.219    0.031    7.168    0.000
    .NegAFF3          0.157    0.026    5.937    0.000
     Agency           1.000
    .Intrinsic        1.000
    .Extrinsic        1.000
    .Positive         1.000
    .Negative         1.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
     ind1             0.202    0.043    4.700    0.000
     ind2             0.055    0.022    2.496    0.013
```

Method 3: Insert “standardized = TRUE” in the summary,

```
summary(mediat.out.2, standardized = TRUE)
```

See far-right column.

A shortcoming is that it does not give us a “model object,” which I want to make a nice table.

Compare Standardized factor loadings ...

```
summary(mediat.out.2, standardized = TRUE)
```

```
lavaan 0.6-3 ended normally after 58 iterations

  Optimization method                           NLMINB
  Number of free parameters                         37

  Number of observations                           380

  Estimator                                         ML
  Model Fit Test Statistic                     109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Parameter Estimates:

  Standard Errors                            Bootstrap
  Number of requested bootstrap draws              500
  Number of successful bootstrap draws             500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.466    0.021   22.563    0.000    0.466    0.905
```

Compare Standardized factor loadings ...

```
    Agency2           0.492    0.022   22.567    0.000    0.492    0.933
    Agency3           0.497    0.024   20.885    0.000    0.497    0.912
  Intrinsic =~
    Intrin1           0.466    0.039   11.831    0.000    0.540    0.703
    Intrin2           0.500    0.035   14.134    0.000    0.580    0.682
    Intrin3           0.531    0.032   16.501    0.000    0.616    0.800
  Extrinsic =~
    Extrin1           0.373    0.023   16.125    0.000    0.388    0.808
    Extrin2           0.440    0.028   15.701    0.000    0.457    0.793
    Extrin3           0.454    0.030   15.022    0.000    0.471    0.827
  Positive =~
    PosAFF1           0.514    0.026   19.694    0.000    0.569    0.854
    PosAFF2           0.545    0.029   19.014    0.000    0.604    0.882
    PosAFF3           0.570    0.028   20.177    0.000    0.632    0.895
  Negative =~
    NegAFF1           0.620    0.040   15.591    0.000    0.633    0.889
    NegAFF2           0.573    0.038   15.159    0.000    0.585    0.883
    NegAFF3           0.586    0.032   18.293    0.000    0.598    0.918

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)    0.343    0.077    4.432    0.000    0.359    0.359
    Agency            0.129    0.072    1.784    0.074    0.116    0.116
  Negative ~
    Extrinsic (b2)    0.199    0.069    2.868    0.004    0.202    0.202
```

Compare Standardized factor loadings ...

```
  Intrinsic ~
    Agency     (a1)    0.588    0.085    6.930    0.000    0.507    0.507
  Extrinsic ~
    Agency     (a2)    0.278    0.063    4.415    0.000    0.268    0.268

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.061   -3.283    0.001   -0.202   -0.202
 .Positive ~~
   .Negative         -0.076    0.073   -1.051    0.293   -0.076   -0.076

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.005    8.914    0.000    0.048    0.181
   .Agency2           0.036    0.005    7.726    0.000    0.036    0.129
   .Agency3           0.050    0.005    9.207    0.000    0.050    0.169
   .Intrin1           0.299    0.033    9.179    0.000    0.299    0.506
   .Intrin2           0.387    0.040    9.796    0.000    0.387    0.535
   .Intrin3           0.213    0.036    5.836    0.000    0.213    0.359
   .Extrin1           0.080    0.011    7.004    0.000    0.080    0.348
   .Extrin2           0.123    0.016    7.734    0.000    0.123    0.372
   .Extrin3           0.103    0.016    6.253    0.000    0.103    0.317
   .PosAFF1           0.121    0.017    7.301    0.000    0.121    0.271
   .PosAFF2           0.104    0.016    6.536    0.000    0.104    0.222
   .PosAFF3           0.099    0.015    6.824    0.000    0.099    0.199
```

Compare Standardized factor loadings ...

```
   .NegAFF1         0.107    0.016    6.571    0.000    0.107    0.210
   .NegAFF2         0.096    0.013    7.157    0.000    0.096    0.219
   .NegAFF3         0.067    0.011    6.104    0.000    0.067    0.158
    Agency          1.000                               1.000    1.000
   .Intrinsic       1.000                               0.743    0.743
   .Extrinsic       1.000                               0.928    0.928
   .Positive        1.000                               0.815    0.815
   .Negative        1.000                               0.959    0.959

Defined Parameters:
                 Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ind1            0.202    0.049    4.092    0.000    0.182    0.182
    ind2            0.055    0.022    2.509    0.012    0.054    0.054
```

Why did we bother to standardize?

- We went into the standardization exercise because one of the GRAs thought that there were “weak” loadings that complicate the estimation of the mediator
  - Possible that measurement error variance translates into low standardized factor loadings
- Perhaps difficulty in latent regressions is due to simple measurement errors in the observed outcomes

Compare fits for standardized and non-standardized

```
mediat.out.2.std = update(mediat.out.2, std.lv =
   TRUE, std.ov = TRUE, meanstructure=TRUE)
semTable(list("Standardized" = mediat.out.2.std,
   "Not Standardized" = mediat.out.2),
 longtable = TRUE, columns = c("estsestars"),
    varLabels = vl)
```

	Standardized	Not Standardized
	Estimate(Std.Err.)	Estimate(Std.Err.)

Compare fits for standardized and non-standardized ...

		Factor Loadings	
Agency			
	Agency1	0.90(0.04)***	0.47(0.02)***
	Agency2	0.93(0.04)***	0.49(0.02)***
	Agency3	0.91(0.04)***	0.50(0.02)***
Intrinsic			
	Intrinsic1	0.61(0.05)***	0.47(0.04)***
	Intrinsic2	0.59(0.04)***	0.50(0.04)***
	Intrinsic3	0.69(0.04)***	0.53(0.03)***
Extrinsic			
	Extrinsic1	0.78(0.05)***	0.37(0.02)***
	Extrinsic2	0.76(0.05)***	0.44(0.03)***
	Extrinsic3	0.80(0.05)***	0.45(0.03)***
Positive			
	Positive1	0.77(0.04)***	0.51(0.03)***
	Positive2	0.80(0.04)***	0.54(0.03)***

Compare fits for standardized and non-standardized ...

	Positive3	0.81(0.04)***	0.57(0.03)***
Negative			
	Negative1	0.87(0.05)***	0.62(0.04)***
	Negative2	0.86(0.06)***	0.57(0.04)***
	Negative3	0.90(0.05)***	0.59(0.03)***
		Regression Slopes	
Positive			
	Intrinsic	0.34(0.08)***	0.34(0.08)***
	Agency	0.13(0.07)	0.13(0.07)
Negative			
	Extrinsic	0.20(0.07)**	0.20(0.07)**
Intrinsic			
	Agency	0.59(0.09)***	0.59(0.08)***
Extrinsic			
	Agency	0.28(0.06)***	0.28(0.06)***
		Intercepts	
	Agency1	0.00(0.05)	

Compare fits for standardized and non-standardized ...

Agency2	0.00(0.05)	
Agency3	0.00(0.05)	
Intrinsic1	0.00(0.05)	
Intrinsic2	0.00(0.05)	
Intrinsic3	0.00(0.05)	
Extrinsic1	0.00(0.05)	
Extrinsic2	0.00(0.05)	
Extrinsic3	0.00(0.05)	
Positive1	0.00(0.05)	
Positive2	0.00(0.05)	
Positive3	0.00(0.05)	
Negative1	0.00(0.05)	
Negative2	0.00(0.05)	
Negative3	0.00(0.05)	
	Residual Variances	
Agency1	0.18(0.02)***	0.05(0.01)***
Agency2	0.13(0.02)***	0.04(0.00)***

Compare fits for standardized and non-standardized ...

Agency3	0.17(0.02)***	0.05(0.01)***
Intrinsic1	0.50(0.05)***	0.30(0.03)***
Intrinsic2	0.53(0.05)***	0.39(0.04)***
Intrinsic3	0.36(0.06)***	0.21(0.04)***
Extrinsic1	0.35(0.05)***	0.08(0.01)***
Extrinsic2	0.37(0.05)***	0.12(0.02)***
Extrinsic3	0.32(0.05)***	0.10(0.02)***
Positive1	0.27(0.03)***	0.12(0.02)***
Positive2	0.22(0.03)***	0.10(0.02)***
Positive3	0.20(0.03)***	0.10(0.01)***
Negative1	0.21(0.03)***	0.11(0.02)***
Negative2	0.22(0.03)***	0.10(0.01)***
Negative3	0.16(0.02)***	0.07(0.01)***
	Latent Intercepts	
Agency	0.00+	
Intrinsic	0.00+	
Extrinsic	0.00+	

Compare fits for standardized and non-standardized ...

Positive	0.00^{+}	
Negative	0.00^{+}	
	Latent Variances	
Agency	1.00^{+}	1.00^{+}
Intrinsic	1.00^{+}	1.00^{+}
Extrinsic	1.00^{+}	1.00^{+}
Positive	1.00^{+}	1.00^{+}
Negative	1.00^{+}	1.00^{+}
	Latent Covariances	
Intrinsic w/Extrinsic	$-0.20(0.06)^{***}$	$-0.20(0.06)^{**}$
Positive w/Negative	$-0.08(0.07)$	$-0.08(0.07)$
	Constructed	
ind1	$0.20(0.05)^{***}$	$0.20(0.05)^{***}$
ind2	$0.06(0.02)^{*}$	$0.06(0.02)^{*}$
	Fit Indices	
$\chi^2(\mathrm{df})$	$109.15(83)^{*}$	$109.15(83)^{*}$
CFI	0.99	0.99

Compare fits for standardized and non-standardized ...

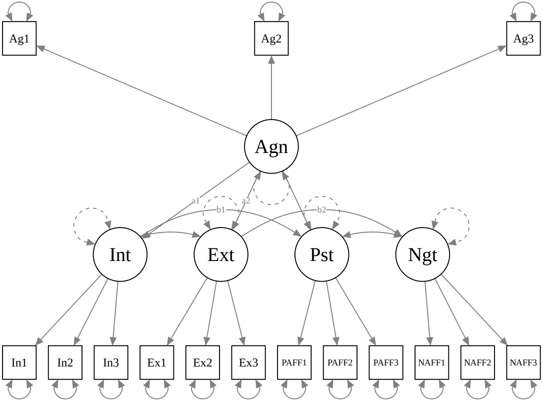
TLI	0.99	0.99
RMSEA	0.03	0.03

+Fixed parameter

* p<0.05, ** p<0.01, ***p<0.001

plot

```
semPaths(mediat.out.2)
```



Check if c1 and c2 are signficantly different

- Did you wonder if c1 is significantly different from c2?
- The assumption that they are the same is another restriction we can put on the model.
- We re-fit, and then run the anova test.

```
# if nesting is not specified, the diff is not
   reported
compareLavaan(list("CFA" = cfa3, "SEM" =
   mediat.out.2), nesting = "CFA > SEM", fitmeas
   = c("chisq", "df", "pvalue", "rmsea", "cfi",
   "bic"),  type="latex")
```

	χ^2	df	*p*-value	rmsea	cfi	bic	$\Delta\chi^2$	Δdf	*p*
CFA	106.85	80.00	0.02	0.03	0.99	7635.83	-	-	-
SEM	109.15	83.00	0.03	0.03	0.99	7620.31	2.305a	3	0.5

a = SEM vs CFA

Check if c1 and c2 are signficantly different ...

```
mediat.mod.3 <- '
 ## measurement model
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 ## structural model
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
 Intrinsic ~ a1*Agency
 Extrinsic ~ a2*Agency
 Intrinsic ~~ Extrinsic
 ## define mediation parameters (indirect effects)
 c1 := a1 * b1
 c2 := a2 * b2
 c1 == c2
```

Check if c1 and c2 are signficantly different ...

```
  ,
```

```
# if nesting is not specified, the diff is not
    reported
compareLavaan(list("CFA" = cfa3, "SEM" =
    mediat.out.2), nesting = "CFA > SEM", fitmeas
    = c("chisq", "df", "pvalue", "rmsea", "cfi",
    "bic"),  type="latex")
```

	χ^2	df	*p*-value	rmsea	cfi	bic	$\Delta\chi^2$	Δdf	*p*
CFA	106.85	80.00	0.02	0.03	0.99	7635.83	-	-	-
SEM	109.15	83.00	0.03	0.03	0.99	7620.31	2.305a	3	0.5[illegible]

a = SEM vs CFA

Check if c1 and c2 are signficantly different ...

```
mediat.mod.3 <- '
 ## measurement model
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 ## structural model
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
 Intrinsic ~ a1*Agency
 Extrinsic ~ a2*Agency
 Intrinsic ~~ Extrinsic
 ## define mediation parameters (indirect effects)
 c1 := a1 * b1
 c2 := a2 * b2
 c1 == c2
```

Check if c1 and c2 are signficantly different ...

```
  '
```

```
Nboot <- 500
mediat.out.3 <- sem(mediat.mod.3, data = dat,
                    std.lv = TRUE, se = "boot",
                       bootstrap = Nboot)
summary(mediat.out.3, standardized = TRUE, fit =
   TRUE)
```

```
lavaan 0.6-3 ended normally after 461 iterations

  Optimization method                           NLMINB
  Number of free parameters                         37

  Number of observations                           380

  Estimator                                         ML
  Model Fit Test Statistic                     118.955
  Degrees of freedom                                84
  P-value (Chi-square)                           0.007

```

Check if c1 and c2 are signficantly different ...

```
Model test baseline model:

  Minimum Function Test Statistic                 3749.411
  Degrees of freedom                                   105
  P-value                                            0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                        0.990
  Tucker-Lewis Index (TLI)                           0.988

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)                  -3705.163
  Loglikelihood unrestricted model (H1)          -3645.686

  Number of free parameters                             36
  Akaike (AIC)                                    7482.326
  Bayesian (BIC)                                  7624.172
  Sample-size adjusted Bayesian (BIC)             7509.951

Root Mean Square Error of Approximation:

  RMSEA                                              0.033
  90 Percent Confidence Interval          0.018      0.046
  P-value RMSEA <= 0.05                              0.986
```

Check if c1 and c2 are signficantly different ...

```
Standardized Root Mean Square Residual:

  SRMR                                           0.051

Parameter Estimates:

  Standard Errors                            Bootstrap
  Number of requested bootstrap draws              500
  Number of successful bootstrap draws             500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.467    0.021   22.277    0.000    0.467    0.905
    Agency2           0.491    0.021   23.539    0.000    0.491    0.933
    Agency3           0.497    0.023   21.758    0.000    0.497    0.911
  Intrinsic =~
    Intrin1           0.468    0.038   12.199    0.000    0.528    0.693
    Intrin2           0.508    0.037   13.809    0.000    0.573    0.679
    Intrin3           0.540    0.032   17.093    0.000    0.609    0.801
  Extrinsic =~
    Extrin1           0.373    0.023   16.540    0.000    0.395    0.813
    Extrin2           0.439    0.029   15.055    0.000    0.465    0.798
    Extrin3           0.452    0.028   15.863    0.000    0.478    0.829
  Positive =~
```

Check if c1 and c2 are signficantly different ...

```
    PosAFF1            0.530    0.024   22.141    0.000    0.564    0.853
    PosAFF2            0.562    0.028   20.274    0.000    0.598    0.881
    PosAFF3            0.587    0.028   21.227    0.000    0.624    0.892
  Negative =~
    NegAFF1            0.614    0.037   16.448    0.000    0.639    0.890
    NegAFF2            0.567    0.038   14.815    0.000    0.590    0.885
    NegAFF3            0.579    0.032   18.377    0.000    0.603    0.919

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)     0.183    0.074    2.485    0.013    0.194    0.194
    Agency             0.218    0.074    2.952    0.003    0.205    0.205
  Negative ~
    Extrinsic (b2)     0.275    0.058    4.745    0.000    0.280    0.280
  Intrinsic ~
    Agency    (a1)     0.523    0.102    5.147    0.000    0.463    0.463
  Extrinsic ~
    Agency    (a2)     0.348    0.066    5.263    0.000    0.328    0.328

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic         -0.202    0.060   -3.388    0.001   -0.202   -0.202
 .Positive ~~
   .Negative          -0.063    0.069   -0.918    0.359   -0.063   -0.063
```

Check if c1 and c2 are signficantly different ...

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.005    8.896    0.000    0.048    0.181
   .Agency2           0.036    0.004    8.028    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.848    0.000    0.050    0.170
   .Intrin1           0.301    0.034    8.886    0.000    0.301    0.519
   .Intrin2           0.383    0.039    9.771    0.000    0.383    0.539
   .Intrin3           0.208    0.038    5.498    0.000    0.208    0.359
   .Extrin1           0.080    0.010    7.751    0.000    0.080    0.339
   .Extrin2           0.123    0.016    7.838    0.000    0.123    0.363
   .Extrin3           0.104    0.017    6.241    0.000    0.104    0.312
   .PosAFF1           0.120    0.015    8.068    0.000    0.120    0.273
   .PosAFF2           0.103    0.016    6.654    0.000    0.103    0.224
   .PosAFF3           0.100    0.014    7.103    0.000    0.100    0.205
   .NegAFF1           0.107    0.016    6.578    0.000    0.107    0.207
   .NegAFF2           0.096    0.014    7.051    0.000    0.096    0.216
   .NegAFF3           0.067    0.011    6.361    0.000    0.067    0.156
    Agency            1.000                               1.000    1.000
   .Intrinsic         1.000                               0.785    0.785
   .Extrinsic         1.000                               0.892    0.892
   .Positive          1.000                               0.883    0.883
   .Negative          1.000                               0.922    0.922

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

Check if c1 and c2 are signficantly different ...

```
    c1                0.096    0.026    3.634    0.000    0.090    0.090
    c2                0.096    0.026    3.634    0.000    0.092    0.092

Constraints:
                                             |Slack|
    c1 - (c2)                                  0.000
```

```
Nboot <- 500
mediat.out.3 <- sem(mediat.mod.3, data = dat,
                    std.lv = TRUE, se = "boot",
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summary(mediat.out.3, standardized = TRUE, fit =
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  Optimization method                           NLMINB
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```

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  P-value RMSEA <= 0.05                         0.986

Standardized Root Mean Square Residual:

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  Number of successful bootstrap draws            500

Latent Variables:
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Check if c1 and c2 are signficantly different ...

```
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    Agency3           0.497    0.023   21.758    0.000    0.497    0.911
  Intrinsic =~
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    Extrin1           0.373    0.023   16.540    0.000    0.395    0.813
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    Extrin3           0.452    0.028   15.863    0.000    0.478    0.829
  Positive =~
    PosAFF1           0.530    0.024   22.141    0.000    0.564    0.853
    PosAFF2           0.562    0.028   20.274    0.000    0.598    0.881
    PosAFF3           0.587    0.028   21.227    0.000    0.624    0.892
  Negative =~
    NegAFF1           0.614    0.037   16.448    0.000    0.639    0.890
    NegAFF2           0.567    0.038   14.815    0.000    0.590    0.885
    NegAFF3           0.579    0.032   18.377    0.000    0.603    0.919

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)    0.183    0.074    2.485    0.013    0.194    0.194
    Agency            0.218    0.074    2.952    0.003    0.205    0.205
```

Check if c1 and c2 are signficantly different ...

```
  Negative ~
    Extrinsic (b2)    0.275    0.058    4.745    0.000    0.280    0.280
  Intrinsic ~
    Agency    (a1)    0.523    0.102    5.147    0.000    0.463    0.463
  Extrinsic ~
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Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.060   -3.388    0.001   -0.202   -0.202
 .Positive ~~
   .Negative         -0.063    0.069   -0.918    0.359   -0.063   -0.063

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.005    8.896    0.000    0.048    0.181
   .Agency2           0.036    0.004    8.028    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.848    0.000    0.050    0.170
   .Intrin1           0.301    0.034    8.886    0.000    0.301    0.519
   .Intrin2           0.383    0.039    9.771    0.000    0.383    0.539
   .Intrin3           0.208    0.038    5.498    0.000    0.208    0.359
   .Extrin1           0.080    0.010    7.751    0.000    0.080    0.339
   .Extrin2           0.123    0.016    7.838    0.000    0.123    0.363
   .Extrin3           0.104    0.017    6.241    0.000    0.104    0.312
   .PosAFF1           0.120    0.015    8.068    0.000    0.120    0.273
```

Check if c1 and c2 are signficantly different ...

```
    .PosAFF2          0.103    0.016    6.654    0.000    0.103    0.224
    .PosAFF3          0.100    0.014    7.103    0.000    0.100    0.205
    .NegAFF1          0.107    0.016    6.578    0.000    0.107    0.207
    .NegAFF2          0.096    0.014    7.051    0.000    0.096    0.216
    .NegAFF3          0.067    0.011    6.361    0.000    0.067    0.156
     Agency           1.000                               1.000    1.000
    .Intrinsic        1.000                               0.785    0.785
    .Extrinsic        1.000                               0.892    0.892
    .Positive         1.000                               0.883    0.883
    .Negative         1.000                               0.922    0.922

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    c1                0.096    0.026    3.634    0.000    0.090    0.090
    c2                0.096    0.026    3.634    0.000    0.092    0.092

Constraints:
                                               |Slack|
    c1 - (c2)                                    0.000
```

```
anova(mediat.out.3, mediat.out.2)
```

Check if c1 and c2 are signficantly different ...

```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
mediat.out.2 83 7474.5 7620.3 109.15
mediat.out.3 84 7482.3 7624.2 118.95     9.8021       1   0.001743 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(mediat.out.3, mediat.out.2)
```


```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
mediat.out.2 83 7474.5 7620.3 109.15
mediat.out.3 84 7482.3 7624.2 118.95     9.8021       1   0.001743 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Mediation is

- Mediation is a comparison of *direct* versus *indirect effects*
- SEM are help in mediation analysis because the integrate
  - analysis of several equations with correlated errors
  - allow for hypothesis testing (bootstrap).
- The "gold standard" software for SEM modeling has been Mplus.
- However, as we see in this example, `lavaan` is completely capable of doing the necessary calculations.

References

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria.

Session

```
sessionInfo()
```

```
R version 3.6.0 (2019-04-26)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 19.04

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/atlas/libblas.so.3.10.3
LAPACK: /usr/lib/x86_64-linux-gnu/atlas/liblapack.so.3.10.3

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
     LC_TIME=en_US.UTF-8
 [4] LC_COLLATE=en_US.UTF-8     LC_MONETARY=en_US.UTF-8
     LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C                  LC_ADDRESS=C
[10] LC_TELEPHONE=C             LC_MEASUREMENT=en_US.UTF-8
    LC_IDENTIFICATION=C

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
[1] semTable_1.4  semPlot_1.1.1 lavaan_0.6-3
```

Session ...

```
loaded via a namespace (and not attached):
 [1] nlme_3.1-140        stationery_0.98.6   RColorBrewer_1.1-2  mi_1.0
 [5] tools_3.6.0         backports_1.1.4     R6_2.4.0
      rpart_4.1-15
 [9] d3Network_0.5.2.1   Hmisc_4.2-0         lazyeval_0.2.2
      colorspace_1.4-1
[13] nnet_7.3-12         tidyselect_0.2.5    gridExtra_2.3
     mnormt_1.5-5
[17] compiler_3.6.0      qgraph_1.6.2        fdrtool_1.2.15
     htmlTable_1.13.1
[21] regsem_1.3.9        scales_1.0.0        checkmate_1.9.3
     psych_1.8.12
[25] pbapply_1.4-0       sem_3.1-9           stringr_1.4.0
     digest_0.6.18
[29] pbivnorm_0.6.0      foreign_0.8-71      minqa_1.2.4
     rmarkdown_1.12
[33] base64enc_0.1-3     jpeg_0.1-8          pkgconfig_2.0.2
     htmltools_0.3.6
[37] lme4_1.1-21         lisrelToR_0.1.4     htmlwidgets_1.3
     rlang_0.3.4
[41] rstudioapi_0.10     huge_1.3.2          gtools_3.8.1
     acepack_1.4.1
[45] dplyr_0.8.1         zip_2.0.2           magrittr_1.5
     OpenMx_2.13.2
```

Session ...

```
[49] Formula_1.2-3        Matrix_1.2-17        Rcpp_1.0.1
     munsell_0.5.0
[53] abind_1.4-5          rockchalk_1.8.144    stringi_1.4.3
     whisker_0.3-2
[57] carData_3.0-2        MASS_7.3-51.4        plyr_1.8.4
     matrixcalc_1.0-3
[61] grid_3.6.0           parallel_3.6.0       crayon_1.3.4
     lattice_0.20-38
[65] kutils_1.69          splines_3.6.0        knitr_1.22
     pillar_1.4.0
[69] igraph_1.2.4.1       boot_1.3-22          rjson_0.2.20
     corpcor_1.6.9
[73] BDgraph_2.59         reshape2_1.4.3       stats4_3.6.0
     XML_3.98-1.19
[77] glue_1.3.1           evaluate_0.13        latticeExtra_0.6-28
     data.table_1.12.2
[81] png_0.1-7            nloptr_1.2.1         gtable_0.3.0
     purrr_0.3.2
[85] assertthat_0.2.1     ggplot2_3.1.1        xfun_0.7
     openxlsx_4.1.0
[89] xtable_1.8-4         semTools_0.5-1       coda_0.19-2
     Rsolnp_1.16
[93] survival_2.44-1.1    glasso_1.10          truncnorm_1.0-8
     tibble_2.1.1
[97] arm_1.10-1           ggm_2.3              cluster_2.0.9
```