CRMDA semexamples in Mplus

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Outline

The Repository

- CRMDA created an archive of SEM working examples.
- To the extent possible, the same models are estimated with
  - Mplus
  - R (R Core Team, 2017)
  - Stata
- The repository can be browsed/downloaded: https://gitlab.crmda.ku.edu/crmda/semexample
- The data folder includes the information that is imported into each of the 3 programs.

Runable examples

1. Exploratory Factor Analysis (EFA)
2. Confirmatory Factor Analysis (CFA)
3. Measurement Invariance (Multi-group CFA)
4. Multiple Regression
5. Structural Equation Model (SEM)
6. Latent Growth Curve (LGC)
7. Modeling strategy changes for Ordinal Data

Outline

The job_placement data set

- We obtained data from job seekers (job_placement.csv)
- Six test score variables that we think could measure latent variables
- Work to do
  1. Identify a factor structure
  2. Determine how our latent variables relate to demographic variables

Excelsior Spring Job Corps center

Items collected from job seekers using the Excelsior Spring Job Corps center

- Variables

  - **id:** Subject identification number
  - **wjcalc:** Subject's score on the WJ calculation subtest. (Numeric)
  - **wjspl:** Subject's score on the WJ spelling subtest. (Numeric)
  - **wratcalc:** Subject's score on the WRAT calculation subtest. (Numeric)
  - **wratspl:** Subject's score on the WRAT spelling subtest. (Numeric)
  - **waiscalc:** Subject's score on the WAIS arithmetic calculations subtest. (Numeric)
  - **waisspl:** Subject's score on the WAIS spelling subtest. (Numeric)
  - **edlevel:** What is the highest level of education completed by the subject? (Ordinal)
  - **newschl:** Did the subject ever change high schools? (Binary: 1=Yes, 0=No)

Excelsior Spring Job Corps center ...

suspend: Has the subject ever been suspended from high school? (Binary: 1=Yes, 0=No)

expelled: Has the subject ever been expelled from high school? (Binary: 1=Yes, 0=No)

haveld: Has the subject been diagnosed with a learning disorder? (Binary: 1=Yes, 0=No)

female: Gender (Binary: 1=Female, 0=Male)

age: Age in years (Numeric)

- Missing data are coded as "99999"

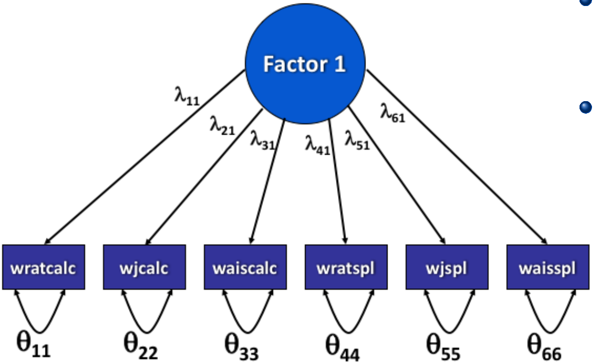
EFA: Questions

- How many latent variables underlie subtest scores (6 items)?
  - Will decide by running various models and comparing the results.
- Mplus can quickly run many EFAs and report the results.
  - Unlike SPSS it does not default to principle components analysis.

The main idea of EFA

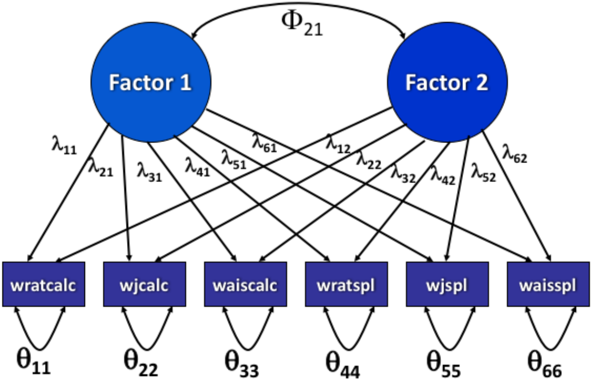
- EFA lets us specify how many factors will be estimated.
- Then it will estimate a loading for *each and every* observed variable on *each* factor.
- Since the 1970s, EFA has been regarded with suspicion by many because it seems to be "atheoretic data-grubbing".
- Nevertheless, we persist in showing you how to do it.

EFA: One Factor



- λ_{11} is the loading of wrtcalc on Factor 1
- θ_{11} is variance of the residual (unexplained part) of wrtcalc
- if θ_{11} were 0, it would mean that wrtcalc is a perfectly accurate reflection of Factor 1.

EFA: Two Factors



- Φ_{21} is covariance between Factor 1 and 2.
- If $\Phi_{21} = 0$, then the factors are uncorrelated ("orthogonal")

EFA in Mplus

```
TITLE:
Example 1 - Exploratory Factor Analysis
DATA:
FILE IS "../../data/job_placement.csv";
VARIABLE:
NAMES ARE
id wjcalc wjspl wratspl wratcalc waiscalc waisspl
edlevel newschl suspend expelled haveld female age;
USEVARIABLES ARE
wjcalc wjspl wratspl wratcalc waiscalc waisspl;
MISSING ARE all (99999);
ANALYSIS: TYPE = EFA 1 2;
!Default estimator is ML;
!Default rotation is Oblique Geomin;
```

EFA: Beyond Defaults

```
ANALYSIS: TYPE = EFA 1 2;
!Default estimator is ML;
!Default rotation is
   Oblique Geomin;
ESTIMATOR = ML;
 MLM;
 MLMV
 MLR;
 MLF;
 MUML;
 WLS;
 WLSM;
 WLSMV;
 ULS;
 ULSMV;
```

```
ROTATION = GEOMIN;
GEOMIN (OBLIQUE);
GEOMIN (ORTHOGONAL);
QUARTIMIN; OBLIQUE
CF-VARIMAX
CF-QUARTIMAX;
CF-EQUAMAX;
CF-PARSIMAX;
CF-FACPARSIM;
CRAWFER; OBLIQUE 1/p
CRAWFER (OBLIQUE or
   ORTHOGONAL value);
OBLIMIN; OBLIQUE 0
OBLIMIN (OBLIQUE or
   ORTHOGONAL value);
VARIMAX;
PROMAX;
```

EFA Results

There are several ways you can access the results.

1. In the semexample/Mplus folders
2. View online: efa-01.out
3. We will import the verbatim output
   Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:31 PM

INPUT INSTRUCTIONS


  TITLE:
  Example 1 - Exploratory Factor Analysis

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
```

EFA Results ...

```
    edlevel newschl suspend expelled haveld female age;

  USEVARIABLES ARE
  wjcalc wjspl wratspl wratcalc waiscalc waisspl;

  MISSING ARE all(99999);

  ANALYSIS:
    TYPE = EFA 1 2;



INPUT READING TERMINATED NORMALLY




Example 1 - Exploratory Factor Analysis

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         322

Number of dependent variables                                    6
Number of independent variables                                  0
```

EFA Results ...

```
Number of continuous latent variables                            0

Observed dependent variables

  Continuous
   WJCALC      WJSPL       WRATSPL     WRATCALC    WAISCALC    WAISSPL


Estimator                                                       ML
Rotation                                                    GEOMIN
Row standardization                                    CORRELATION
Type of rotation                                           OBLIQUE
Epsilon value                                               Varies
Information matrix                                        OBSERVED
Maximum number of iterations                                  1000
Convergence criterion                                    0.500D-04
Maximum number of steepest descent iterations                   20
Maximum number of iterations for H1                           2000
Convergence criterion for H1                             0.100D-03
Optimization Specifications for the Exploratory Factor Analysis
Rotation Algorithm
  Number of random starts                                       30
  Maximum number of iterations                               10000
  Derivative convergence criterion                       0.100D-04

Input data file(s)
```

EFA Results ...

```
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT


           Covariance Coverage
              WJCALC        WJSPL         WRATSPL       WRATCALC      WAISCALC
              ________      ________      ________      ________      ________
 WJCALC         0.994
 WJSPL          0.994         1.000
 WRATSPL        0.991         0.997         0.997
 WRATCALC       0.994         1.000         0.997         1.000
 WAISCALC       0.988         0.994         0.991         0.994         0.994
```

EFA Results ...

```
WAISSPL       0.988         0.994         0.991         0.994         0.994


          Covariance Coverage
             WAISSPL
             ________
WAISSPL        0.994



SUMMARY OF MODEL FIT INFORMATION


                    Number of                      Degrees of
     Model          Parameters      Chi-Square     Freedom      P-Value

     1-factor           18             476.495          9        0.0000
     2-factor           23               3.690          4        0.4496

                                                   Degrees of
     Models Compared                Chi-Square     Freedom      P-Value

     1-factor against 2-factor         472.805          5        0.0000
```

EFA Results ...

```
RESULTS FOR EXPLORATORY FACTOR ANALYSIS


           EIGENVALUES FOR SAMPLE CORRELATION MATRIX
                  1             2             3             4             5
              ________      ________      ________      ________      ________
      1         4.041         1.219         0.397         0.142         0.107


           EIGENVALUES FOR SAMPLE CORRELATION MATRIX
                  6
              ________
      1         0.095


EXPLORATORY FACTOR ANALYSIS WITH 1 FACTOR(S):


MODEL FIT INFORMATION

Number of Free Parameters                       18

Loglikelihood

          H0 Value                       -5361.307
          H1 Value                       -5123.060
```

EFA Results ...

```
Information Criteria

          Akaike (AIC)                    10758.615
          Bayesian (BIC)                  10826.557
          Sample-Size Adjusted BIC        10769.463
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             476.495
          Degrees of Freedom                      9
          P-Value                            0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.402
          90 Percent C.I.                     0.371  0.433
          Probability RMSEA <= .05            0.000

CFI/TLI

          CFI                                 0.750
          TLI                                 0.583

Chi-Square Test of Model Fit for the Baseline Model
```

EFA Results ...

```
          Value                              1882.335
          Degrees of Freedom                       15
          P-Value                              0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                 0.156



MINIMUM ROTATION FUNCTION VALUE       3.49664



           GEOMIN ROTATED LOADINGS (* significant at 5% level)
                  1
              --------
 WJCALC         0.516*
 WJSPL          0.953*
 WRATSPL        0.944*
 WRATCALC       0.545*
 WAISCALC       0.500*
 WAISSPL        0.940*
```

EFA Results ...

```
        GEOMIN FACTOR CORRELATIONS (* significant at 5% level)
               1
           ________
 1            1.000


        ESTIMATED RESIDUAL VARIANCES
           WJCALC        WJSPL         WRATSPL       WRATCALC      WAISCALC
           ________      ________      ________      ________      ________
 1            0.734         0.092         0.108         0.703         0.750


        ESTIMATED RESIDUAL VARIANCES
           WAISSPL
           ________
 1            0.117


        S.E. GEOMIN ROTATED LOADINGS
               1
           ________
 WJCALC       0.042
 WJSPL        0.007
 WRATSPL      0.008
 WRATCALC     0.040
 WAISCALC     0.043
```

EFA Results ...

```
WAISSPL        0.008


          S.E. GEOMIN FACTOR CORRELATIONS
                 1
              ________
     1           0.000


          S.E. ESTIMATED RESIDUAL VARIANCES
             WJCALC        WJSPL         WRATSPL       WRATCALC      WAISCALC
              ________      ________      ________      ________      ________
     1           0.043         0.014         0.015         0.044         0.043


          S.E. ESTIMATED RESIDUAL VARIANCES
             WAISSPL
              ________
     1           0.015


          Est./S.E. GEOMIN ROTATED LOADINGS
                 1
              ________
WJCALC          12.251
WJSPL          133.177
```

EFA Results ...

```
 WRATSPL      120.556
 WRATCALC      13.487
 WAISCALC      11.620
 WAISSPL      114.214


           Est./S.E. GEOMIN FACTOR CORRELATIONS
                  1
              ________
      1         0.000


           Est./S.E. ESTIMATED RESIDUAL VARIANCES
              WJCALC        WJSPL         WRATSPL       WRATCALC      WAISCALC
              ________      ________      ________      ________      ________
      1        16.879         6.759         7.298        15.974        17.427


           Est./S.E. ESTIMATED RESIDUAL VARIANCES
              WAISSPL
              ________
      1         7.564


EXPLORATORY FACTOR ANALYSIS WITH 2 FACTOR(S):
```

EFA Results ...

```
MODEL FIT INFORMATION

Number of Free Parameters                       23

Loglikelihood

          H0 Value                       -5124.905
          H1 Value                       -5123.060

Information Criteria

          Akaike (AIC)                   10295.810
          Bayesian (BIC)                 10382.625
          Sample-Size Adjusted BIC       10309.672
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              3.690
          Degrees of Freedom                     4
          P-Value                           0.4496

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
```

EFA Results ...

```
          90 Percent C.I.                    0.000  0.081
          Probability RMSEA <= .05           0.763

CFI/TLI

          CFI                                1.000
          TLI                                1.001

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1882.335
          Degrees of Freedom                    15
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.003



MINIMUM ROTATION FUNCTION VALUE       0.12976



           GEOMIN ROTATED LOADINGS (* significant at 5% level)
                  1             2
```

EFA Results ...

```
              --------      --------
WJCALC         0.908*       -0.005
WJSPL         -0.001         0.955*
WRATSPL        0.008         0.941*
WRATCALC       0.950*        0.000
WAISCALC       0.653*        0.118*
WAISSPL       -0.006         0.945*


          GEOMIN FACTOR CORRELATIONS (* significant at 5% level)
                 1             2
              --------      --------
     1         1.000
     2         0.544*        1.000


          ESTIMATED RESIDUAL VARIANCES
             WJCALC        WJSPL         WRATSPL       WRATCALC      WAISCALC
              --------      --------      --------      --------      --------
     1         0.182         0.089         0.107         0.097         0.476


          ESTIMATED RESIDUAL VARIANCES
             WAISSPL
              --------
     1         0.113
```

EFA Results ...

```
          S.E. GEOMIN ROTATED LOADINGS
                 1              2
              ________       ________
 WJCALC         0.029          0.027
 WJSPL          0.014          0.011
 WRATSPL        0.022          0.015
 WRATCALC       0.024          0.017
 WAISCALC       0.044          0.050
 WAISSPL        0.020          0.014


          S.E. GEOMIN FACTOR CORRELATIONS
                 1              2
              ________       ________
      1         0.000
      2         0.043          0.000


          S.E. ESTIMATED RESIDUAL VARIANCES
              WJCALC         WJSPL          WRATSPL        WRATCALC       WAISCALC
              ________       ________       ________       ________       ________
      1         0.034          0.014          0.015          0.033          0.041
```

EFA Results ...

```
          S.E. ESTIMATED RESIDUAL VARIANCES
             WAISSPL
             --------
     1          0.015


          Est./S.E. GEOMIN ROTATED LOADINGS
                 1              2
             --------       --------
 WJCALC        31.690         -0.200
 WJSPL         -0.052         83.613
 WRATSPL        0.367         60.996
 WRATCALC      40.076          0.013
 WAISCALC      15.009          2.348
 WAISSPL       -0.287         65.440


          Est./S.E. GEOMIN FACTOR CORRELATIONS
                 1              2
             --------       --------
     1          0.000
     2         12.796          0.000


          Est./S.E. ESTIMATED RESIDUAL VARIANCES
             WJCALC        WJSPL         WRATSPL       WRATCALC      WAISCALC
```

EFA Results ...

```
             --------      --------      --------      --------      --------
 1              5.298         6.407         7.148         2.916        11.497


           Est./S.E. ESTIMATED RESIDUAL VARIANCES
              WAISSPL
              --------
 1              7.308


           FACTOR STRUCTURE
                 1             2
              --------      --------
 WJCALC         0.905         0.488
 WJSPL          0.519         0.955
 WRATSPL        0.520         0.945
 WRATCALC       0.950         0.517
 WAISCALC       0.717         0.473
 WAISSPL        0.508         0.942


     Beginning Time:  21:31:59
        Ending Time:  21:31:59
       Elapsed Time:  00:00:00
```

EFA Results ...

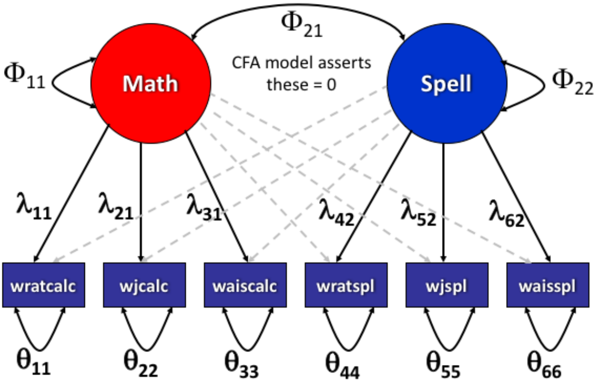
```
MUTHEN & MUTHEN
3463 Stoner Ave.
Los Angeles, CA  90066

Tel: (310) 391-9971
Fax: (310) 391-8971
Web: www.StatModel.com
Support: Support@StatModel.com

Copyright (c) 1998-2015 Muthen & Muthen
```

Outline

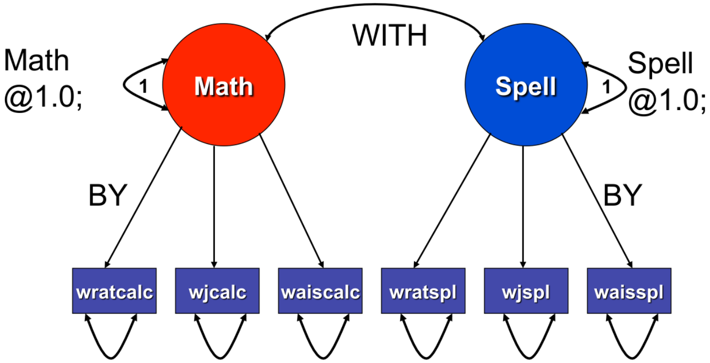
CFA = Confirmatory Factor Analysis

- Some factor loadings are set at 0, on theoretical grounds
  - Put another way, FA where only certain parameters are estimated.
- We believe (on theoretical grounds) that Math is indicated by:
  - wratcalc, wjcalc, waiscalc
- We believe that Spelling is indicated by:
  - wratspl, wjspl, waisspl

CFA: Grayed arrows are fixed at 0



CFA: With Mplus Terminology



About the identification

- All models with latent variables run into the "identification problem". They have some coefficients that cannot be estimated from the data
  - The usual approach is to fix some coefficients
  - Because some are fixed, the other coefficients "rescale themselves", absorb the assumed scaling.

Method 1: marker variable approach

- Mplus defaults to use the "**marker variable**" identification.
  - Fix the first indicator for each latent variable to have $\lambda = 1$.
  - The variance of each latent variable is estimated.
    - The latent variable's scale is thus the scale of the first indicator variable.
  - The loading estimates for other variables are relative to 1.0.
    - A variable that is more sensitive to the latent factor will have a coefficient > 1.0.
  - This method is preferred by adherents of the LISREL method of factor analysis.

Method 2: standardized latent variable approach

- An alternative identification approach, which is also widely used, is to
  - Fix the variances of each of the latent variables to 1.0 (but usually allow covariance to be estimated)
  - Estimate the λ for each of the indicators.
    - Now the λ estimates are understood as proportions of the factor which are translated into the indicators

CFA Mplus Syntax

```
TITLE:
  Example 2 - Confirmatory Factor Analysis
DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
     age;
  USEVARIABLES ARE
  wratspl wjspl waisspl wratcalc wjcalc waiscalc;
MISSING ARE all(99999);
MODEL:
  MATH BY wratcalc* wjcalc waiscalc;
  SPELL BY wratspl* wjspl waisspl;
  MATH@1 SPELL@1;!Set variance at 1;
```

CFA Mplus Syntax ...

```
OUTPUT:
  TECH1;
  STDYX;
```

- To alter identification from Mplus defaults, we insert “ * ” with wratcalc and wratspl

CFA Results

View online: cfa-01.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:35 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 2 - Confirmatory Factor Analysis

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wratspl wjspl waisspl wratcalc wjcalc waiscalc;
```

CFA Results ...

```
    MISSING ARE all(99999);

  MODEL:
    MATH BY wratcalc* wjcalc waiscalc;
    SPELL BY wratspl* wjspl waisspl;

    MATH@1 SPELL@1;

  OUTPUT:
    TECH1;
    STDYX;



INPUT READING TERMINATED NORMALLY




Example 2 - Confirmatory Factor Analysis

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         322
```

CFA Results ...

```
Number of dependent variables                                6
Number of independent variables                              0
Number of continuous latent variables                        2

Observed dependent variables

  Continuous
   WRATSPL     WJSPL       WAISSPL     WRATCALC    WJCALC      WAISCALC

Continuous latent variables
   MATH        SPELL


Estimator                                                   ML
Information matrix                                    OBSERVED
Maximum number of iterations                              1000
Convergence criterion                                0.500D-04
Maximum number of steepest descent iterations               20
Maximum number of iterations for H1                       2000
Convergence criterion for H1                         0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE
```

CFA Results ...

```
SUMMARY OF DATA

     Number of missing data patterns             4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT


           Covariance Coverage
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
 WRATSPL        0.997
 WJSPL          0.997         1.000
 WAISSPL        0.991         0.994         0.994
 WRATCALC       0.997         1.000         0.994         1.000
 WJCALC         0.991         0.994         0.988         0.994         0.994
 WAISCALC       0.991         0.994         0.994         0.994         0.988


           Covariance Coverage
```

CFA Results ...

```
              WAISCALC
              --------
 WAISCALC       0.994


THE MODEL ESTIMATION TERMINATED NORMALLY



MODEL FIT INFORMATION

Number of Free Parameters                         19

Loglikelihood

          H0 Value                         -5127.830
          H1 Value                         -5123.060

Information Criteria

          Akaike (AIC)                     10293.661
          Bayesian (BIC)                   10365.377
          Sample-Size Adjusted BIC         10305.112
            (n* = (n + 2) / 24)
```

CFA Results ...

```
Chi-Square Test of Model Fit

          Value                              9.540
          Degrees of Freedom                     8
          P-Value                           0.2988

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.024
          90 Percent C.I.                    0.000  0.073
          Probability RMSEA <= .05           0.761

CFI/TLI

          CFI                                0.999
          TLI                                0.998

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1882.335
          Degrees of Freedom                    15
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.024
```

CFA Results ...

```
MODEL RESULTS

                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 MATH     BY
    WRATCALC           6.041      0.276     21.921      0.000
    WJCALC             4.144      0.203     20.370      0.000
    WAISCALC           2.410      0.165     14.636      0.000

 SPELL    BY
    WRATSPL            6.532      0.288     22.645      0.000
    WJSPL              6.809      0.296     23.025      0.000
    WAISSPL            6.354      0.283     22.463      0.000

 SPELL    WITH
    MATH               0.553      0.042     13.191      0.000

 Intercepts
    WRATSPL           36.484      0.385     94.751      0.000
    WJSPL             41.674      0.398    104.808      0.000
    WAISSPL           37.163      0.376     98.788      0.000
    WRATCALC          38.922      0.355    109.514      0.000
```

CFA Results ...

```
   WJCALC          23.812      0.255     93.298      0.000
   WAISCALC        11.022      0.186     59.230      0.000

 Variances
   MATH             1.000      0.000    999.000    999.000
   SPELL            1.000      0.000    999.000    999.000

 Residual Variances
   WRATSPL          5.053      0.612      8.256      0.000
   WJSPL            4.541      0.616      7.369      0.000
   WAISSPL          5.156      0.599      8.605      0.000
   WRATCALC         4.179      1.014      4.122      0.000
   WJCALC           3.769      0.537      7.017      0.000
   WAISCALC         5.304      0.457     11.596      0.000


STANDARDIZED MODEL RESULTS


STDYX Standardization

                                                Two-Tailed
                 Estimate       S.E.  Est./S.E.    P-Value

 MATH     BY
   WRATCALC         0.947      0.014     69.150      0.000
```

CFA Results ...

```
   WJCALC             0.906      0.015     58.479      0.000
   WAISCALC           0.723      0.029     24.919      0.000

SPELL    BY
   WRATSPL            0.946      0.008    120.751      0.000
   WJSPL              0.954      0.007    133.161      0.000
   WAISSPL            0.942      0.008    115.722      0.000

SPELL    WITH
   MATH               0.553      0.042     13.191      0.000

Intercepts
   WRATSPL            5.282      0.215     24.513      0.000
   WJSPL              5.841      0.237     24.664      0.000
   WAISSPL            5.508      0.224     24.555      0.000
   WRATCALC           6.103      0.247     24.722      0.000
   WJCALC             5.203      0.213     24.447      0.000
   WAISCALC           3.306      0.142     23.269      0.000

Variances
   MATH               1.000      0.000    999.000    999.000
   SPELL              1.000      0.000    999.000    999.000

Residual Variances
   WRATSPL            0.106      0.015      7.150      0.000
   WJSPL              0.089      0.014      6.521      0.000
```

CFA Results ...

```
    WAISSPL          0.113      0.015      7.390      0.000
    WRATCALC         0.103      0.026      3.959      0.000
    WJCALC           0.180      0.028      6.417      0.000
    WAISCALC         0.477      0.042     11.374      0.000


R-SQUARE

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    WRATSPL          0.894      0.015     60.375      0.000
    WJSPL            0.911      0.014     66.580      0.000
    WAISSPL          0.887      0.015     57.861      0.000
    WRATCALC         0.897      0.026     34.575      0.000
    WJCALC           0.820      0.028     29.240      0.000
    WAISCALC         0.523      0.042     12.459      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.135E-03
       (ratio of smallest to largest eigenvalue)


TECHNICAL 1 OUTPUT
```

CFA Results ...

```
PARAMETER SPECIFICATION

       NU
          WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
          --------      --------      --------      --------      --------
  1            1             2             3             4             5

       NU
          WAISCALC
          --------
  1            6

       LAMBDA
          MATH          SPELL
          --------      --------
WRATSPL         0             7
WJSPL           0             8
WAISSPL         0             9
WRATCALC       10             0
WJCALC         11             0
WAISCALC       12             0
```

CFA Results ...

```
          THETA
             WRATSPL        WJSPL          WAISSPL        WRATCALC       WJCALC
             --------       --------       --------       --------       --------
WRATSPL           13
WJSPL              0             14
WAISSPL            0              0             15
WRATCALC           0              0              0             16
WJCALC             0              0              0              0             17
WAISCALC           0              0              0              0              0


          THETA
             WAISCALC
             --------
WAISCALC          18


          ALPHA
             MATH           SPELL
             --------       --------
      1           0              0


          BETA
```

CFA Results ...

```
              MATH          SPELL
              --------      --------
 MATH                0             0
 SPELL               0             0


           PSI
              MATH          SPELL
              --------      --------
 MATH                0
 SPELL              19             0


     STARTING VALUES


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
      1         36.483        41.674        37.197        38.922        23.822


           NU
              WAISCALC
              --------
      1         11.016
```

CFA Results ...

```
         LAMBDA
            MATH          SPELL
          --------      --------
WRATSPL     0.000         1.000
WJSPL       0.000         1.000
WAISSPL     0.000         1.000
WRATCALC    1.000         0.000
WJCALC      1.000         0.000
WAISCALC    1.000         0.000


         THETA
            WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
          --------      --------      --------      --------      --------
WRATSPL    23.922
WJSPL       0.000        25.455
WAISSPL     0.000         0.000        22.785
WRATCALC    0.000         0.000         0.000        20.337
WJCALC      0.000         0.000         0.000         0.000        10.464
WAISCALC    0.000         0.000         0.000         0.000         0.000


         THETA
            WAISCALC
```

CFA Results ...

```
                 --------
WAISCALC           5.545


          ALPHA
             MATH          SPELL
             --------      --------
     1         0.000         0.000


          BETA
             MATH          SPELL
             --------      --------
MATH           0.000         0.000
SPELL          0.000         0.000


          PSI
             MATH          SPELL
             --------      --------
MATH           1.000
SPELL          0.000         1.000


    Beginning Time:  21:35:18
       Ending Time:  21:35:18
```

CFA Results ...

```
      Elapsed Time:  00:00:00


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```

Outline

Should 2 groups be included in same model?

- Goal is to compare males and female on Math and Spelling
- If groups cannot be included in same model, then formal comparison is impossible.
- Focus on measurement parameters.
  - loadings λ, which indicate an indicator depends on the latent ability (Math or Spelling)
  - item intercepts, which we will call τ

This is called:

Measurement Invariance Analysis is the attempt to determine if the measurement parameters (λ_j , τ_j) are equal for both groups. If they are, then the values of the latent variables might be compared between groups.

Measurement Invariance Analysis

- Quantitative psychologists developed a sequence of steps where we can test for the similarity of the models for the two groups.

  Step 1. Configural invariance (same variables, same number of factors)

  Step 2. Metric invariance (equal factor loadings, λ)

  Step 3. Scalar invariance (equal factor loadings λ and item intercepts τ)

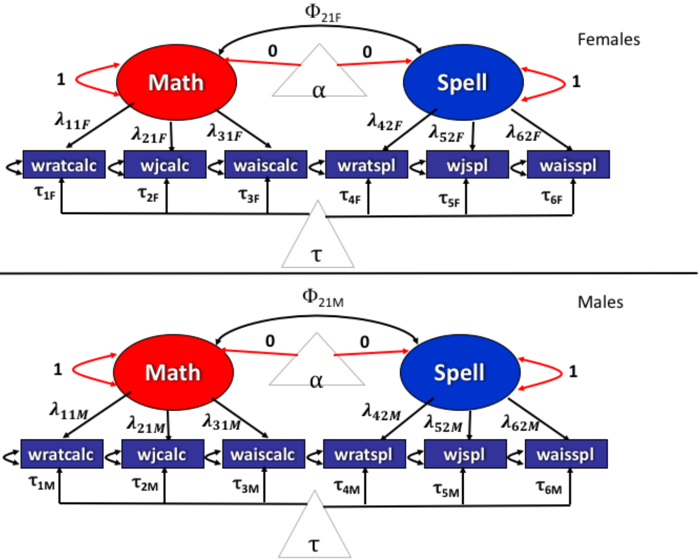
These three models are treating the groups in increasingly similar ways.

Configural Invariance Model

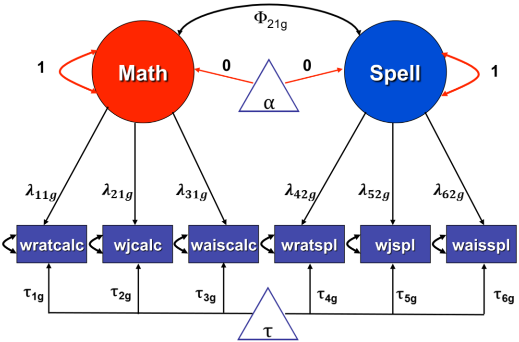
Configural: The same model applies for $g = Males$ and $g = Females$, but...

We allow the loadings λ_{ijg} and intercepts τ_{ig} to differ between the groups.

Configural Invariance Model ...



Another Graphic, perhaps more common type



Mplus code for Configural Analysis

```
TITLE:
  Example 3 - Multiple Group Confirmatory Factor
      Analysis (Configural Invariance)
DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
      age;
  USEVARIABLES ARE
  wratspl wjspl waisspl wratcalc wjcalc waiscalc;
  MISSING ARE all(99999);
  GROUPING IS female (0=male 1 =female);
MODEL:
  MATH BY wratcalc* wjcalc waiscalc;
```

Mplus code for Configural Analysis ...

```
  SPELL BY wratspl* wjspl waisspl;
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
  MATH@1 SPELL@1;
  [MATH@0 SPELL@0];
MODEL female:
  MATH BY wratcalc* wjcalc waiscalc;
  SPELL BY wratspl* wjspl waisspl;
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
OUTPUT:
  TECH1;
  STDYX;
```

Configural Model Results

View Online: cfa-01-1-configural.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:42 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 3 - Multiple Group Confirmatory Factor Analysis (Configural Invariance)

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wratspl wjspl waisspl wratcalc wjcalc waiscalc;
    MISSING ARE all(99999);
```

Configural Model Results ...

```
GROUPING IS female(0=male 1=female);

MODEL:
  MATH BY wratcalc* wjcalc waiscalc;
  SPELL BY wratspl* wjspl waisspl;
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
  MATH@1 SPELL@1;
  [MATH@0 SPELL@0];

  MODEL female:
  MATH BY wratcalc* wjcalc waiscalc;
  SPELL BY wratspl* wjspl waisspl;
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];

OUTPUT:
  TECH1;
  STDYX;


INPUT READING TERMINATED NORMALLY
```

Configural Model Results ...

```
Example 3 - Multiple Group Confirmatory Factor Analysis (Configural Invariance)

SUMMARY OF ANALYSIS

Number of groups                                                 2
Number of observations
   Group MALE                                                  221
   Group FEMALE                                                101
   Total sample size                                           322

Number of dependent variables                                    6
Number of independent variables                                  0
Number of continuous latent variables                            2

Observed dependent variables

  Continuous
   WRATSPL     WJSPL       WAISSPL     WRATCALC    WJCALC      WAISCALC

Continuous latent variables
   MATH        SPELL

Variables with special functions

  Grouping variable     FEMALE
```

Configural Model Results ...

```
Estimator                                                 ML
Information matrix                                  OBSERVED
Maximum number of iterations                            1000
Convergence criterion                              0.500D-04
Maximum number of steepest descent iterations             20
Maximum number of iterations for H1                     2000
Convergence criterion for H1                       0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA

   Group MALE
     Number of missing data patterns             2

   Group FEMALE
     Number of missing data patterns             4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100
```

Configural Model Results ...

```
    PROPORTION OF DATA PRESENT FOR MALE


           Covariance Coverage
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
 WRATSPL        1.000
 WJSPL          1.000         1.000
 WAISSPL        0.995         0.995         0.995
 WRATCALC       1.000         1.000         0.995         1.000
 WJCALC         1.000         1.000         0.995         1.000         1.000
 WAISCALC       0.995         0.995         0.995         0.995         0.995


           Covariance Coverage
              WAISCALC
              ________
 WAISCALC       0.995


    PROPORTION OF DATA PRESENT FOR FEMALE


           Covariance Coverage
```

Configural Model Results ...

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	0.990				
WJSPL	0.990	1.000			
WAISSPL	0.980	0.990	0.990		
WRATCALC	0.990	1.000	0.990	1.000	
WJCALC	0.970	0.980	0.970	0.980	0.980
WAISCALC	0.980	0.990	0.990	0.990	0.970

Covariance Coverage

	WAISCALC
WAISCALC	0.990

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 38

Loglikelihood

Configural Model Results ...

```
          H0 Value                       -5112.407
          H1 Value                       -5102.800

Information Criteria

          Akaike (AIC)                   10300.815
          Bayesian (BIC)                 10444.248
          Sample-Size Adjusted BIC       10323.717
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             19.215
          Degrees of Freedom                    16
          P-Value                           0.2577

Chi-Square Contribution From Each Group

          MALE                               7.660
          FEMALE                            11.555

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.035
          90 Percent C.I.                    0.000  0.085
```

Configural Model Results ...

```
          Probability RMSEA <= .05              0.631

CFI/TLI

          CFI                                   0.998
          TLI                                   0.997

Chi-Square Test of Model Fit for the Baseline Model

          Value                              1905.113
          Degrees of Freedom                       30
          P-Value                              0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                 0.029



MODEL RESULTS

                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE
```

Configural Model Results ...

```
MATH     BY
   WRATCALC         5.764      0.330     17.492      0.000
   WJCALC           4.123      0.244     16.926      0.000
   WAISCALC         2.446      0.200     12.212      0.000

SPELL    BY
   WRATSPL          6.278      0.337     18.646      0.000
   WJSPL            6.788      0.359     18.929      0.000
   WAISSPL          6.177      0.335     18.441      0.000

SPELL    WITH
   MATH             0.520      0.053      9.761      0.000

Means
   MATH             0.000      0.000    999.000    999.000
   SPELL            0.000      0.000    999.000    999.000

Intercepts
   WRATSPL         36.172      0.448     80.762      0.000
   WJSPL           41.371      0.480     86.157      0.000
   WAISSPL         36.767      0.444     82.899      0.000
   WRATCALC        39.222      0.417     93.961      0.000
   WJCALC          23.910      0.305     78.419      0.000
   WAISCALC        11.367      0.226     50.321      0.000

Variances
```

Configural Model Results ...

```
    MATH               1.000      0.000    999.000    999.000
    SPELL              1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL            4.915      0.726      6.772      0.000
    WJSPL              4.881      0.795      6.138      0.000
    WAISSPL            5.285      0.736      7.176      0.000
    WRATCALC           5.281      1.232      4.286      0.000
    WJCALC             3.548      0.664      5.341      0.000
    WAISCALC           5.269      0.557      9.463      0.000

Group FEMALE

 MATH      BY
    WRATCALC           6.545      0.496     13.187      0.000
    WJCALC             4.215      0.366     11.530      0.000
    WAISCALC           2.290      0.276      8.306      0.000

 SPELL     BY
    WRATSPL            7.010      0.547     12.817      0.000
    WJSPL              6.833      0.518     13.182      0.000
    WAISSPL            6.638      0.520     12.763      0.000

 SPELL     WITH
    MATH               0.634      0.064      9.914      0.000
```

Configural Model Results ...

```
 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000

 Intercepts
    WRATSPL           37.171      0.735     50.589      0.000
    WJSPL             42.337      0.705     60.073      0.000
    WAISSPL           38.030      0.697     54.578      0.000
    WRATCALC          38.267      0.666     57.464      0.000
    WJCALC            23.603      0.465     50.781      0.000
    WAISCALC          10.266      0.316     32.528      0.000

 Variances
    MATH               1.000      0.000    999.000    999.000
    SPELL              1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL            5.307      1.105      4.804      0.000
    WJSPL              3.474      0.910      3.818      0.000
    WAISSPL            4.900      1.007      4.865      0.000
    WRATCALC           1.956      1.621      1.207      0.227
    WJCALC             3.960      0.848      4.672      0.000
    WAISCALC           4.766      0.714      6.678      0.000


STANDARDIZED MODEL RESULTS
```

Configural Model Results ...

```
STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE

 MATH     BY
    WRATCALC           0.929      0.018     50.941      0.000
    WJCALC             0.910      0.019     47.330      0.000
    WAISCALC           0.729      0.035     20.921      0.000

 SPELL    BY
    WRATSPL            0.943      0.010     94.427      0.000
    WJSPL              0.951      0.009    102.619      0.000
    WAISSPL            0.937      0.011     89.110      0.000

 SPELL    WITH
    MATH               0.520      0.053      9.761      0.000

 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000
```

Configural Model Results ...

```
 Intercepts
    WRATSPL            5.433      0.267     20.346      0.000
    WJSPL              5.796      0.284     20.425      0.000
    WAISSPL            5.579      0.274     20.368      0.000
    WRATCALC           6.320      0.308     20.516      0.000
    WJCALC             5.275      0.260     20.306      0.000
    WAISCALC           3.389      0.175     19.365      0.000

 Variances
    MATH               1.000      0.000    999.000    999.000
    SPELL              1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL            0.111      0.019      5.887      0.000
    WJSPL              0.096      0.018      5.435      0.000
    WAISSPL            0.122      0.020      6.171      0.000
    WRATCALC           0.137      0.034      4.048      0.000
    WJCALC             0.173      0.035      4.939      0.000
    WAISCALC           0.468      0.051      9.214      0.000

Group FEMALE

 MATH     BY
    WRATCALC           0.978      0.019     52.179      0.000
    WJCALC             0.904      0.024     37.149      0.000
    WAISCALC           0.724      0.050     14.423      0.000
```

Configural Model Results ...

```
SPELL    BY
   WRATSPL            0.950      0.012     76.115      0.000
   WJSPL              0.965      0.011     91.847      0.000
   WAISSPL            0.949      0.013     74.596      0.000

SPELL    WITH
   MATH               0.634      0.064      9.914      0.000

Means
   MATH               0.000      0.000    999.000    999.000
   SPELL              0.000      0.000    999.000    999.000

Intercepts
   WRATSPL            5.037      0.368     13.682      0.000
   WJSPL              5.977      0.432     13.831      0.000
   WAISSPL            5.434      0.396     13.728      0.000
   WRATCALC           5.718      0.414     13.797      0.000
   WJCALC             5.064      0.371     13.633      0.000
   WAISCALC           3.245      0.250     12.985      0.000

Variances
   MATH               1.000      0.000    999.000    999.000
   SPELL              1.000      0.000    999.000    999.000

Residual Variances
```

Configural Model Results ...

WRATSPL	0.097	0.024	4.110	0.000
WJSPL	0.069	0.020	3.417	0.001
WAISSPL	0.100	0.024	4.147	0.000
WRATCALC	0.044	0.037	1.191	0.234
WJCALC	0.182	0.044	4.141	0.000
WAISCALC	0.476	0.073	6.552	0.000

R-SQUARE

Group MALE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
WRATSPL	0.889	0.019	47.213	0.000
WJSPL	0.904	0.018	51.310	0.000
WAISSPL	0.878	0.020	44.555	0.000
WRATCALC	0.863	0.034	25.471	0.000
WJCALC	0.827	0.035	23.665	0.000
WAISCALC	0.532	0.051	10.460	0.000

Group FEMALE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value

Configural Model Results ...

```
    WRATSPL            0.903      0.024     38.058      0.000
    WJSPL              0.931      0.020     45.924      0.000
    WAISSPL            0.900      0.024     37.298      0.000
    WRATCALC           0.956      0.037     26.089      0.000
    WJCALC             0.818      0.044     18.575      0.000
    WAISCALC           0.524      0.073      7.212      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.211E-04
       (ratio of smallest to largest eigenvalue)


TECHNICAL 1 OUTPUT


     PARAMETER SPECIFICATION FOR MALE


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
      1           1             2             3             4             5
```

Configural Model Results ...

```
        NU
           WAISCALC
           --------
     1          6


        LAMBDA
           MATH          SPELL
           --------      --------
WRATSPL           0             7
WJSPL             0             8
WAISSPL           0             9
WRATCALC         10             0
WJCALC           11             0
WAISCALC         12             0


        THETA
           WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
           --------      --------      --------      --------      --------
WRATSPL          13
WJSPL             0            14
WAISSPL           0             0            15
WRATCALC          0             0             0            16
WJCALC            0             0             0             0            17
```

Configural Model Results ...

```
WAISCALC           0             0             0             0             0


          THETA
             WAISCALC
             --------
WAISCALC          18


          ALPHA
             MATH          SPELL
             --------      --------
     1          0             0


          BETA
             MATH          SPELL
             --------      --------
MATH               0             0
SPELL              0             0


          PSI
             MATH          SPELL
             --------      --------
MATH               0
```

Configural Model Results ...

```
SPELL              19             0


    PARAMETER SPECIFICATION FOR FEMALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
     1          20            21            22            23            24


          NU
             WAISCALC
             --------
     1          25


          LAMBDA
             MATH          SPELL
             --------      --------
WRATSPL            0            26
WJSPL              0            27
WAISSPL            0            28
WRATCALC          29             0
WJCALC            30             0
```

Configural Model Results ...

```
WAISCALC          31             0


         THETA
            WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
            --------      --------      --------      --------      --------
WRATSPL           32
WJSPL              0            33
WAISSPL            0             0            34
WRATCALC           0             0             0            35
WJCALC             0             0             0             0            36
WAISCALC           0             0             0             0             0


         THETA
            WAISCALC
            --------
WAISCALC          37


         ALPHA
            MATH          SPELL
            --------      --------
     1           0             0
```

Configural Model Results ...

```
          BETA
             MATH          SPELL
             --------      --------
 MATH               0             0
 SPELL              0             0


          PSI
             MATH          SPELL
             --------      --------
 MATH               0
 SPELL             38             0


     STARTING VALUES FOR MALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
      1        36.172        41.371        36.786        39.222        23.910


          NU
             WAISCALC
             --------
```

Configural Model Results ...

```
      1         11.350


             LAMBDA
                MATH          SPELL
                --------      --------
WRATSPL           0.000         1.000
WJSPL             0.000         1.000
WAISSPL           0.000         1.000
WRATCALC          1.000         0.000
WJCALC            1.000         0.000
WAISCALC          1.000         0.000


             THETA
                WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
                --------      --------      --------      --------      --------
WRATSPL          22.166
WJSPL             0.000        25.479
WAISSPL           0.000         0.000        21.761
WRATCALC          0.000         0.000         0.000        19.254
WJCALC            0.000         0.000         0.000         0.000        10.272
WAISCALC          0.000         0.000         0.000         0.000         0.000


             THETA
```

Configural Model Results ...

```
             WAISCALC
             ________
 WAISCALC       5.605


          ALPHA
             MATH          SPELL
             ________      ________
      1         0.000         0.000


          BETA
             MATH          SPELL
             ________      ________
 MATH           0.000         0.000
 SPELL          0.000         0.000


          PSI
             MATH          SPELL
             ________      ________
 MATH           1.000
 SPELL          0.000         1.000


     STARTING VALUES FOR FEMALE
```

Configural Model Results ...

```
        NU
           WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
           --------      --------      --------      --------      --------
 1          36.172        41.371        36.786        39.222        23.910


        NU
           WAISCALC
           --------
 1          11.350


        LAMBDA
           MATH          SPELL
           --------      --------
 WRATSPL      0.000         1.000
 WJSPL        0.000         1.000
 WAISSPL      0.000         1.000
 WRATCALC     1.000         0.000
 WJCALC       1.000         0.000
 WAISCALC     1.000         0.000


        THETA
```

Configural Model Results ...

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
	--------	--------	--------	--------	--------
WRATSPL	22.166				
WJSPL	0.000	25.479			
WAISSPL	0.000	0.000	21.761		
WRATCALC	0.000	0.000	0.000	19.254	
WJCALC	0.000	0.000	0.000	0.000	10.272
WAISCALC	0.000	0.000	0.000	0.000	0.000

THETA

	WAISCALC

WAISCALC	5.605

ALPHA

	MATH	SPELL
	--------	--------
1	0.000	0.000

BETA

	MATH	SPELL
	--------	--------
MATH	0.000	0.000

Configural Model Results ...

```
 SPELL          0.000         0.000


          PSI
             MATH          SPELL
             --------      --------
 MATH           1.000
 SPELL          0.000         1.000


    Beginning Time:  21:42:53
       Ending Time:  21:42:53
      Elapsed Time:  00:00:00
```

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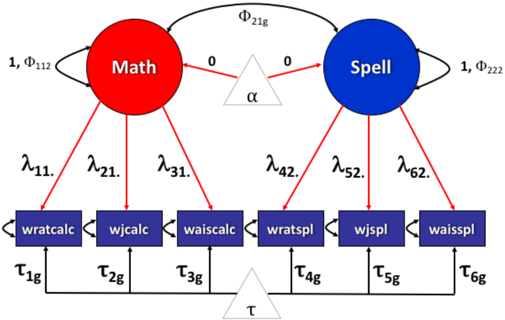
Step 2: Metric Invariance

- We fit the Configural Model because we need something to compare against.
- The first model we will compare is the Metric Invariance model.
- In the Metric Invariance analysis, we remove the subscript g from the **loadings**, to assert that loadings are same for both groups

Metric Invariance Illustrated



- The λ coefficients have a "." in place of "g", a reminder of the fact they are not differentiated between groups
- still differing among groups:
  - τ_{ig}
  - Φ

Metric Model: Syntax

```
TITLE:
Example 3 - Multiple Group Confirmatory Factor
   Analysis (Weak Invariance)
DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
     age;
  USEVARIABLES ARE
  wratspl wjspl waisspl wratcalc wjcalc waiscalc;
  MISSING ARE all(99999);
  GROUPING IS female (0=male 1 =female);
MODEL:
  MATH BY wratcalc* wjcalc waiscalc;
```

Metric Model: Syntax ...

```
  SPELL BY wratspl* wjspl waisspl;
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
  MATH@1 SPELL@1;
  [MATH@0 SPELL@0];
MODEL female:
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
  MATH SPELL;
OUTPUT:
  TECH1;
  STDYX;
```

Metric Model Results

View Online: cfa-01-2-metric.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:42 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 3 - Multiple Group Confirmatory Factor Analysis (Metric Invariance)

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wratspl wjspl waisspl wratcalc wjcalc waiscalc;
```

Metric Model Results ...

```
    MISSING ARE all(99999);

    GROUPING IS female(0=male 1=female);

  MODEL:
    MATH BY wratcalc* wjcalc waiscalc;
    SPELL BY wratspl* wjspl waisspl;
    [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
    MATH@1 SPELL@1;
    [MATH@0 SPELL@0];

    MODEL female:
    [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
    MATH SPELL;

  OUTPUT:
    TECH1;
    STDYX;



INPUT READING TERMINATED NORMALLY
```

Metric Model Results ...

```
Example 3 - Multiple Group Confirmatory Factor Analysis (Metric Invariance)

SUMMARY OF ANALYSIS

Number of groups                                                 2
Number of observations
   Group MALE                                                  221
   Group FEMALE                                                101
   Total sample size                                           322

Number of dependent variables                                    6
Number of independent variables                                  0
Number of continuous latent variables                            2

Observed dependent variables

  Continuous
   WRATSPL     WJSPL       WAISSPL     WRATCALC    WJCALC      WAISCALC

Continuous latent variables
   MATH        SPELL

Variables with special functions

  Grouping variable     FEMALE
```

Metric Model Results ...

```
Estimator                                                  ML
Information matrix                                   OBSERVED
Maximum number of iterations                             1000
Convergence criterion                               0.500D-04
Maximum number of steepest descent iterations              20
Maximum number of iterations for H1                      2000
Convergence criterion for H1                        0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA

   Group MALE
     Number of missing data patterns             2

   Group FEMALE
     Number of missing data patterns             4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100
```

Metric Model Results ...

```
    PROPORTION OF DATA PRESENT FOR MALE


           Covariance Coverage
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
 WRATSPL        1.000
 WJSPL          1.000         1.000
 WAISSPL        0.995         0.995         0.995
 WRATCALC       1.000         1.000         0.995         1.000
 WJCALC         1.000         1.000         0.995         1.000         1.000
 WAISCALC       0.995         0.995         0.995         0.995         0.995


           Covariance Coverage
              WAISCALC
              ________
 WAISCALC       0.995


    PROPORTION OF DATA PRESENT FOR FEMALE


           Covariance Coverage
```

Metric Model Results ...

```
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
 WRATSPL        0.990
 WJSPL          0.990         1.000
 WAISSPL        0.980         0.990         0.990
 WRATCALC       0.990         1.000         0.990         1.000
 WJCALC         0.970         0.980         0.970         0.980         0.980
 WAISCALC       0.980         0.990         0.990         0.990         0.970


           Covariance Coverage
              WAISCALC
              --------
 WAISCALC       0.990



THE MODEL ESTIMATION TERMINATED NORMALLY



MODEL FIT INFORMATION

Number of Free Parameters                        34

Loglikelihood
```

Metric Model Results ...

```
        H0 Value                         -5115.735
        H1 Value                         -5102.800

Information Criteria

        Akaike (AIC)                     10299.471
        Bayesian (BIC)                   10427.805
        Sample-Size Adjusted BIC         10319.962
          (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

        Value                               25.871
        Degrees of Freedom                      20
        P-Value                             0.1701

Chi-Square Contribution From Each Group

        MALE                                10.290
        FEMALE                              15.581

RMSEA (Root Mean Square Error Of Approximation)

        Estimate                             0.043
        90 Percent C.I.                      0.000  0.085
```

Metric Model Results ...

```
          Probability RMSEA <= .05           0.566

CFI/TLI

          CFI                                0.997
          TLI                                0.995

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1905.113
          Degrees of Freedom                    30
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.054



MODEL RESULTS

                                                 Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE
```

Metric Model Results ...

```
MATH     BY
   WRATCALC          5.898      0.317     18.614      0.000
   WJCALC            4.039      0.232     17.414      0.000
   WAISCALC          2.321      0.173     13.389      0.000

SPELL    BY
   WRATSPL           6.387      0.332     19.245      0.000
   WJSPL             6.640      0.346     19.186      0.000
   WAISSPL           6.213      0.326     19.054      0.000

SPELL    WITH
   MATH              0.519      0.053      9.733      0.000

Means
   MATH              0.000      0.000    999.000    999.000
   SPELL             0.000      0.000    999.000    999.000

Intercepts
   WRATSPL          36.172      0.454     79.673      0.000
   WJSPL            41.371      0.472     87.637      0.000
   WAISSPL          36.767      0.446     82.522      0.000
   WRATCALC         39.222      0.422     92.880      0.000
   WJCALC           23.910      0.302     79.192      0.000
   WAISCALC         11.367      0.221     51.462      0.000

Variances
```

Metric Model Results ...

```
    MATH               1.000      0.000    999.000    999.000
    SPELL              1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL            4.764      0.721      6.607      0.000
    WJSPL              5.154      0.790      6.526      0.000
    WAISSPL            5.233      0.732      7.153      0.000
    WRATCALC           4.619      1.205      3.834      0.000
    WJCALC             3.835      0.654      5.864      0.000
    WAISCALC           5.370      0.564      9.525      0.000

Group FEMALE

 MATH     BY
    WRATCALC           5.898      0.317     18.614      0.000
    WJCALC             4.039      0.232     17.414      0.000
    WAISCALC           2.321      0.173     13.389      0.000

 SPELL    BY
    WRATSPL            6.387      0.332     19.245      0.000
    WJSPL              6.640      0.346     19.186      0.000
    WAISSPL            6.213      0.326     19.054      0.000

 SPELL    WITH
    MATH               0.731      0.150      4.857      0.000
```

Metric Model Results ...

```
 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000

 Intercepts
    WRATSPL           37.172      0.713     52.136      0.000
    WJSPL             42.337      0.721     58.719      0.000
    WAISSPL           38.031      0.691     55.052      0.000
    WRATCALC          38.267      0.656     58.319      0.000
    WJCALC            23.602      0.474     49.832      0.000
    WAISCALC          10.263      0.330     31.085      0.000

 Variances
    MATH               1.162      0.211      5.501      0.000
    SPELL              1.117      0.197      5.676      0.000

 Residual Variances
    WRATSPL            5.686      1.123      5.061      0.000
    WJSPL              3.230      0.929      3.478      0.001
    WAISSPL            4.995      1.023      4.883      0.000
    WRATCALC           3.047      1.413      2.157      0.031
    WJCALC             3.603      0.778      4.632      0.000
    WAISCALC           4.700      0.714      6.587      0.000


STANDARDIZED MODEL RESULTS
```

Metric Model Results ...

```
STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE

 MATH     BY
    WRATCALC           0.940      0.016     57.271      0.000
    WJCALC             0.900      0.019     46.925      0.000
    WAISCALC           0.708      0.034     20.796      0.000

 SPELL    BY
    WRATSPL            0.946      0.009    102.287      0.000
    WJSPL              0.946      0.010     97.916      0.000
    WAISSPL            0.938      0.010     93.327      0.000

 SPELL    WITH
    MATH               0.519      0.053      9.733      0.000

 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000
```

Metric Model Results ...

```
 Intercepts
    WRATSPL            5.359      0.260     20.603      0.000
    WJSPL              5.895      0.282     20.922      0.000
    WAISSPL            5.553      0.267     20.809      0.000
    WRATCALC           6.248      0.304     20.582      0.000
    WJCALC             5.327      0.254     20.933      0.000
    WAISCALC           3.466      0.163     21.210      0.000

 Variances
    MATH               1.000      0.000    999.000    999.000
    SPELL              1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL            0.105      0.018      5.973      0.000
    WJSPL              0.105      0.018      5.723      0.000
    WAISSPL            0.119      0.019      6.325      0.000
    WRATCALC           0.117      0.031      3.802      0.000
    WJCALC             0.190      0.035      5.517      0.000
    WAISCALC           0.499      0.048     10.368      0.000

Group FEMALE

 MATH     BY
    WRATCALC           0.964      0.018     54.629      0.000
    WJCALC             0.917      0.020     46.823      0.000
    WAISCALC           0.756      0.038     19.875      0.000
```

Metric Model Results ...

```
SPELL    BY
   WRATSPL           0.943      0.014     69.743      0.000
   WJSPL             0.969      0.010    101.347      0.000
   WAISSPL           0.947      0.013     73.865      0.000

SPELL    WITH
   MATH              0.641      0.063     10.165      0.000

Means
   MATH              0.000      0.000    999.000    999.000
   SPELL             0.000      0.000    999.000    999.000

Intercepts
   WRATSPL           5.192      0.361     14.385      0.000
   WJSPL             5.843      0.420     13.926      0.000
   WAISSPL           5.482      0.383     14.308      0.000
   WRATCALC          5.803      0.416     13.950      0.000
   WJCALC            4.969      0.353     14.068      0.000
   WAISCALC          3.100      0.218     14.229      0.000

Variances
   MATH              1.000      0.000    999.000    999.000
   SPELL             1.000      0.000    999.000    999.000

Residual Variances
```

Metric Model Results ...

WRATSPL	0.111	0.025	4.350	0.000
WJSPL	0.062	0.019	3.322	0.001
WAISSPL	0.104	0.024	4.277	0.000
WRATCALC	0.070	0.034	2.058	0.040
WJCALC	0.160	0.036	4.449	0.000
WAISCALC	0.429	0.057	7.461	0.000

R-SQUARE

Group MALE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
WRATSPL	0.895	0.018	51.143	0.000
WJSPL	0.895	0.018	48.958	0.000
WAISSPL	0.881	0.019	46.663	0.000
WRATCALC	0.883	0.031	28.635	0.000
WJCALC	0.810	0.035	23.462	0.000
WAISCALC	0.501	0.048	10.398	0.000

Group FEMALE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value

Metric Model Results ...

```
    WRATSPL            0.889      0.025     34.872      0.000
    WJSPL              0.938      0.019     50.673      0.000
    WAISSPL            0.896      0.024     36.933      0.000
    WRATCALC           0.930      0.034     27.314      0.000
    WJCALC             0.840      0.036     23.411      0.000
    WAISCALC           0.571      0.057      9.938      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.626E-04
       (ratio of smallest to largest eigenvalue)


TECHNICAL 1 OUTPUT


     PARAMETER SPECIFICATION FOR MALE


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
      1            1             2             3             4             5
```

Metric Model Results ...

```
         NU
            WAISCALC
            --------
     1           6


         LAMBDA
            MATH          SPELL
            --------      --------
 WRATSPL           0             7
 WJSPL             0             8
 WAISSPL           0             9
 WRATCALC         10             0
 WJCALC           11             0
 WAISCALC         12             0


         THETA
            WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
            --------      --------      --------      --------      --------
 WRATSPL          13
 WJSPL             0            14
 WAISSPL           0             0            15
 WRATCALC          0             0             0            16
 WJCALC            0             0             0             0            17
```

Metric Model Results ...

```
WAISCALC          0             0             0             0             0


          THETA
             WAISCALC
             --------
WAISCALC         18


          ALPHA
             MATH          SPELL
             --------      --------
     1            0             0


          BETA
             MATH          SPELL
             --------      --------
MATH              0             0
SPELL             0             0


          PSI
             MATH          SPELL
             --------      --------
MATH              0
```

Metric Model Results ...

```
SPELL             19            0


    PARAMETER SPECIFICATION FOR FEMALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
     1          20            21            22            23            24


          NU
             WAISCALC
             --------
     1          25


          LAMBDA
             MATH          SPELL
             --------      --------
WRATSPL            0             7
WJSPL              0             8
WAISSPL            0             9
WRATCALC          10             0
WJCALC            11             0
```

Metric Model Results ...

```
WAISCALC          12              0


           THETA
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
WRATSPL          26
WJSPL             0            27
WAISSPL           0             0            28
WRATCALC          0             0             0            29
WJCALC            0             0             0             0            30
WAISCALC          0             0             0             0             0


           THETA
              WAISCALC
              --------
WAISCALC         31


           ALPHA
              MATH          SPELL
              --------      --------
     1            0             0
```

Metric Model Results ...

```
        BETA
           MATH          SPELL
           --------      --------
MATH              0             0
SPELL             0             0


        PSI
           MATH          SPELL
           --------      --------
MATH             32
SPELL            33            34


    STARTING VALUES FOR MALE


        NU
           WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
           --------      --------      --------      --------      --------
     1       36.172        41.371        36.786        39.222        23.910


        NU
           WAISCALC
           --------
```

Metric Model Results ...

```
      1         11.350


              LAMBDA
                 MATH          SPELL
                 --------      --------
    WRATSPL        0.000         1.000
    WJSPL          0.000         1.000
    WAISSPL        0.000         1.000
    WRATCALC       1.000         0.000
    WJCALC         1.000         0.000
    WAISCALC       1.000         0.000


              THETA
                 WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
                 --------      --------      --------      --------      --------
    WRATSPL       22.166
    WJSPL          0.000        25.479
    WAISSPL        0.000         0.000        21.761
    WRATCALC       0.000         0.000         0.000        19.254
    WJCALC         0.000         0.000         0.000         0.000        10.272
    WAISCALC       0.000         0.000         0.000         0.000         0.000


              THETA
```

Metric Model Results ...

```
              WAISCALC
              ________
 WAISCALC       5.605


           ALPHA
              MATH          SPELL
              ________      ________
     1          0.000         0.000


           BETA
              MATH          SPELL
              ________      ________
 MATH           0.000         0.000
 SPELL          0.000         0.000


           PSI
              MATH          SPELL
              ________      ________
 MATH           1.000
 SPELL          0.000         1.000


    STARTING VALUES FOR FEMALE
```

Metric Model Results ...

```
        NU
           WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
           ________      ________      ________      ________      ________
 1          36.172        41.371        36.786        39.222        23.910


        NU
           WAISCALC
           ________
 1          11.350


        LAMBDA
           MATH          SPELL
           ________      ________
 WRATSPL      0.000         7.010
 WJSPL        0.000         6.828
 WAISSPL      0.000         6.653
 WRATCALC     6.633         0.000
 WJCALC       4.167         0.000
 WAISCALC     2.259         0.000


        THETA
```

Metric Model Results ...

```
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
WRATSPL       22.166
WJSPL          0.000        25.479
WAISSPL        0.000         0.000        21.761
WRATCALC       0.000         0.000         0.000        19.254
WJCALC         0.000         0.000         0.000         0.000        10.272
WAISCALC       0.000         0.000         0.000         0.000         0.000


          THETA
             WAISCALC
             --------
WAISCALC       5.605


          ALPHA
             MATH          SPELL
             --------      --------
     1         0.000         0.000


          BETA
             MATH          SPELL
             --------      --------
MATH           0.000         0.000
```

Metric Model Results ...

```
 SPELL          0.000         0.000


          PSI
             MATH          SPELL
             --------      --------
 MATH           1.000
 SPELL          0.000         1.000


     Beginning Time:  21:42:53
        Ending Time:  21:42:53
       Elapsed Time:  00:00:00



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```
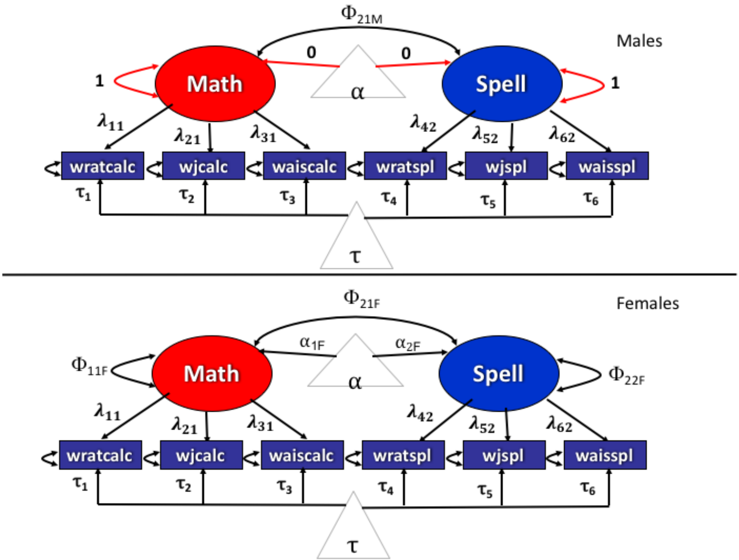
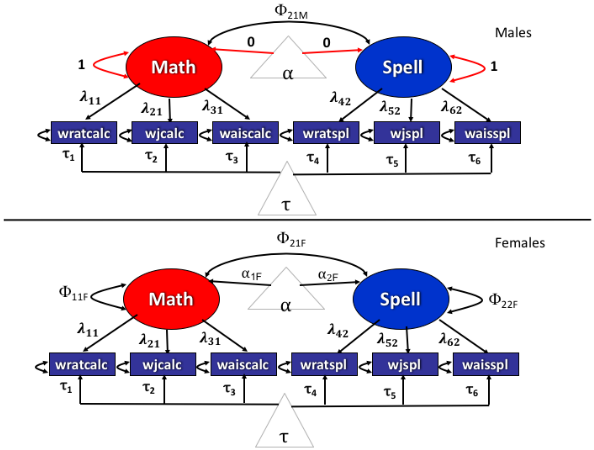

Step 3: Scalar Invariance Model

- Assume not only that the loadings are the same between groups, but also
- the intercepts on the measurement indicators (τ_i) are the same

Scalar Invariance Illustrated



Scalar Invariance Model



- The last subscript disappeared on λ and τ. No more group differentiation
- Only remaining group differences are the latent variable's
  - intercept, 0 for M, α for F, reflecting the average group difference
  - And variances, Φ_{11g} Φ_{22g}, and covariance Φ_{21g}

Scalar Invariance Model Syntax

```
TITLE:
Example 3 - Multiple Group Confirmatory Factor
    Analysis (Strong Invariance)
DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
      age;
  USEVARIABLES ARE
  wratspl wjspl waisspl wratcalc wjcalc waiscalc;
  MISSING ARE all(99999);
  GROUPING IS female (0=male 1 =female);
MODEL:
  MATH BY wratcalc* wjcalc waiscalc;
```

Scalar Invariance Model Syntax ...

```
  SPELL BY wratspl* wjspl waisspl;
  [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
  MATH@1 SPELL@1;
  [MATH@0 SPELL@0];

MODEL female:
  MATH SPELL;
  [MATH SPELL];

OUTPUT:
    TECH1;
  STDYX;
```

Scalar Model Estimates

View Online: cfa-01-3-scalar.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:42 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 3 - Multiple Group Confirmatory Factor Analysis (Scalar Invariance)

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wratspl wjspl waisspl wratcalc wjcalc waiscalc;
```

Scalar Model Estimates ...

```
    MISSING ARE all(99999);

    GROUPING IS female(0=male 1=female);

  MODEL:
    MATH BY wratcalc* wjcalc waiscalc;
    SPELL BY wratspl* wjspl waisspl;
    [wratspl wjspl waisspl wratcalc wjcalc waiscalc];
    MATH@1 SPELL@1;
    [MATH@0 SPELL@0];

    MODEL female:
    MATH SPELL;
    [MATH SPELL];

  OUTPUT:
    TECH1;
    STDYX;



INPUT READING TERMINATED NORMALLY
```

Scalar Model Estimates ...

```
Example 3 - Multiple Group Confirmatory Factor Analysis (Scalar Invariance)

SUMMARY OF ANALYSIS

Number of groups                                                 2
Number of observations
   Group MALE                                                  221
   Group FEMALE                                                101
   Total sample size                                           322

Number of dependent variables                                    6
Number of independent variables                                  0
Number of continuous latent variables                            2

Observed dependent variables

  Continuous
   WRATSPL     WJSPL       WAISSPL     WRATCALC    WJCALC      WAISCALC

Continuous latent variables
   MATH        SPELL

Variables with special functions

  Grouping variable     FEMALE
```

Scalar Model Estimates ...

```
Estimator                                                  ML
Information matrix                                   OBSERVED
Maximum number of iterations                             1000
Convergence criterion                               0.500D-04
Maximum number of steepest descent iterations              20
Maximum number of iterations for H1                      2000
Convergence criterion for H1                        0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA

   Group MALE
     Number of missing data patterns             2

   Group FEMALE
     Number of missing data patterns             4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100
```

Scalar Model Estimates ...

```
    PROPORTION OF DATA PRESENT FOR MALE


          Covariance Coverage
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             ________      ________      ________      ________      ________
 WRATSPL        1.000
 WJSPL          1.000         1.000
 WAISSPL        0.995         0.995         0.995
 WRATCALC       1.000         1.000         0.995         1.000
 WJCALC         1.000         1.000         0.995         1.000         1.000
 WAISCALC       0.995         0.995         0.995         0.995         0.995


          Covariance Coverage
             WAISCALC
             ________
 WAISCALC       0.995


    PROPORTION OF DATA PRESENT FOR FEMALE


          Covariance Coverage
```

Scalar Model Estimates ...

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	0.990				
WJSPL	0.990	1.000			
WAISSPL	0.980	0.990	0.990		
WRATCALC	0.990	1.000	0.990	1.000	
WJCALC	0.970	0.980	0.970	0.980	0.980
WAISCALC	0.980	0.990	0.990	0.990	0.970

Covariance Coverage

	WAISCALC
WAISCALC	0.990

```
THE MODEL ESTIMATION TERMINATED NORMALLY



MODEL FIT INFORMATION

Number of Free Parameters                        30

Loglikelihood
```

Scalar Model Estimates ...

```
          H0 Value                      -5120.981
          H1 Value                      -5102.800

Information Criteria

          Akaike (AIC)                  10301.961
          Bayesian (BIC)                10415.198
          Sample-Size Adjusted BIC      10320.042
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                            36.362
          Degrees of Freedom                   24
          P-Value                          0.0506

Chi-Square Contribution From Each Group

          MALE                             13.816
          FEMALE                           22.546

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                          0.057
          90 Percent C.I.                   0.000  0.092
```

Scalar Model Estimates ...

```
          Probability RMSEA <= .05              0.356

CFI/TLI

          CFI                                   0.993
          TLI                                   0.992

Chi-Square Test of Model Fit for the Baseline Model

          Value                              1905.113
          Degrees of Freedom                       30
          P-Value                              0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                 0.053



MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE
```

Scalar Model Estimates ...

```
MATH     BY
   WRATCALC           5.907       0.317      18.658       0.000
   WJCALC             4.017       0.232      17.352       0.000
   WAISCALC           2.341       0.175      13.388       0.000

SPELL    BY
   WRATSPL            6.385       0.332      19.249       0.000
   WJSPL              6.634       0.346      19.183       0.000
   WAISSPL            6.221       0.326      19.064       0.000

SPELL    WITH
   MATH               0.519       0.053       9.732       0.000

Means
   MATH               0.000       0.000     999.000     999.000
   SPELL              0.000       0.000     999.000     999.000

Intercepts
   WRATSPL           36.157       0.449      80.484       0.000
   WJSPL             41.317       0.466      88.688       0.000
   WAISSPL           36.843       0.440      83.721       0.000
   WRATCALC          39.217       0.419      93.584       0.000
   WJCALC            24.016       0.295      81.470       0.000
   WAISCALC          11.125       0.207      53.742       0.000

Variances
```

Scalar Model Estimates ...

```
    MATH              1.000      0.000    999.000    999.000
    SPELL             1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL           4.761      0.721      6.603      0.000
    WJSPL             5.173      0.791      6.541      0.000
    WAISSPL           5.235      0.734      7.134      0.000
    WRATCALC          4.518      1.204      3.753      0.000
    WJCALC            3.911      0.655      5.970      0.000
    WAISCALC          5.431      0.576      9.435      0.000

Group FEMALE

 MATH     BY
    WRATCALC          5.907      0.317     18.658      0.000
    WJCALC            4.017      0.232     17.352      0.000
    WAISCALC          2.341      0.175     13.388      0.000

 SPELL    BY
    WRATSPL           6.385      0.332     19.249      0.000
    WJSPL             6.634      0.346     19.183      0.000
    WAISSPL           6.221      0.326     19.064      0.000

 SPELL    WITH
    MATH              0.731      0.151      4.855      0.000
```

Scalar Model Estimates ...

Means				
MATH	-0.160	0.131	-1.223	0.221
SPELL	0.165	0.127	1.296	0.195
Intercepts				
WRATSPL	36.157	0.449	80.484	0.000
WJSPL	41.317	0.466	88.688	0.000
WAISSPL	36.843	0.440	83.721	0.000
WRATCALC	39.217	0.419	93.584	0.000
WJCALC	24.016	0.295	81.470	0.000
WAISCALC	11.125	0.207	53.742	0.000
Variances				
MATH	1.167	0.212	5.502	0.000
SPELL	1.118	0.197	5.676	0.000
Residual Variances				
WRATSPL	5.681	1.123	5.056	0.000
WJSPL	3.237	0.930	3.481	0.000
WAISSPL	5.035	1.032	4.880	0.000
WRATCALC	2.841	1.424	1.995	0.046
WJCALC	3.743	0.793	4.723	0.000
WAISCALC	4.977	0.766	6.496	0.000

STANDARDIZED MODEL RESULTS

Scalar Model Estimates ...

```
STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE

 MATH     BY
    WRATCALC           0.941      0.016     57.567      0.000
    WJCALC             0.897      0.019     46.272      0.000
    WAISCALC           0.709      0.034     20.811      0.000

 SPELL    BY
    WRATSPL            0.946      0.009    102.225      0.000
    WJSPL              0.946      0.010     97.559      0.000
    WAISSPL            0.939      0.010     93.435      0.000

 SPELL    WITH
    MATH               0.519      0.053      9.732      0.000

 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000
```

Scalar Model Estimates ...

```
 Intercepts
    WRATSPL            5.359      0.260     20.603      0.000
    WJSPL              5.892      0.282     20.925      0.000
    WAISSPL            5.558      0.267     20.819      0.000
    WRATCALC           6.247      0.304     20.582      0.000
    WJCALC             5.363      0.254     21.086      0.000
    WAISCALC           3.368      0.160     21.063      0.000

 Variances
    MATH               1.000      0.000    999.000    999.000
    SPELL              1.000      0.000    999.000    999.000

 Residual Variances
    WRATSPL            0.105      0.018      5.970      0.000
    WJSPL              0.105      0.018      5.733      0.000
    WAISSPL            0.119      0.019      6.319      0.000
    WRATCALC           0.115      0.031      3.727      0.000
    WJCALC             0.195      0.035      5.607      0.000
    WAISCALC           0.498      0.048     10.312      0.000

Group FEMALE

 MATH     BY
    WRATCALC           0.967      0.018     54.795      0.000
    WJCALC             0.913      0.020     45.824      0.000
    WAISCALC           0.750      0.039     19.366      0.000
```

Scalar Model Estimates ...

```
SPELL    BY
   WRATSPL            0.943       0.014      69.796       0.000
   WJSPL              0.969       0.010     101.046       0.000
   WAISSPL            0.946       0.013      73.563       0.000

SPELL    WITH
   MATH               0.640       0.063      10.128       0.000

Means
   MATH              -0.148       0.121      -1.220       0.222
   SPELL              0.156       0.121       1.293       0.196

Intercepts
   WRATSPL            5.050       0.343      14.709       0.000
   WJSPL              5.705       0.403      14.156       0.000
   WAISSPL            5.301       0.363      14.585       0.000
   WRATCALC           5.942       0.419      14.183       0.000
   WJCALC             5.054       0.350      14.428       0.000
   WAISCALC           3.299       0.210      15.730       0.000

Variances
   MATH               1.000       0.000     999.000     999.000
   SPELL              1.000       0.000     999.000     999.000

Residual Variances
```

Scalar Model Estimates ...

WRATSPL	0.111	0.025	4.349	0.000
WJSPL	0.062	0.019	3.324	0.001
WAISSPL	0.104	0.024	4.280	0.000
WRATCALC	0.065	0.034	1.912	0.056
WJCALC	0.166	0.036	4.553	0.000
WAISCALC	0.438	0.058	7.536	0.000

R-SQUARE

Group MALE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
WRATSPL	0.895	0.018	51.112	0.000
WJSPL	0.895	0.018	48.779	0.000
WAISSPL	0.881	0.019	46.717	0.000
WRATCALC	0.885	0.031	28.784	0.000
WJCALC	0.805	0.035	23.136	0.000
WAISCALC	0.502	0.048	10.406	0.000

Group FEMALE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value

Scalar Model Estimates ...

```
    WRATSPL            0.889      0.025     34.898      0.000
    WJSPL              0.938      0.019     50.523      0.000
    WAISSPL            0.896      0.024     36.781      0.000
    WRATCALC           0.935      0.034     27.398      0.000
    WJCALC             0.834      0.036     22.912      0.000
    WAISCALC           0.562      0.058      9.683      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.345E-04
       (ratio of smallest to largest eigenvalue)


TECHNICAL 1 OUTPUT


     PARAMETER SPECIFICATION FOR MALE


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
      1           1             2             3             4             5
```

Scalar Model Estimates ...

```
        NU
           WAISCALC
           --------
     1          6


        LAMBDA
           MATH          SPELL
           --------      --------
WRATSPL         0             7
WJSPL           0             8
WAISSPL         0             9
WRATCALC       10             0
WJCALC         11             0
WAISCALC       12             0


        THETA
           WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
           --------      --------      --------      --------      --------
WRATSPL        13
WJSPL           0            14
WAISSPL         0             0            15
WRATCALC        0             0             0            16
WJCALC          0             0             0             0            17
```

Scalar Model Estimates ...

```
WAISCALC          0             0             0             0             0


          THETA
             WAISCALC
             --------
WAISCALC          18


          ALPHA
             MATH          SPELL
             --------      --------
     1          0             0


          BETA
             MATH          SPELL
             --------      --------
MATH              0             0
SPELL             0             0


          PSI
             MATH          SPELL
             --------      --------
MATH              0
```

Scalar Model Estimates ...

```
SPELL              19             0


    PARAMETER SPECIFICATION FOR FEMALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
     1           1             2             3             4             5


          NU
             WAISCALC
             --------
     1           6


          LAMBDA
             MATH          SPELL
             --------      --------
WRATSPL            0             7
WJSPL              0             8
WAISSPL            0             9
WRATCALC          10             0
WJCALC            11             0
```

Scalar Model Estimates ...

```
WAISCALC        12            0


         THETA
            WRATSPL       WJSPL       WAISSPL      WRATCALC      WJCALC
            --------    --------    --------     --------     --------
WRATSPL        20
WJSPL           0          21
WAISSPL         0           0          22
WRATCALC        0           0           0           23
WJCALC          0           0           0            0           24
WAISCALC        0           0           0            0            0


         THETA
            WAISCALC
            --------
WAISCALC       25


         ALPHA
            MATH          SPELL
            --------    --------
     1         26          27
```

Scalar Model Estimates ...

```
           BETA
              MATH          SPELL
              --------      --------
 MATH                0             0
 SPELL               0             0


           PSI
              MATH          SPELL
              --------      --------
 MATH               28
 SPELL              29            30


     STARTING VALUES FOR MALE


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
      1         36.172        41.371        36.786        39.222        23.910


           NU
              WAISCALC
              --------
```

Scalar Model Estimates ...

```
     1        11.350


           LAMBDA
              MATH          SPELL
              --------      --------
 WRATSPL        0.000         1.000
 WJSPL          0.000         1.000
 WAISSPL        0.000         1.000
 WRATCALC       1.000         0.000
 WJCALC         1.000         0.000
 WAISCALC       1.000         0.000


           THETA
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
 WRATSPL       22.166
 WJSPL          0.000        25.479
 WAISSPL        0.000         0.000        21.761
 WRATCALC       0.000         0.000         0.000        19.254
 WJCALC         0.000         0.000         0.000         0.000        10.272
 WAISCALC       0.000         0.000         0.000         0.000         0.000


           THETA
```

Scalar Model Estimates ...

```
             WAISCALC
             --------
 WAISCALC      5.605


          ALPHA
             MATH          SPELL
             --------      --------
      1        0.000         0.000


          BETA
             MATH          SPELL
             --------      --------
 MATH          0.000         0.000
 SPELL         0.000         0.000


          PSI
             MATH          SPELL
             --------      --------
 MATH          1.000
 SPELL         0.000         1.000


     STARTING VALUES FOR FEMALE
```

Scalar Model Estimates ...

```
         NU
            WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
            --------      --------      --------      --------      --------
 1           36.172        41.371        36.786        39.222        23.910


         NU
            WAISCALC
            --------
 1           11.350


         LAMBDA
            MATH          SPELL
            --------      --------
 WRATSPL       0.000         7.010
 WJSPL         0.000         6.828
 WAISSPL       0.000         6.653
 WRATCALC      6.633         0.000
 WJCALC        4.167         0.000
 WAISCALC      2.259         0.000


         THETA
```

Scalar Model Estimates ...

```
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
WRATSPL        22.166
WJSPL           0.000        25.479
WAISSPL         0.000         0.000        21.761
WRATCALC        0.000         0.000         0.000        19.254
WJCALC          0.000         0.000         0.000         0.000        10.272
WAISCALC        0.000         0.000         0.000         0.000         0.000


           THETA
              WAISCALC
              --------
WAISCALC        5.605


           ALPHA
              MATH          SPELL
              --------      --------
     1          0.000         0.000


           BETA
              MATH          SPELL
              --------      --------
MATH            0.000         0.000
```

Scalar Model Estimates ...

```
 SPELL          0.000         0.000


          PSI
             MATH          SPELL
             ________      ________
 MATH           1.000
 SPELL          0.000         1.000


     Beginning Time:  21:42:53
        Ending Time:  21:42:53
       Elapsed Time:  00:00:00



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```

Nested Model Comparisons

- The most general model is the Configural Model, the two groups are treated differently
- We want to know if the “same loadings” requirement caused the quality of the fitted model to be reduced.
  - This is called a “chi square difference test”. The estimated χ^2stats from the 2 models are used

$$\chi^2_{diff} = \chi^2_{metric} - \chi^2_{config}$$

  The degrees of freedom for

$$\chi^2_{diff} = df_{metric} - df_{config}$$

- If that test says “there’s no difference”, then we proceed to compare the scalar model.

This is a special Mplus feature

- Mplus 7.1 introduced an automated way to conduct the difference of models test
- The Measurement Invariance Testing offered by Mplus, is considered to be one of the value-added components in the program.
- Related to the "difftest", another way Mplus offers to test the difference between two models.

Request a Nested Model Comparison

- Modify the MODEL and ANALYSIS stanzas in the Mplus program.
- The first part of program is the same

```
TITLE:
Example 3 - Multiple Group Confirmatory Factor
   Analysis (Strong Invariance)
DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
     age;
  USEVARIABLES ARE
  wratspl wjspl waisspl wratcalc wjcalc waiscalc;
  MISSING ARE all(99999);
  GROUPING IS female (0=male 1 =female);
```

Request a Nested Model Comparison ...

- Here is the part that is different:

```
ANALYSIS:
    Model = configural metric scalar;
MODEL:
  MATH BY wratcalc wjcalc waiscalc;
  SPELL BY wratspl wjspl waisspl;
```

- Same ending stanza

```
OUTPUT:
    TECH1;
  STDYX;
```

Scalar Invariance result

View Online: cfa-01-4-shortcut.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:42 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 3 - Multiple Group Confirmatory Factor Analysis

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wratspl wjspl waisspl wratcalc wjcalc waiscalc;
```

Scalar Invariance result ...

```
    MISSING ARE all(99999);

    GROUPING IS female(0=male 1=female);

  ANALYSIS:
    Model = configural metric scalar;

  MODEL:
    MATH BY wratcalc wjcalc waiscalc;
    SPELL BY wratspl wjspl waisspl;

  OUTPUT:
    TECH1;



INPUT READING TERMINATED NORMALLY




Example 3 - Multiple Group Confirmatory Factor Analysis

SUMMARY OF ANALYSIS

Number of groups                                          2
```

Scalar Invariance result ...

```
Number of observations
   Group MALE                                                 221
   Group FEMALE                                               101
   Total sample size                                          322

Number of dependent variables                                   6
Number of independent variables                                 0
Number of continuous latent variables                           2

Observed dependent variables

  Continuous
   WRATSPL     WJSPL       WAISSPL     WRATCALC    WJCALC      WAISCALC

Continuous latent variables
   MATH        SPELL

Variables with special functions

  Grouping variable     FEMALE

Estimator                                                      ML
Information matrix                                       OBSERVED
Maximum number of iterations                                 1000
Convergence criterion                                   0.500D-04
Maximum number of steepest descent iterations                  20
```

Scalar Invariance result ...

```
Maximum number of iterations for H1                          2000
Convergence criterion for H1                            0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA

   Group MALE
     Number of missing data patterns            2

   Group FEMALE
     Number of missing data patterns            4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT FOR MALE
```

Scalar Invariance result ...

```
           Covariance Coverage
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
 WRATSPL        1.000
 WJSPL          1.000         1.000
 WAISSPL        0.995         0.995         0.995
 WRATCALC       1.000         1.000         0.995         1.000
 WJCALC         1.000         1.000         0.995         1.000         1.000
 WAISCALC       0.995         0.995         0.995         0.995         0.995


           Covariance Coverage
              WAISCALC
              ________
 WAISCALC       0.995


     PROPORTION OF DATA PRESENT FOR FEMALE


           Covariance Coverage
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
 WRATSPL        0.990
 WJSPL          0.990         1.000
 WAISSPL        0.980         0.990         0.990
```

Scalar Invariance result ...

```
WRATCALC      0.990       1.000       0.990       1.000
WJCALC        0.970       0.980       0.970       0.980       0.980
WAISCALC      0.980       0.990       0.990       0.990       0.970


          Covariance Coverage
             WAISCALC
             ________
WAISCALC      0.990


MODEL FIT INFORMATION

Invariance Testing

                   Number of                   Degrees of
     Model         Parameters      Chi-Square    Freedom      P-Value

     Configural        38              19.215        16         0.2577
     Metric            34              25.871        20         0.1701
     Scalar            30              36.362        24         0.0506

                                               Degrees of
     Models Compared               Chi-Square    Freedom      P-Value

     Metric against Configural          6.656         4         0.1552
```

Scalar Invariance result ...

```
    Scalar against Configural        17.146          8         0.0286
    Scalar against Metric            10.490          4         0.0329


MODEL FIT INFORMATION FOR THE CONFIGURAL MODEL

Number of Free Parameters                        38

Loglikelihood

         H0 Value                         -5112.407
         H1 Value                         -5102.800

Information Criteria

         Akaike (AIC)                     10300.815
         Bayesian (BIC)                   10444.248
         Sample-Size Adjusted BIC         10323.717
           (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

         Value                               19.215
         Degrees of Freedom                      16
         P-Value                             0.2577
```

Scalar Invariance result ...

```
Chi-Square Contribution and P-Value From Each Group (degrees of freedom = 8)

         MALE                                7.660     0.467
         FEMALE                             11.555     0.172

RMSEA (Root Mean Square Error Of Approximation)

         Estimate                            0.035
         90 Percent C.I.                     0.000  0.085
         Probability RMSEA <= .05            0.631

CFI/TLI

         CFI                                 0.998
         TLI                                 0.997

Chi-Square Test of Model Fit for the Baseline Model

         Value                            1905.113
         Degrees of Freedom                     30
         P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

         Value                               0.029
```

Scalar Invariance result ...

```
MODEL FIT INFORMATION FOR THE METRIC MODEL

Number of Free Parameters                       34

Loglikelihood

          H0 Value                       -5115.735
          H1 Value                       -5102.800

Information Criteria

          Akaike (AIC)                   10299.471
          Bayesian (BIC)                 10427.805
          Sample-Size Adjusted BIC       10319.962
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             25.871
          Degrees of Freedom                    20
          P-Value                           0.1701

Chi-Square Contribution From Each Group

          MALE                              10.290
```

Scalar Invariance result ...

```
          FEMALE                                 15.581

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                                0.043
          90 Percent C.I.                         0.000  0.085
          Probability RMSEA <= .05                0.566

CFI/TLI

          CFI                                     0.997
          TLI                                     0.995

Chi-Square Test of Model Fit for the Baseline Model

          Value                                1905.113
          Degrees of Freedom                         30
          P-Value                                0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                   0.054


MODEL FIT INFORMATION FOR THE SCALAR MODEL
```

Scalar Invariance result ...

```
Number of Free Parameters                       30

Loglikelihood

          H0 Value                       -5120.981
          H1 Value                       -5102.800

Information Criteria

          Akaike (AIC)                   10301.961
          Bayesian (BIC)                 10415.198
          Sample-Size Adjusted BIC       10320.042
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                             36.362
          Degrees of Freedom                    24
          P-Value                           0.0506

Chi-Square Contribution From Each Group

          MALE                              13.815
          FEMALE                            22.547

RMSEA (Root Mean Square Error Of Approximation)
```

Scalar Invariance result ...

```
          Estimate                                  0.057
          90 Percent C.I.                           0.000  0.092
          Probability RMSEA <= .05                  0.356

CFI/TLI

          CFI                                       0.993
          TLI                                       0.992

Chi-Square Test of Model Fit for the Baseline Model

          Value                                  1905.113
          Degrees of Freedom                           30
          P-Value                                  0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                     0.053



MODEL RESULTS FOR THE CONFIGURAL MODEL

                                                     Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
```

Scalar Invariance result ...

```
Group MALE

 MATH     BY
    WRATCALC          1.000        0.000      999.000      999.000
    WJCALC            0.715        0.037       19.444        0.000
    WAISCALC          0.424        0.032       13.334        0.000

 SPELL    BY
    WRATSPL           1.000        0.000      999.000      999.000
    WJSPL             1.081        0.037       29.078        0.000
    WAISSPL           0.984        0.036       27.585        0.000

 SPELL    WITH
    MATH             18.836        2.940        6.407        0.000

 Means
    MATH              0.000        0.000      999.000      999.000
    SPELL             0.000        0.000      999.000      999.000

 Intercepts
    WRATSPL          36.172        0.448       80.762        0.000
    WJSPL            41.371        0.480       86.157        0.000
    WAISSPL          36.767        0.444       82.899        0.000
    WRATCALC         39.222        0.417       93.962        0.000
    WJCALC           23.910        0.305       78.420        0.000
```

Scalar Invariance result ...

```
    WAISCALC          11.367       0.226      50.321       0.000

 Variances
    MATH              33.227       3.799       8.746       0.000
    SPELL             39.417       4.228       9.323       0.000

 Residual Variances
    WRATSPL            4.915       0.726       6.772       0.000
    WJSPL              4.881       0.795       6.138       0.000
    WAISSPL            5.285       0.736       7.176       0.000
    WRATCALC           5.280       1.232       4.285       0.000
    WJCALC             3.549       0.664       5.342       0.000
    WAISCALC           5.269       0.557       9.463       0.000

Group FEMALE

 MATH     BY
    WRATCALC           1.000       0.000     999.000     999.000
    WJCALC             0.644       0.040      15.980       0.000
    WAISCALC           0.350       0.036       9.607       0.000

 SPELL    BY
    WRATSPL            1.000       0.000     999.000     999.000
    WJSPL              0.975       0.044      22.359       0.000
    WAISSPL            0.947       0.046      20.668       0.000
```

Scalar Invariance result ...

```
SPELL    WITH
   MATH              29.101      5.608      5.189      0.000

Means
   MATH               0.000      0.000    999.000    999.000
   SPELL              0.000      0.000    999.000    999.000

Intercepts
   WRATSPL           37.171      0.735     50.589      0.000
   WJSPL             42.337      0.705     60.073      0.000
   WAISSPL           38.030      0.697     54.578      0.000
   WRATCALC          38.267      0.666     57.464      0.000
   WJCALC            23.603      0.465     50.781      0.000
   WAISCALC          10.266      0.316     32.528      0.000

Variances
   MATH              42.833      6.496      6.594      0.000
   SPELL             49.143      7.669      6.408      0.000

Residual Variances
   WRATSPL            5.307      1.105      4.804      0.000
   WJSPL              3.474      0.910      3.818      0.000
   WAISSPL            4.900      1.007      4.865      0.000
   WRATCALC           1.957      1.621      1.208      0.227
   WJCALC             3.960      0.848      4.672      0.000
   WAISCALC           4.766      0.714      6.678      0.000
```

Scalar Invariance result ...

```
MODEL RESULTS FOR THE METRIC MODEL

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE

 MATH     BY
    WRATCALC           1.000      0.000    999.000    999.000
    WJCALC             0.685      0.027     24.915      0.000
    WAISCALC           0.393      0.024     16.236      0.000

 SPELL    BY
    WRATSPL            1.000      0.000    999.000    999.000
    WJSPL              1.040      0.029     35.953      0.000
    WAISSPL            0.973      0.028     34.330      0.000

 SPELL    WITH
    MATH              19.557      3.018      6.479      0.000

 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000
```

Scalar Invariance result ...

```
 Intercepts
    WRATSPL           36.172      0.454     79.673      0.000
    WJSPL             41.371      0.472     87.637      0.000
    WAISSPL           36.767      0.446     82.522      0.000
    WRATCALC          39.222      0.422     92.879      0.000
    WJCALC            23.910      0.302     79.191      0.000
    WAISCALC          11.367      0.221     51.462      0.000

 Variances
    MATH              34.791      3.738      9.307      0.000
    SPELL             40.789      4.239      9.622      0.000

 Residual Variances
    WRATSPL            4.764      0.721      6.607      0.000
    WJSPL              5.154      0.790      6.526      0.000
    WAISSPL            5.233      0.732      7.153      0.000
    WRATCALC           4.620      1.205      3.834      0.000
    WJCALC             3.835      0.654      5.864      0.000
    WAISCALC           5.370      0.564      9.525      0.000

Group FEMALE

 MATH     BY
    WRATCALC           1.000      0.000    999.000    999.000
    WJCALC             0.685      0.027     24.915      0.000
    WAISCALC           0.393      0.024     16.236      0.000
```

Scalar Invariance result ...

```
SPELL    BY
   WRATSPL          1.000      0.000    999.000    999.000
   WJSPL            1.040      0.029     35.953      0.000
   WAISSPL          0.973      0.028     34.330      0.000

SPELL    WITH
   MATH            27.515      5.266      5.225      0.000

Means
   MATH             0.000      0.000    999.000    999.000
   SPELL            0.000      0.000    999.000    999.000

Intercepts
   WRATSPL         37.172      0.713     52.138      0.000
   WJSPL           42.337      0.721     58.721      0.000
   WAISSPL         38.031      0.691     55.053      0.000
   WRATCALC        38.267      0.656     58.322      0.000
   WJCALC          23.602      0.474     49.834      0.000
   WAISCALC        10.263      0.330     31.086      0.000

Variances
   MATH            40.435      6.233      6.487      0.000
   SPELL           45.576      6.856      6.648      0.000

Residual Variances
```

Scalar Invariance result ...

```
    WRATSPL            5.686      1.124      5.061      0.000
    WJSPL              3.230      0.929      3.478      0.001
    WAISSPL            4.995      1.023      4.883      0.000
    WRATCALC           3.048      1.413      2.157      0.031
    WJCALC             3.603      0.778      4.631      0.000
    WAISCALC           4.700      0.714      6.586      0.000


MODEL RESULTS FOR THE SCALAR MODEL

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Group MALE

 MATH     BY
    WRATCALC           1.000      0.000    999.000    999.000
    WJCALC             0.680      0.027     24.897      0.000
    WAISCALC           0.396      0.024     16.225      0.000

 SPELL    BY
    WRATSPL            1.000      0.000    999.000    999.000
    WJSPL              1.039      0.029     36.045      0.000
    WAISSPL            0.974      0.028     34.431      0.000

 SPELL    WITH
```

Scalar Invariance result ...

```
    MATH              19.579      3.020      6.482      0.000

 Means
    MATH               0.000      0.000    999.000    999.000
    SPELL              0.000      0.000    999.000    999.000

 Intercepts
    WRATSPL           36.157      0.449     80.484      0.000
    WJSPL             41.316      0.466     88.687      0.000
    WAISSPL           36.843      0.440     83.720      0.000
    WRATCALC          39.217      0.419     93.583      0.000
    WJCALC            24.016      0.295     81.469      0.000
    WAISCALC          11.125      0.207     53.742      0.000

 Variances
    MATH              34.893      3.740      9.329      0.000
    SPELL             40.764      4.236      9.624      0.000

 Residual Variances
    WRATSPL            4.762      0.721      6.603      0.000
    WJSPL              5.173      0.791      6.541      0.000
    WAISSPL            5.235      0.734      7.134      0.000
    WRATCALC           4.518      1.204      3.753      0.000
    WJCALC             3.911      0.655      5.970      0.000
    WAISCALC           5.431      0.576      9.435      0.000
```

Scalar Invariance result ...

```
Group FEMALE

 MATH     BY
    WRATCALC          1.000      0.000    999.000    999.000
    WJCALC            0.680      0.027     24.897      0.000
    WAISCALC          0.396      0.024     16.225      0.000

 SPELL    BY
    WRATSPL           1.000      0.000    999.000    999.000
    WJSPL             1.039      0.029     36.045      0.000
    WAISSPL           0.974      0.028     34.431      0.000

 SPELL    WITH
    MATH             27.567      5.276      5.225      0.000

 Means
    MATH             -0.944      0.771     -1.225      0.221
    SPELL             1.054      0.811      1.298      0.194

 Intercepts
    WRATSPL          36.157      0.449     80.484      0.000
    WJSPL            41.316      0.466     88.687      0.000
    WAISSPL          36.843      0.440     83.720      0.000
    WRATCALC         39.217      0.419     93.583      0.000
    WJCALC           24.016      0.295     81.469      0.000
    WAISCALC         11.125      0.207     53.742      0.000
```

Scalar Invariance result ...

```
 Variances
    MATH              40.717      6.267      6.497      0.000
    SPELL             45.577      6.854      6.650      0.000

 Residual Variances
    WRATSPL            5.681      1.123      5.056      0.000
    WJSPL              3.237      0.930      3.481      0.000
    WAISSPL            5.035      1.032      4.880      0.000
    WRATCALC           2.841      1.424      1.995      0.046
    WJCALC             3.743      0.793      4.723      0.000
    WAISCALC           4.977      0.766      6.496      0.000


QUALITY OF NUMERICAL RESULTS FOR THE CONFIGURAL MODEL

     Condition Number for the Information Matrix              0.211E-04
       (ratio of smallest to largest eigenvalue)


QUALITY OF NUMERICAL RESULTS FOR THE METRIC MODEL

     Condition Number for the Information Matrix              0.622E-04
       (ratio of smallest to largest eigenvalue)
```

Scalar Invariance result ...

```
QUALITY OF NUMERICAL RESULTS FOR THE SCALAR MODEL

     Condition Number for the Information Matrix              0.342E-04
       (ratio of smallest to largest eigenvalue)


TECHNICAL 1 OUTPUT FOR THE CONFIGURAL MODEL


     PARAMETER SPECIFICATION FOR MALE


           NU
              WRATSPL        WJSPL          WAISSPL        WRATCALC       WJCALC
              ________       ________       ________       ________       ________
      1           1              2              3              4              5


           NU
              WAISCALC
              ________
      1           6


           LAMBDA
              MATH           SPELL
```

Scalar Invariance result ...

```
            --------   --------
WRATSPL            0          0
WJSPL              0          7
WAISSPL            0          8
WRATCALC           0          0
WJCALC             9          0
WAISCALC          10          0


         THETA
            WRATSPL    WJSPL      WAISSPL    WRATCALC   WJCALC
            --------   --------   --------   --------   --------
WRATSPL           11
WJSPL              0         12
WAISSPL            0          0         13
WRATCALC           0          0          0         14
WJCALC             0          0          0          0         15
WAISCALC           0          0          0          0          0


         THETA
            WAISCALC
            --------
WAISCALC          16
```

Scalar Invariance result ...

```
          ALPHA
             MATH          SPELL
             --------      --------
     1          0             0


          BETA
             MATH          SPELL
             --------      --------
 MATH             0             0
 SPELL            0             0


          PSI
             MATH          SPELL
             --------      --------
 MATH            17
 SPELL           18            19


     PARAMETER SPECIFICATION FOR FEMALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
```

Scalar Invariance result ...

```
      1          20          21          22          23          24


          NU
             WAISCALC
             --------
      1         25


          LAMBDA
             MATH          SPELL
             --------      --------
 WRATSPL            0             0
 WJSPL              0            26
 WAISSPL            0            27
 WRATCALC           0             0
 WJCALC            28             0
 WAISCALC          29             0


          THETA
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
 WRATSPL           30
 WJSPL              0            31
 WAISSPL            0             0            32
```

Scalar Invariance result ...

```
WRATCALC          0          0          0         33
WJCALC            0          0          0          0         34
WAISCALC          0          0          0          0          0


         THETA
            WAISCALC
            --------
WAISCALC         35


         ALPHA
            MATH          SPELL
            --------      --------
     1            0             0


         BETA
            MATH          SPELL
            --------      --------
MATH              0             0
SPELL             0             0


         PSI
            MATH          SPELL
```

Scalar Invariance result ...

```
               --------      --------
MATH              36
SPELL             37            38


    STARTING VALUES FOR MALE


          NU
             WRATSPL       WJSPL        WAISSPL      WRATCALC      WJCALC
             --------      --------     --------     --------      --------
     1        36.172        41.371       36.786       39.222        23.910


          NU
             WAISCALC
             --------
     1        11.350


          LAMBDA
             MATH          SPELL
             --------      --------
WRATSPL        0.000         1.000
WJSPL          0.000         1.082
WAISSPL        0.000         0.984
```

Scalar Invariance result ...

```
WRATCALC      1.000         0.000
WJCALC        0.719         0.000
WAISCALC      0.421         0.000


          THETA
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC

             --------      --------      --------      --------      --------
WRATSPL      22.166
WJSPL         0.000        25.479
WAISSPL       0.000         0.000        21.761
WRATCALC      0.000         0.000         0.000        19.254
WJCALC        0.000         0.000         0.000         0.000        10.272
WAISCALC      0.000         0.000         0.000         0.000         0.000


          THETA
             WAISCALC

             --------
WAISCALC      5.605


          ALPHA
             MATH          SPELL

             --------      --------
     1        0.000         0.000
```

Scalar Invariance result ...

```
          BETA
             MATH          SPELL
             --------      --------
MATH           0.000         0.000
SPELL          0.000         0.000


          PSI
             MATH          SPELL
             --------      --------
MATH           0.050
SPELL          0.000         0.050


    STARTING VALUES FOR FEMALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
     1        36.172        41.371        36.786        39.222        23.910


          NU
```

Scalar Invariance result ...

	WAISCALC
1	11.350

LAMBDA

	MATH	SPELL
WRATSPL	0.000	1.000
WJSPL	0.000	0.974
WAISSPL	0.000	0.949
WRATCALC	1.000	0.000
WJCALC	0.628	0.000
WAISCALC	0.341	0.000

THETA

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	22.166				
WJSPL	0.000	25.479			
WAISSPL	0.000	0.000	21.761		
WRATCALC	0.000	0.000	0.000	19.254	
WJCALC	0.000	0.000	0.000	0.000	10.272
WAISCALC	0.000	0.000	0.000	0.000	0.000

Scalar Invariance result ...

```
         THETA
            WAISCALC
            --------
WAISCALC       5.605


         ALPHA
            MATH          SPELL
            --------      --------
     1         0.000         0.000


         BETA
            MATH          SPELL
            --------      --------
MATH           0.000         0.000
SPELL          0.000         0.000


         PSI
            MATH          SPELL
            --------      --------
MATH           0.050
SPELL          0.000         0.050
```

Scalar Invariance result ...

```
TECHNICAL 1 OUTPUT FOR THE METRIC MODEL


     PARAMETER SPECIFICATION FOR MALE


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
      1           1             2             3             4             5


           NU
              WAISCALC
              ________
      1           6


           LAMBDA
              MATH          SPELL
              ________      ________
 WRATSPL            0             0
 WJSPL              0             7
 WAISSPL            0             8
 WRATCALC           0             0
```

Scalar Invariance result ...

```
WJCALC             9             0
WAISCALC          10             0


          THETA
             WRATSPL       WJSPL       WAISSPL      WRATCALC      WJCALC
            --------     --------     --------     --------     --------
WRATSPL           11
WJSPL              0           12
WAISSPL            0            0           13
WRATCALC           0            0            0           14
WJCALC             0            0            0            0           15
WAISCALC           0            0            0            0            0


          THETA
             WAISCALC
            --------
WAISCALC          16


          ALPHA
             MATH          SPELL
            --------     --------
     1           0            0
```

Scalar Invariance result ...

```
         BETA
            MATH          SPELL
            --------      --------
MATH               0             0
SPELL              0             0


         PSI
            MATH          SPELL
            --------      --------
MATH              17
SPELL             18            19


    PARAMETER SPECIFICATION FOR FEMALE


         NU
            WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
            --------      --------      --------      --------      --------
     1            20            21            22            23            24


         NU
            WAISCALC
```

Scalar Invariance result ...

1	25

LAMBDA

	MATH	SPELL
WRATSPL	0	0
WJSPL	0	7
WAISSPL	0	8
WRATCALC	0	0
WJCALC	9	0
WAISCALC	10	0

THETA

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	26				
WJSPL	0	27			
WAISSPL	0	0	28		
WRATCALC	0	0	0	29	
WJCALC	0	0	0	0	30
WAISCALC	0	0	0	0	0

Scalar Invariance result ...

```
         THETA
            WAISCALC
            --------
WAISCALC          31


         ALPHA
            MATH          SPELL
            --------      --------
     1            0             0


         BETA
            MATH          SPELL
            --------      --------
MATH              0             0
SPELL             0             0


         PSI
            MATH          SPELL
            --------      --------
MATH             32
SPELL            33            34
```

Scalar Invariance result ...

```
 STARTING VALUES FOR MALE


         NU
            WRATSPL       WJSPL      WAISSPL     WRATCALC      WJCALC
            --------    --------    --------    --------    --------
  1          36.172      41.371      36.786      39.222      23.910


         NU
            WAISCALC
            --------
  1          11.350


         LAMBDA
            MATH        SPELL
            --------    --------
 WRATSPL       0.000       1.000
 WJSPL         0.000       1.082
 WAISSPL       0.000       0.984
 WRATCALC      1.000       0.000
 WJCALC        0.719       0.000
 WAISCALC      0.421       0.000
```

Scalar Invariance result ...

```
         THETA
            WRATSPL        WJSPL          WAISSPL        WRATCALC       WJCALC
            --------       --------       --------       --------       --------
WRATSPL       22.166
WJSPL          0.000         25.479
WAISSPL        0.000          0.000         21.761
WRATCALC       0.000          0.000          0.000         19.254
WJCALC         0.000          0.000          0.000          0.000         10.272
WAISCALC       0.000          0.000          0.000          0.000          0.000


         THETA
            WAISCALC
            --------
WAISCALC       5.605


         ALPHA
            MATH           SPELL
            --------       --------
     1         0.000          0.000


         BETA
            MATH           SPELL
            --------       --------
```

Scalar Invariance result ...

```
MATH           0.000         0.000
SPELL          0.000         0.000


          PSI
             MATH          SPELL
             --------      --------
MATH           0.050
SPELL          0.000         0.050


    STARTING VALUES FOR FEMALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
     1        36.172        41.371        36.786        39.222        23.910


          NU
             WAISCALC
             --------
     1        11.350
```

Scalar Invariance result ...

LAMBDA

	MATH	SPELL
WRATSPL	0.000	1.000
WJSPL	0.000	0.974
WAISSPL	0.000	0.949
WRATCALC	1.000	0.000
WJCALC	0.628	0.000
WAISCALC	0.341	0.000

THETA

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	22.166				
WJSPL	0.000	25.479			
WAISSPL	0.000	0.000	21.761		
WRATCALC	0.000	0.000	0.000	19.254	
WJCALC	0.000	0.000	0.000	0.000	10.272
WAISCALC	0.000	0.000	0.000	0.000	0.000

THETA

	WAISCALC
WAISCALC	5.605

Scalar Invariance result ...

```
           ALPHA
              MATH          SPELL
              --------      --------
     1          0.000         0.000


           BETA
              MATH          SPELL
              --------      --------
 MATH           0.000         0.000
 SPELL          0.000         0.000


           PSI
              MATH          SPELL
              --------      --------
 MATH           0.050
 SPELL          0.000         0.050


TECHNICAL 1 OUTPUT FOR THE SCALAR MODEL


     PARAMETER SPECIFICATION FOR MALE
```

Scalar Invariance result ...

```
         NU
            WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
            --------      --------      --------      --------      --------
 1              1             2             3             4             5


         NU
            WAISCALC
            --------
 1              6


         LAMBDA
            MATH          SPELL
            --------      --------
 WRATSPL            0             0
 WJSPL              0             7
 WAISSPL            0             8
 WRATCALC           0             0
 WJCALC             9             0
 WAISCALC          10             0


         THETA
```

Scalar Invariance result ...

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
	--------	--------	--------	--------	--------
WRATSPL	11				
WJSPL	0	12			
WAISSPL	0	0	13		
WRATCALC	0	0	0	14	
WJCALC	0	0	0	0	15
WAISCALC	0	0	0	0	0

THETA

	WAISCALC

WAISCALC	16

ALPHA

	MATH	SPELL
	--------	--------
1	0	0

BETA

	MATH	SPELL
	--------	--------
MATH	0	0

Scalar Invariance result ...

```
SPELL                 0              0


         PSI
            MATH           SPELL
            --------       --------
MATH          17
SPELL         18             19


    PARAMETER SPECIFICATION FOR FEMALE


         NU
            WRATSPL        WJSPL          WAISSPL        WRATCALC       WJCALC
            --------       --------       --------       --------       --------
     1         1              2              3              4              5


         NU
            WAISCALC
            --------
     1         6


         LAMBDA
```

Scalar Invariance result ...

	MATH	SPELL
WRATSPL	0	0
WJSPL	0	7
WAISSPL	0	8
WRATCALC	0	0
WJCALC	9	0
WAISCALC	10	0

THETA

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	20				
WJSPL	0	21			
WAISSPL	0	0	22		
WRATCALC	0	0	0	23	
WJCALC	0	0	0	0	24
WAISCALC	0	0	0	0	0

THETA

	WAISCALC
WAISCALC	25

Scalar Invariance result ...

```
          ALPHA
             MATH          SPELL
             --------      --------
     1          26            27


          BETA
             MATH          SPELL
             --------      --------
 MATH              0             0
 SPELL             0             0


          PSI
             MATH          SPELL
             --------      --------
 MATH             28
 SPELL            29            30


     STARTING VALUES FOR MALE


          NU
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
```

Scalar Invariance result ...

```
             --------     --------     --------     --------     --------
       1      36.172       41.371       36.786       39.222       23.910

           NU
              WAISCALC
             --------
       1      11.350

           LAMBDA
              MATH         SPELL
             --------     --------
 WRATSPL        0.000        1.000
 WJSPL          0.000        1.082
 WAISSPL        0.000        0.984
 WRATCALC       1.000        0.000
 WJCALC         0.719        0.000
 WAISCALC       0.421        0.000

           THETA
              WRATSPL      WJSPL        WAISSPL      WRATCALC     WJCALC
             --------     --------     --------     --------     --------
 WRATSPL       22.166
 WJSPL          0.000       25.479
```

Scalar Invariance result ...

```
WAISSPL        0.000        0.000       21.761
WRATCALC       0.000        0.000        0.000       19.254
WJCALC         0.000        0.000        0.000        0.000       10.272
WAISCALC       0.000        0.000        0.000        0.000        0.000


          THETA
             WAISCALC
             --------
WAISCALC       5.605


          ALPHA
             MATH          SPELL
             --------      --------
     1         0.000         0.000


          BETA
             MATH          SPELL
             --------      --------
MATH           0.000         0.000
SPELL          0.000         0.000


          PSI
```

Scalar Invariance result ...

```
              MATH          SPELL
              --------      --------
MATH           0.050
SPELL          0.000         0.050


    STARTING VALUES FOR FEMALE


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
     1         36.172        41.371        36.786        39.222        23.910


           NU
              WAISCALC
              --------
     1         11.350


           LAMBDA
              MATH          SPELL
              --------      --------
WRATSPL        0.000         1.000
WJSPL          0.000         0.974
```

Scalar Invariance result ...

```
WAISSPL       0.000      0.949
WRATCALC      1.000      0.000
WJCALC        0.628      0.000
WAISCALC      0.341      0.000


          THETA
             WRATSPL      WJSPL        WAISSPL      WRATCALC     WJCALC
             --------     --------     --------     --------     --------
WRATSPL       22.166
WJSPL          0.000      25.479
WAISSPL        0.000       0.000       21.761
WRATCALC       0.000       0.000        0.000       19.254
WJCALC         0.000       0.000        0.000        0.000       10.272
WAISCALC       0.000       0.000        0.000        0.000        0.000


          THETA
             WAISCALC
             --------
WAISCALC       5.605


          ALPHA
             MATH         SPELL
             --------     --------
```

Scalar Invariance result ...

```
      1          0.000         0.000


             BETA
                MATH          SPELL
                --------      --------
 MATH            0.000         0.000
 SPELL           0.000         0.000


             PSI
                MATH          SPELL
                --------      --------
 MATH            0.050
 SPELL           0.000         0.050


TECHNICAL 9 OUTPUT

Error messages for the Configural Model:

THE MODEL ESTIMATION TERMINATED NORMALLY

Error messages for the Metric Model:

THE MODEL ESTIMATION TERMINATED NORMALLY
```

Scalar Invariance result ...

```
Error messages for the Scalar Model:

THE MODEL ESTIMATION TERMINATED NORMALLY



     Beginning Time:  21:42:53
        Ending Time:  21:42:53
       Elapsed Time:  00:00:00



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```

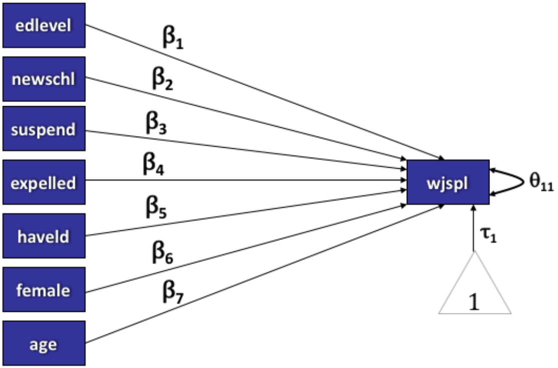
Outline

Incorporate predictors

- Mplus does same ordinary regression model as other programs might do.
- We can start from common building blocks as regression, and then add features
- Predict WJ spelling scores (wjspl) using predictor variables

Multiple Regression Diagram

Multiple Regression Diagram ...



Mplus Regression Syntax

- Note the difference is in the “ON” usage of the MODEL stanza

```
TITLE:
  Example 4 - Multiple Linear Regression
DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
     age;
  USEVARIABLES ARE
  wjspl edlevel newschl suspend expelled haveld
     female age;
  MISSING ARE all(99999);
MODEL:
```

Mplus Regression Syntax ...

```
wjspl ON edlevel newschl suspend expelled haveld
    female age;
```

Regression Results

View Online: reg-01.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:45 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 4 - Multiple Linear Regression

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wjspl edlevel newschl suspend expelled haveld female age;
```

Regression Results ...

```
    MISSING ARE all(99999);

  MODEL:
    wjspl ON edlevel newschl suspend expelled haveld female age;



*** WARNING
  Data set contains cases with missing on x-variables.
  These cases were not included in the analysis.
  Number of cases with missing on x-variables:  9
   1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS




Example 4 - Multiple Linear Regression

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         313

Number of dependent variables                                    1
Number of independent variables                                  7
Number of continuous latent variables                            0
```

Regression Results ...

```
Observed dependent variables

  Continuous
   WJSPL

Observed independent variables
   EDLEVEL     NEWSCHL     SUSPEND     EXPELLED    HAVELD      FEMALE
   AGE


Estimator                                                  ML
Information matrix                                   OBSERVED
Maximum number of iterations                             1000
Convergence criterion                               0.500D-04
Maximum number of steepest descent iterations              20
Maximum number of iterations for H1                      2000
Convergence criterion for H1                        0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA
```

Regression Results ...

```
     Number of missing data patterns             1


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT


           Covariance Coverage
              WJSPL         EDLEVEL       NEWSCHL       SUSPEND       EXPELLED
              ________      ________      ________      ________      ________
 WJSPL          1.000
 EDLEVEL        1.000         1.000
 NEWSCHL        1.000         1.000         1.000
 SUSPEND        1.000         1.000         1.000         1.000
 EXPELLED       1.000         1.000         1.000         1.000         1.000
 HAVELD         1.000         1.000         1.000         1.000         1.000
 FEMALE         1.000         1.000         1.000         1.000         1.000
 AGE            1.000         1.000         1.000         1.000         1.000


           Covariance Coverage
```

Regression Results ...

```
              HAVELD        FEMALE        AGE
              ________      ________      ________
 HAVELD         1.000
 FEMALE         1.000         1.000
 AGE            1.000         1.000         1.000


THE MODEL ESTIMATION TERMINATED NORMALLY



MODEL FIT INFORMATION

Number of Free Parameters                        9

Loglikelihood

          H0 Value                       -1021.060
          H1 Value                       -1021.060

Information Criteria

          Akaike (AIC)                    2060.120
          Bayesian (BIC)                  2093.836
          Sample-Size Adjusted BIC        2065.291
```

Regression Results ...

```
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                              0.000
          Degrees of Freedom                     0
          P-Value                           0.0000

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                           0.000
          90 Percent C.I.                    0.000  0.000
          Probability RMSEA <= .05           0.000

CFI/TLI

          CFI                                1.000
          TLI                                1.000

Chi-Square Test of Model Fit for the Baseline Model

          Value                             78.031
          Degrees of Freedom                     7
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)
```

Regression Results ...

```
      Value                                    0.000


MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 WJSPL    ON
    EDLEVEL            1.162      0.324      3.584      0.000
    NEWSCHL            0.063      0.747      0.085      0.932
    SUSPEND           -0.052      0.773     -0.067      0.947
    EXPELLED          -2.758      1.098     -2.510      0.012
    HAVELD            -6.974      0.988     -7.063      0.000
    FEMALE             0.720      0.792      0.909      0.363
    AGE                0.412      0.207      1.994      0.046

 Intercepts
    WJSPL             21.951      4.414      4.973      0.000

 Residual Variances
    WJSPL             39.905      3.190     12.510      0.000
```

Regression Results ...

```
QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.257E-03
       (ratio of smallest to largest eigenvalue)


     Beginning Time:  21:45:53
        Ending Time:  21:45:53
       Elapsed Time:  00:00:00



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```

Outline

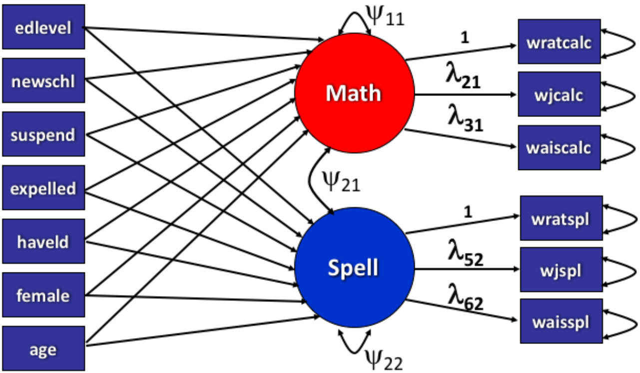
SEM = Structural Equation Model

- SEM Combines CFA and Regression Concepts
  - A latent variable can be a predictor or an outcome variable
- Observed variables can be either
  - indicators (used to identify the latent constructs)
  - exogenous predicors or outcomes that are treated as "correctly measured" variables (e.g., no measurement model)

SEM terminology

- Originally dubbed LISREL (linear structural relations) by creator Karl Joreskog
- Two components
  1. A measurement model, the CFA we did previously
  2. A regression component that treats latent variables as predictors or outcomes

SEM Diagram


edlevel
newschl
suspend
expelled
haveld
female
age
Math
Spell
ψ_{11}
ψ_{21}
ψ_{22}
1
λ_{21}
λ_{31}
1
λ_{52}
λ_{62}
wratcalc
wjcalc
waiscalc
wratspl
wjspl
waisspl

SEM Syntax

```
TITLE:
Example 5 - Structural Equation Model;

DATA:
  FILE IS "../../data/job_placement.csv";
VARIABLE:
  NAMES ARE
  id wjcalc wjspl wratspl wratcalc waiscalc waisspl
  edlevel newschl suspend expelled haveld female
     age;
  USEVARIABLES ARE
  wratspl wjspl waisspl wratcalc wjcalc waiscalc
  edlevel newschl suspend expelled haveld female
     age;
  MISSING ARE all(99999);
MODEL:
```

SEM Syntax ...

```
  MATH BY wratcalc wjcalc waiscalc;
  SPELL BY wratspl wjspl waisspl;
  MATH ON edlevel newschl suspend expelled haveld
      female age;
  SPELL ON edlevel newschl suspend expelled haveld
      female age;
  MATH WITH SPELL;
OUTPUT:
  SAMPSTAT;
  TECH1;
  STDYX;
```

SEM Results

View Online: sem-01.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:48 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 5 - Structural Equation Model

  DATA:
    FILE IS "../../data/job_placement.csv";

  VARIABLE:
    NAMES ARE
    id wjcalc wjspl wratspl wratcalc waiscalc waisspl
    edlevel newschl suspend expelled haveld female age;

    USEVARIABLES ARE
    wratspl wjspl waisspl wratcalc wjcalc waiscalc
    edlevel newschl suspend expelled haveld female age;
```

SEM Results ...

```
    MISSING ARE all(99999);

  MODEL:
    MATH BY wratcalc wjcalc waiscalc;
    SPELL BY wratspl wjspl waisspl;
    MATH ON edlevel newschl suspend expelled haveld female age;
    SPELL ON edlevel newschl suspend expelled haveld female age;
    MATH WITH SPELL;

  OUTPUT:
    SAMPSTAT;
    TECH1;
    STDYX;



*** WARNING
  Data set contains cases with missing on x-variables.
  These cases were not included in the analysis.
  Number of cases with missing on x-variables:  9
   1 WARNING(S) FOUND IN THE INPUT INSTRUCTIONS
```

SEM Results ...

```
Example 5 - Structural Equation Model

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         313

Number of dependent variables                                    6
Number of independent variables                                  7
Number of continuous latent variables                            2

Observed dependent variables

  Continuous
   WRATSPL     WJSPL       WAISSPL     WRATCALC    WJCALC      WAISCALC

Observed independent variables
   EDLEVEL     NEWSCHL     SUSPEND     EXPELLED    HAVELD      FEMALE
   AGE

Continuous latent variables
   MATH        SPELL


Estimator                                                       ML
Information matrix                                        OBSERVED
```

SEM Results ...

```
Maximum number of iterations                                   1000
Convergence criterion                                     0.500D-04
Maximum number of steepest descent iterations                    20
Maximum number of iterations for H1                            2000
Convergence criterion for H1                              0.100D-03

Input data file(s)
  ../../data/job_placement.csv

Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             4


COVARIANCE COVERAGE OF DATA

Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT


           Covariance Coverage
```

SEM Results ...

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
	--------	--------	--------	--------	--------
WRATSPL	0.997				
WJSPL	0.997	1.000			
WAISSPL	0.990	0.994	0.994		
WRATCALC	0.997	1.000	0.994	1.000	
WJCALC	0.990	0.994	0.987	0.994	0.994
WAISCALC	0.990	0.994	0.994	0.994	0.987
EDLEVEL	0.997	1.000	0.994	1.000	0.994
NEWSCHL	0.997	1.000	0.994	1.000	0.994
SUSPEND	0.997	1.000	0.994	1.000	0.994
EXPELLED	0.997	1.000	0.994	1.000	0.994
HAVELD	0.997	1.000	0.994	1.000	0.994
FEMALE	0.997	1.000	0.994	1.000	0.994
AGE	0.997	1.000	0.994	1.000	0.994

Covariance Coverage

	WAISCALC	EDLEVEL	NEWSCHL	SUSPEND	EXPELLED
	--------	--------	--------	--------	--------
WAISCALC	0.994				
EDLEVEL	0.994	1.000			
NEWSCHL	0.994	1.000	1.000		
SUSPEND	0.994	1.000	1.000	1.000	
EXPELLED	0.994	1.000	1.000	1.000	1.000
HAVELD	0.994	1.000	1.000	1.000	1.000

SEM Results ...

```
FEMALE        0.994      1.000      1.000      1.000      1.000
AGE           0.994      1.000      1.000      1.000      1.000


          Covariance Coverage
             HAVELD     FEMALE     AGE
             ________   ________   ________
HAVELD        1.000
FEMALE        1.000      1.000
AGE           1.000      1.000      1.000


SAMPLE STATISTICS


     ESTIMATED SAMPLE STATISTICS


          Means
             WRATSPL    WJSPL      WAISSPL    WRATCALC   WJCALC
             ________   ________   ________   ________   ________
     1       36.575     41.789     37.310     38.930     23.844


          Means
             WAISCALC   EDLEVEL    NEWSCHL    SUSPEND    EXPELLED
```

SEM Results ...

1	11.022	11.153	0.546	0.518	0.141

Means

	HAVELD	FEMALE	AGE
1	0.157	0.316	19.719

Covariances

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	48.101				
WJSPL	44.763	51.202			
WAISSPL	41.698	43.272	45.292		
WRATCALC	22.641	22.816	21.201	41.292	
WJCALC	14.816	15.467	14.420	25.536	21.338
WAISCALC	10.784	11.343	10.170	14.964	10.276
EDLEVEL	2.139	2.182	2.081	3.190	2.390
NEWSCHL	-0.135	-0.003	-0.072	-0.022	-0.033
SUSPEND	-0.355	-0.277	-0.239	-0.689	-0.374
EXPELLED	-0.356	-0.370	-0.353	-0.201	-0.119
HAVELD	-0.854	-0.932	-0.868	-0.177	-0.168
FEMALE	0.198	0.191	0.236	-0.185	-0.059
AGE	2.638	2.650	2.021	4.051	2.962

SEM Results ...

Covariances

	WAISCALC	EDLEVEL	NEWSCHL	SUSPEND	EXPELLED
WAISCALC	11.278				
EDLEVEL	1.166	1.472			
NEWSCHL	0.064	-0.026	0.248		
SUSPEND	-0.031	-0.057	0.046	0.250	
EXPELLED	-0.006	-0.031	0.035	0.052	0.121
HAVELD	-0.131	-0.008	-0.015	-0.004	-0.003
FEMALE	-0.237	-0.064	0.019	-0.033	-0.019
AGE	1.404	0.912	-0.022	-0.158	-0.015

Covariances

	HAVELD	FEMALE	AGE
HAVELD	0.132		
FEMALE	-0.008	0.216	
AGE	-0.007	-0.001	3.627

Correlations

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC

SEM Results ...

WRATSPL	1.000				
WJSPL	0.902	1.000			
WAISSPL	0.893	0.899	1.000		
WRATCALC	0.508	0.496	0.490	1.000	
WJCALC	0.462	0.468	0.464	0.860	1.000
WAISCALC	0.463	0.472	0.450	0.693	0.662
EDLEVEL	0.254	0.251	0.255	0.409	0.427
NEWSCHL	-0.039	-0.001	-0.022	-0.007	-0.014
SUSPEND	-0.102	-0.078	-0.071	-0.215	-0.162
EXPELLED	-0.148	-0.149	-0.151	-0.090	-0.074
HAVELD	-0.339	-0.358	-0.355	-0.076	-0.100
FEMALE	0.061	0.057	0.075	-0.062	-0.027
AGE	0.200	0.194	0.158	0.331	0.337

Correlations

	WAISCALC	EDLEVEL	NEWSCHL	SUSPEND	EXPELLED
WAISCALC	1.000				
EDLEVEL	0.286	1.000			
NEWSCHL	0.038	-0.044	1.000		
SUSPEND	-0.018	-0.094	0.186	1.000	
EXPELLED	-0.005	-0.074	0.202	0.298	1.000
HAVELD	-0.108	-0.018	-0.084	-0.024	-0.022
FEMALE	-0.152	-0.114	0.082	-0.141	-0.117
AGE	0.219	0.395	-0.023	-0.166	-0.022

SEM Results ...

Correlations

	HAVELD	FEMALE	AGE
HAVELD	1.000		
FEMALE	-0.047	1.000	
AGE	-0.010	-0.001	1.000

MAXIMUM LOG-LIKELIHOOD VALUE FOR THE UNRESTRICTED (H1) MODEL IS -6866.105

UNIVARIATE SAMPLE STATISTICS

UNIVARIATE HIGHER-ORDER MOMENT DESCRIPTIVE STATISTICS

Variable/ Sample Size	Mean/ Variance	Skewness/ Kurtosis	Minimum/ Maximum	% with Min/Max	Percentiles 20%/60%	40%/80%	Me
WRATSPL	36.574	-0.048	19.000	0.32%	30.000	35.000	3
312.000	48.232	-0.499	57.000	0.32%	38.000	43.000	
WJSPL	41.789	-0.209	19.000	0.32%	36.000	40.000	4
313.000	51.202	-0.290	57.000	0.64%	44.000	48.000	
WAISSPL	37.344	-0.438	16.000	0.32%	31.000	36.000	3

SEM Results ...

	311.000	45.345	-0.128	52.000	0.32%	39.000	43.000	
WRATCALC		38.930	0.191	23.000	0.32%	33.000	37.000	38
	313.000	41.292	-0.593	54.000	0.32%	40.000	45.000	
WJCALC		23.855	0.085	13.000	0.64%	20.000	23.000	24
	311.000	21.301	-0.521	36.000	0.32%	25.000	28.000	
WAISCALC		11.019	0.749	6.000	1.29%	8.000	9.000	10
	311.000	11.241	-0.146	21.000	0.64%	11.000	14.000	
EDLEVEL		11.153	-0.639	8.000	1.60%	10.000	11.000	11
	313.000	1.472	-0.525	13.000	7.35%	12.000	12.000	
NEWSCHL		0.546	-0.186	0.000	45.37%	0.000	0.000	
	313.000	0.248	-1.965	1.000	54.63%	1.000	1.000	
SUSPEND		0.518	-0.070	0.000	48.24%	0.000	0.000	
	313.000	0.250	-1.995	1.000	51.76%	1.000	1.000	
EXPELLED		0.141	2.068	0.000	85.94%	0.000	0.000	0
	313.000	0.121	2.277	1.000	14.06%	0.000	0.000	
HAVELD		0.157	1.890	0.000	84.35%	0.000	0.000	0
	313.000	0.132	1.573	1.000	15.65%	0.000	0.000	
FEMALE		0.316	0.790	0.000	68.37%	0.000	0.000	0
	313.000	0.216	-1.376	1.000	31.63%	0.000	1.000	
AGE		19.719	0.802	16.000	0.96%	18.000	19.000	19
	313.000	3.627	0.530	27.000	0.32%	20.000	21.000	

THE MODEL ESTIMATION TERMINATED NORMALLY

SEM Results ...

```
MODEL FIT INFORMATION

Number of Free Parameters                       33

Loglikelihood

         H0 Value                        -4909.694
         H1 Value                        -4884.805

Information Criteria

         Akaike (AIC)                     9885.389
         Bayesian (BIC)                  10009.014
         Sample-Size Adjusted BIC         9904.348
           (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

         Value                              49.780
         Degrees of Freedom                     36
         P-Value                            0.0630

RMSEA (Root Mean Square Error Of Approximation)

         Estimate                            0.035
```

SEM Results ...

```
          90 Percent C.I.                    0.000  0.057
          Probability RMSEA <= .05           0.859

CFI/TLI

          CFI                                0.993
          TLI                                0.989

Chi-Square Test of Model Fit for the Baseline Model

          Value                           2035.834
          Degrees of Freedom                    57
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.024



MODEL RESULTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 MATH     BY
```

SEM Results ...

```
    WRATCALC          1.000      0.000    999.000    999.000
    WJCALC            0.684      0.027     25.116      0.000
    WAISCALC          0.403      0.024     16.552      0.000

 SPELL     BY
    WRATSPL           1.000      0.000    999.000    999.000
    WJSPL             1.038      0.029     35.933      0.000
    WAISSPL           0.967      0.028     34.549      0.000

 MATH      ON
    EDLEVEL           1.720      0.282      6.097      0.000
    NEWSCHL           0.535      0.649      0.824      0.410
    SUSPEND          -1.727      0.677     -2.550      0.011
    EXPELLED         -0.524      0.954     -0.549      0.583
    HAVELD           -1.495      0.858     -1.743      0.081
    FEMALE           -0.757      0.689     -1.098      0.272
    AGE               0.617      0.179      3.441      0.001

 SPELL     ON
    EDLEVEL           1.169      0.302      3.874      0.000
    NEWSCHL          -0.212      0.694     -0.305      0.760
    SUSPEND          -0.111      0.718     -0.155      0.877
    EXPELLED         -2.540      1.021     -2.488      0.013
    HAVELD           -6.630      0.923     -7.182      0.000
    FEMALE            0.843      0.736      1.145      0.252
    AGE               0.349      0.192      1.819      0.069
```

SEM Results ...

```
MATH     WITH
   SPELL            15.401      2.048      7.520      0.000

Intercepts
   WRATSPL          17.949      4.113      4.364      0.000
   WJSPL            22.456      4.267      5.263      0.000
   WAISSPL          19.289      3.978      4.849      0.000
   WRATCALC          8.718      3.847      2.266      0.023
   WJCALC            3.169      2.710      1.169      0.242
   WAISCALC         -1.141      1.686     -0.677      0.498

Residual Variances
   WRATSPL           4.985      0.612      8.151      0.000
   WJSPL             4.751      0.626      7.587      0.000
   WAISSPL           4.960      0.591      8.385      0.000
   WRATCALC          4.035      0.966      4.175      0.000
   WJCALC            3.876      0.528      7.335      0.000
   WAISCALC          5.221      0.457     11.426      0.000
   MATH             27.561      2.600     10.601      0.000
   SPELL            32.786      2.961     11.074      0.000


STANDARDIZED MODEL RESULTS
```

SEM Results ...

```
STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 MATH     BY
    WRATCALC           0.950      0.013     73.867      0.000
    WJCALC             0.905      0.015     59.784      0.000
    WAISCALC           0.732      0.029     25.661      0.000

 SPELL    BY
    WRATSPL            0.947      0.008    121.996      0.000
    WJSPL              0.952      0.007    130.144      0.000
    WAISSPL            0.944      0.008    117.377      0.000

 MATH     ON
    EDLEVEL            0.342      0.054      6.381      0.000
    NEWSCHL            0.044      0.053      0.825      0.409
    SUSPEND           -0.141      0.055     -2.576      0.010
    EXPELLED          -0.030      0.054     -0.550      0.583
    HAVELD            -0.089      0.051     -1.746      0.081
    FEMALE            -0.058      0.052     -1.100      0.271
    AGE                0.193      0.055      3.487      0.000

 SPELL    ON
    EDLEVEL            0.216      0.055      3.942      0.000
```

SEM Results ...

```
    NEWSCHL          -0.016      0.053     -0.305      0.760
    SUSPEND          -0.008      0.055     -0.155      0.877
    EXPELLED         -0.134      0.054     -2.505      0.012
    HAVELD           -0.367      0.048     -7.689      0.000
    FEMALE            0.060      0.052      1.147      0.251
    AGE               0.101      0.055      1.826      0.068

 MATH     WITH
    SPELL             0.512      0.046     11.178      0.000

 Intercepts
    WRATSPL           2.588      0.630      4.106      0.000
    WJSPL             3.138      0.643      4.879      0.000
    WAISSPL           2.866      0.633      4.525      0.000
    WRATCALC          1.357      0.627      2.165      0.030
    WJCALC            0.686      0.601      1.143      0.253
    WAISCALC         -0.340      0.496     -0.686      0.493

 Residual Variances
    WRATSPL           0.104      0.015      7.052      0.000
    WJSPL             0.093      0.014      6.656      0.000
    WAISSPL           0.109      0.015      7.214      0.000
    WRATCALC          0.098      0.024      3.999      0.000
    WJCALC            0.182      0.027      6.641      0.000
    WAISCALC          0.464      0.042     11.088      0.000
    MATH              0.740      0.045     16.551      0.000
```

SEM Results ...

```
    SPELL              0.760      0.043     17.632      0.000


R-SQUARE

    Observed                                        Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    WRATSPL            0.896      0.015     60.998      0.000
    WJSPL              0.907      0.014     65.072      0.000
    WAISSPL            0.891      0.015     58.689      0.000
    WRATCALC           0.902      0.024     36.933      0.000
    WJCALC             0.818      0.027     29.892      0.000
    WAISCALC           0.536      0.042     12.830      0.000

     Latent                                         Two-Tailed
    Variable        Estimate       S.E.  Est./S.E.    P-Value

    MATH               0.260      0.045      5.823      0.000
    SPELL              0.240      0.043      5.556      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.247E-05
       (ratio of smallest to largest eigenvalue)
```

SEM Results ...

```
TECHNICAL 1 OUTPUT


     PARAMETER SPECIFICATION


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              ________      ________      ________      ________      ________
    1             1             2             3             4             5


           NU
              WAISCALC      EDLEVEL       NEWSCHL       SUSPEND       EXPELLED
              ________      ________      ________      ________      ________
    1             6             0             0             0             0


           NU
              HAVELD        FEMALE        AGE
              ________      ________      ________
    1             0             0             0
```

SEM Results ...

LAMBDA

	MATH	SPELL	EDLEVEL	NEWSCHL	SUSPEND
WRATSPL	0	0	0	0	0
WJSPL	0	7	0	0	0
WAISSPL	0	8	0	0	0
WRATCALC	0	0	0	0	0
WJCALC	9	0	0	0	0
WAISCALC	10	0	0	0	0
EDLEVEL	0	0	0	0	0
NEWSCHL	0	0	0	0	0
SUSPEND	0	0	0	0	0
EXPELLED	0	0	0	0	0
HAVELD	0	0	0	0	0
FEMALE	0	0	0	0	0
AGE	0	0	0	0	0

LAMBDA

	EXPELLED	HAVELD	FEMALE	AGE
WRATSPL	0	0	0	0
WJSPL	0	0	0	0
WAISSPL	0	0	0	0
WRATCALC	0	0	0	0
WJCALC	0	0	0	0

SEM Results ...

```
WAISCALC         0         0         0         0
EDLEVEL          0         0         0         0
NEWSCHL          0         0         0         0
SUSPEND          0         0         0         0
EXPELLED         0         0         0         0
HAVELD           0         0         0         0
FEMALE           0         0         0         0
AGE              0         0         0         0


          THETA
             WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
             --------      --------      --------      --------      --------
WRATSPL           11
WJSPL              0            12
WAISSPL            0             0            13
WRATCALC           0             0             0            14
WJCALC             0             0             0             0            15
WAISCALC           0             0             0             0             0
EDLEVEL            0             0             0             0             0
NEWSCHL            0             0             0             0             0
SUSPEND            0             0             0             0             0
EXPELLED           0             0             0             0             0
HAVELD             0             0             0             0             0
FEMALE             0             0             0             0             0
AGE                0             0             0             0             0
```

SEM Results ...

```
         THETA
            WAISCALC      EDLEVEL       NEWSCHL       SUSPEND       EXPELLED
            --------      --------      --------      --------      --------
WAISCALC         16
EDLEVEL           0             0
NEWSCHL           0             0             0
SUSPEND           0             0             0             0
EXPELLED          0             0             0             0             0
HAVELD            0             0             0             0             0
FEMALE            0             0             0             0             0
AGE               0             0             0             0             0


         THETA
            HAVELD        FEMALE        AGE
            --------      --------      --------
HAVELD            0
FEMALE            0             0
AGE               0             0             0


         ALPHA
            MATH          SPELL         EDLEVEL       NEWSCHL       SUSPEND
            --------      --------      --------      --------      --------
```

SEM Results ...

```
      1        0        0        0        0        0


         ALPHA
            EXPELLED      HAVELD        FEMALE        AGE
            ________      ________      ________      ________
      1        0             0             0             0


         BETA
            MATH          SPELL         EDLEVEL       NEWSCHL       SUSPEND
            ________      ________      ________      ________      ________
 MATH              0             0            17            18            19
 SPELL             0             0            24            25            26
 EDLEVEL           0             0             0             0             0
 NEWSCHL           0             0             0             0             0
 SUSPEND           0             0             0             0             0
 EXPELLED          0             0             0             0             0
 HAVELD            0             0             0             0             0
 FEMALE            0             0             0             0             0
 AGE               0             0             0             0             0


         BETA
            EXPELLED      HAVELD        FEMALE        AGE
            ________      ________      ________      ________
```

SEM Results ...

MATH	20	21	22	23
SPELL	27	28	29	30
EDLEVEL	0	0	0	0
NEWSCHL	0	0	0	0
SUSPEND	0	0	0	0
EXPELLED	0	0	0	0
HAVELD	0	0	0	0
FEMALE	0	0	0	0
AGE	0	0	0	0

PSI

	MATH	SPELL	EDLEVEL	NEWSCHL	SUSPEND
MATH	31				
SPELL	32	33			
EDLEVEL	0	0	0		
NEWSCHL	0	0	0	0	
SUSPEND	0	0	0	0	0
EXPELLED	0	0	0	0	0
HAVELD	0	0	0	0	0
FEMALE	0	0	0	0	0
AGE	0	0	0	0	0

PSI

SEM Results ...

```
             EXPELLED      HAVELD        FEMALE        AGE
             --------      --------      --------      --------
 EXPELLED           0
 HAVELD             0             0
 FEMALE             0             0             0
 AGE                0             0             0             0


     STARTING VALUES


           NU
              WRATSPL       WJSPL         WAISSPL       WRATCALC      WJCALC
              --------      --------      --------      --------      --------
      1         36.574        41.789        37.344        38.930        23.855


           NU
              WAISCALC      EDLEVEL       NEWSCHL       SUSPEND       EXPELLED
              --------      --------      --------      --------      --------
      1         11.019         0.000         0.000         0.000         0.000


           NU
              HAVELD        FEMALE        AGE
              --------      --------      --------
```

SEM Results ...

```
      1          0.000         0.000         0.000


               LAMBDA
                  MATH          SPELL         EDLEVEL       NEWSCHL       SUSPEND
                  --------      --------      --------      --------      --------
 WRATSPL         0.000         1.000         0.000         0.000         0.000
 WJSPL           0.000         1.038         0.000         0.000         0.000
 WAISSPL         0.000         0.967         0.000         0.000         0.000
 WRATCALC        1.000         0.000         0.000         0.000         0.000
 WJCALC          0.687         0.000         0.000         0.000         0.000
 WAISCALC        0.402         0.000         0.000         0.000         0.000
 EDLEVEL         0.000         0.000         1.000         0.000         0.000
 NEWSCHL         0.000         0.000         0.000         1.000         0.000
 SUSPEND         0.000         0.000         0.000         0.000         1.000
 EXPELLED        0.000         0.000         0.000         0.000         0.000
 HAVELD          0.000         0.000         0.000         0.000         0.000
 FEMALE          0.000         0.000         0.000         0.000         0.000
 AGE             0.000         0.000         0.000         0.000         0.000


               LAMBDA
                  EXPELLED      HAVELD        FEMALE        AGE
                  --------      --------      --------      --------
 WRATSPL         0.000         0.000         0.000         0.000
 WJSPL           0.000         0.000         0.000         0.000
```

SEM Results ...

WAISSPL	0.000	0.000	0.000	0.000
WRATCALC	0.000	0.000	0.000	0.000
WJCALC	0.000	0.000	0.000	0.000
WAISCALC	0.000	0.000	0.000	0.000
EDLEVEL	0.000	0.000	0.000	0.000
NEWSCHL	0.000	0.000	0.000	0.000
SUSPEND	0.000	0.000	0.000	0.000
EXPELLED	1.000	0.000	0.000	0.000
HAVELD	0.000	1.000	0.000	0.000
FEMALE	0.000	0.000	1.000	0.000
AGE	0.000	0.000	0.000	1.000

THETA

	WRATSPL	WJSPL	WAISSPL	WRATCALC	WJCALC
WRATSPL	24.116				
WJSPL	0.000	25.601			
WAISSPL	0.000	0.000	22.672		
WRATCALC	0.000	0.000	0.000	20.646	
WJCALC	0.000	0.000	0.000	0.000	10.650
WAISCALC	0.000	0.000	0.000	0.000	0.000
EDLEVEL	0.000	0.000	0.000	0.000	0.000
NEWSCHL	0.000	0.000	0.000	0.000	0.000
SUSPEND	0.000	0.000	0.000	0.000	0.000
EXPELLED	0.000	0.000	0.000	0.000	0.000

SEM Results ...

```
 HAVELD         0.000        0.000        0.000        0.000        0.000
 FEMALE         0.000        0.000        0.000        0.000        0.000
 AGE            0.000        0.000        0.000        0.000        0.000


           THETA
              WAISCALC     EDLEVEL      NEWSCHL      SUSPEND      EXPELLED
              --------     --------     --------     --------     --------
 WAISCALC       5.620
 EDLEVEL        0.000        0.000
 NEWSCHL        0.000        0.000        0.000
 SUSPEND        0.000        0.000        0.000        0.000
 EXPELLED       0.000        0.000        0.000        0.000        0.000
 HAVELD         0.000        0.000        0.000        0.000        0.000
 FEMALE         0.000        0.000        0.000        0.000        0.000
 AGE            0.000        0.000        0.000        0.000        0.000


           THETA
              HAVELD       FEMALE       AGE
              --------     --------     --------
 HAVELD         0.000
 FEMALE         0.000        0.000
 AGE            0.000        0.000        0.000
```

SEM Results ...

ALPHA

	MATH	SPELL	EDLEVEL	NEWSCHL	SUSPEND
	--------	--------	--------	--------	--------
1	0.000	0.000	11.153	0.546	0.518

ALPHA

	EXPELLED	HAVELD	FEMALE	AGE
	--------	--------	--------	--------
1	0.141	0.157	0.316	19.719

BETA

	MATH	SPELL	EDLEVEL	NEWSCHL	SUSPEND
	--------	--------	--------	--------	--------
MATH	0.000	0.000	0.000	0.000	0.000
SPELL	0.000	0.000	0.000	0.000	0.000
EDLEVEL	0.000	0.000	0.000	0.000	0.000
NEWSCHL	0.000	0.000	0.000	0.000	0.000
SUSPEND	0.000	0.000	0.000	0.000	0.000
EXPELLED	0.000	0.000	0.000	0.000	0.000
HAVELD	0.000	0.000	0.000	0.000	0.000
FEMALE	0.000	0.000	0.000	0.000	0.000
AGE	0.000	0.000	0.000	0.000	0.000

SEM Results ...

BETA

	EXPELLED	HAVELD	FEMALE	AGE
MATH	0.000	0.000	0.000	0.000
SPELL	0.000	0.000	0.000	0.000
EDLEVEL	0.000	0.000	0.000	0.000
NEWSCHL	0.000	0.000	0.000	0.000
SUSPEND	0.000	0.000	0.000	0.000
EXPELLED	0.000	0.000	0.000	0.000
HAVELD	0.000	0.000	0.000	0.000
FEMALE	0.000	0.000	0.000	0.000
AGE	0.000	0.000	0.000	0.000

PSI

	MATH	SPELL	EDLEVEL	NEWSCHL	SUSPEND
MATH	0.050				
SPELL	0.000	0.050			
EDLEVEL	0.000	0.000	1.472		
NEWSCHL	0.000	0.000	-0.026	0.248	
SUSPEND	0.000	0.000	-0.057	0.046	0.250
EXPELLED	0.000	0.000	-0.031	0.035	0.052
HAVELD	0.000	0.000	-0.008	-0.015	-0.004
FEMALE	0.000	0.000	-0.064	0.019	-0.033
AGE	0.000	0.000	0.912	-0.022	-0.158

SEM Results ...

```
          PSI
             EXPELLED      HAVELD        FEMALE        AGE
             --------      --------      --------      --------
 EXPELLED       0.121
 HAVELD        -0.003         0.132
 FEMALE        -0.019        -0.008         0.216
 AGE           -0.015        -0.007        -0.001         3.627


     Beginning Time:  21:48:08
        Ending Time:  21:48:08
       Elapsed Time:  00:00:00



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Los Angeles, CA  90066

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Fax: (310) 391-8971
Web: www.StatModel.com
Support: Support@StatModel.com
```

SEM Results ...

```
Copyright (c) 1998-2015 Muthen & Muthen
```

Outline

Insomnia Project

- Data file is “insomnia.dat” has 7 indicators of sleep difficulty
- Question: How do the separate dimensions of insomnia predict the quality of life
- Here we are focused on the regression model, not on the measurement model (which, we assume, has been investigated previously).
    - Estimate regression coefficients for 3 insomnia factors predicting Quality of life

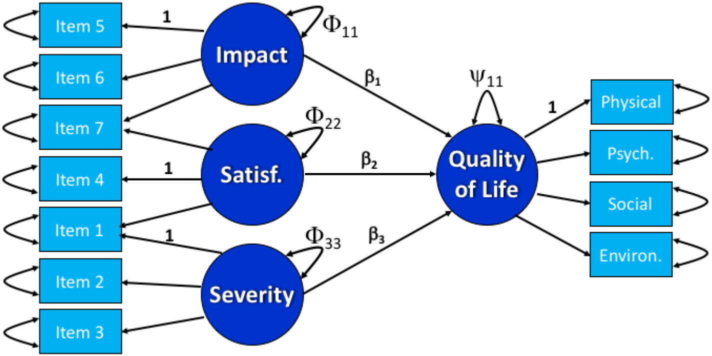
Insomnia SEM

- Three latent variables about insomnia (7 observed indicators altogether)
  1. Impact of insomnia
  2. Severity of insomnia
  3. Satisfaction with sleep
- Four items that are indicators of Quality of Life

About the Model

- We used the “marker variable” approach for identification
  - the loading (λ) of the FIRST indicator of each factor is fixed at 1
  - This means the other estimates for that factor are scaled relative to the first item

Insomnia SEM Diagram



About the Model

- We used the "marker variable" approach for identification
  - the loading (λ) of the FIRST indicator of each factor is fixed at 1
  - This means the other estimates for that factor are scaled relative to the first item

Mplus Syntax

```
TITLE: Example 5B
DATA:
  FILE IS "../../data/insomnia.dat";
VARIABLE:
  names are insom1-insom7 cesd1-cesd20 phy psy soc
     env;
  usevariables are insom1-insom7 phy psy soc env;
  missing all (999);
MODEL:
  Impact by insom5 insom6 insom7;
  Severity by insom1 insom2 insom3;
  Satisf by insom4 insom1 insom7;
  Qol by phy psy soc env;
  Qol on Impact Severity Satisf;
```

Results

View Online: sem-02.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:48 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 5b

  DATA:
    FILE IS "../../data/insomnia.dat";

  VARIABLE:
    names are insom1-insom7 cesd1-cesd20 phy psy soc env;
    usevariables are insom1-insom7 phy psy soc env;
    missing all (999);

  MODEL:
    Impact by insom5 insom6 insom7;
    Severity by insom1 insom2 insom3;
```

Results ...

```
    Satisf by insom4 insom1 insom7;
    Qol by phy psy soc env;
    Qol on Impact Severity Satisf;



INPUT READING TERMINATED NORMALLY




Example 5b

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         103

Number of dependent variables                                   11
Number of independent variables                                  0
Number of continuous latent variables                            4

Observed dependent variables

  Continuous
   INSOM1      INSOM2      INSOM3      INSOM4      INSOM5      INSOM6
```

Results ...

```
   INSOM7      PHY         PSY         SOC         ENV

Continuous latent variables
   IMPACT      SEVERITY    SATISF      QOL


Estimator                                                  ML
Information matrix                                   OBSERVED
Maximum number of iterations                             1000
Convergence criterion                               0.500D-04
Maximum number of steepest descent iterations              20
Maximum number of iterations for H1                      2000
Convergence criterion for H1                        0.100D-03

Input data file(s)
  ../../data/insomnia.dat

Input data format  FREE


SUMMARY OF DATA

     Number of missing data patterns             3


COVARIANCE COVERAGE OF DATA
```

Results ...

```
Minimum covariance coverage value   0.100


     PROPORTION OF DATA PRESENT


           Covariance Coverage
              INSOM1        INSOM2        INSOM3        INSOM4        INSOM5
              ________      ________      ________      ________      ________
 INSOM1         1.000
 INSOM2         1.000         1.000
 INSOM3         1.000         1.000         1.000
 INSOM4         1.000         1.000         1.000         1.000
 INSOM5         1.000         1.000         1.000         1.000         1.000
 INSOM6         1.000         1.000         1.000         1.000         1.000
 INSOM7         1.000         1.000         1.000         1.000         1.000
 PHY            1.000         1.000         1.000         1.000         1.000
 PSY            0.990         0.990         0.990         0.990         0.990
 SOC            0.990         0.990         0.990         0.990         0.990
 ENV            1.000         1.000         1.000         1.000         1.000


           Covariance Coverage
              INSOM6        INSOM7        PHY           PSY           SOC
              ________      ________      ________      ________      ________
```

Results ...

```
 INSOM6         1.000
 INSOM7         1.000         1.000
 PHY            1.000         1.000         1.000
 PSY            0.990         0.990         0.990         0.990
 SOC            0.990         0.990         0.990         0.981         0.990
 ENV            1.000         1.000         1.000         0.990         0.990


          Covariance Coverage
             ENV
             --------
 ENV          1.000



THE MODEL ESTIMATION TERMINATED NORMALLY



MODEL FIT INFORMATION

Number of Free Parameters                       41

Loglikelihood

          H0 Value                        -1044.951
```

Results ...

```
          H1 Value                          -1023.210

Information Criteria

          Akaike (AIC)                       2171.901
          Bayesian (BIC)                     2279.925
          Sample-Size Adjusted BIC           2150.413
            (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

          Value                                43.481
          Degrees of Freedom                       36
          P-Value                              0.1829

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                              0.045
          90 Percent C.I.                       0.000  0.087
          Probability RMSEA <= .05              0.539

CFI/TLI

          CFI                                   0.986
          TLI                                   0.979
```

Results ...

```
Chi-Square Test of Model Fit for the Baseline Model

          Value                              602.272
          Degrees of Freedom                      55
          P-Value                             0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                0.045



MODEL RESULTS

                                                  Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 IMPACT   BY
    INSOM5             1.000      0.000    999.000    999.000
    INSOM6             0.650      0.110      5.923      0.000
    INSOM7             0.317      0.118      2.676      0.007

 SEVERITY BY
    INSOM1             1.000      0.000    999.000    999.000
    INSOM2             6.966      6.248      1.115      0.265
    INSOM3             5.562      5.055      1.100      0.271
```

Results ...

```
SATISF   BY
   INSOM4            1.000      0.000    999.000    999.000
   INSOM1            0.756      0.174      4.354      0.000
   INSOM7            0.801      0.187      4.275      0.000

QOL      BY
   PHY               1.000      0.000    999.000    999.000
   PSY               1.176      0.115     10.196      0.000
   SOC               0.803      0.118      6.778      0.000
   ENV               0.942      0.101      9.301      0.000

QOL      ON
   IMPACT           -0.041      0.065     -0.629      0.530
   SEVERITY         -0.189      0.595     -0.318      0.750
   SATISF           -0.302      0.140     -2.156      0.031

SEVERITY WITH
   IMPACT            0.045      0.044      1.035      0.301

SATISF   WITH
   IMPACT            0.492      0.107      4.612      0.000
   SEVERITY          0.058      0.052      1.120      0.263

Intercepts
   INSOM1            0.718      0.082      8.792      0.000
```

Results ...

```
   INSOM2          0.670     0.090      7.407     0.000
   INSOM3          0.738     0.091      8.105     0.000
   INSOM4          1.738     0.087     20.023     0.000
   INSOM5          1.621     0.112     14.523     0.000
   INSOM6          0.951     0.100      9.485     0.000
   INSOM7          1.184     0.103     11.554     0.000
   PHY             3.659     0.050     72.783     0.000
   PSY             3.413     0.060     56.813     0.000
   SOC             3.523     0.054     64.839     0.000
   ENV             3.520     0.050     69.751     0.000

Variances
   IMPACT          1.109     0.228      4.857     0.000
   SEVERITY        0.015     0.026      0.556     0.578
   SATISF          0.509     0.114      4.482     0.000

Residual Variances
   INSOM1          0.295     0.054      5.485     0.000
   INSOM2          0.136     0.095      1.428     0.153
   INSOM3          0.403     0.082      4.924     0.000
   INSOM4          0.267     0.063      4.228     0.000
   INSOM5          0.174     0.146      1.195     0.232
   INSOM6          0.568     0.100      5.676     0.000
   INSOM7          0.394     0.071      5.591     0.000
   PHY             0.068     0.015      4.443     0.000
   PSY             0.104     0.021      4.830     0.000
```

Results ...

```
    SOC                0.178      0.028      6.469      0.000
    ENV                0.091      0.017      5.370      0.000
    QOL                0.124      0.026      4.804      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.399E-06
       (ratio of smallest to largest eigenvalue)


     Beginning Time:  21:48:08
        Ending Time:  21:48:08
       Elapsed Time:  00:00:00



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```

Results ...

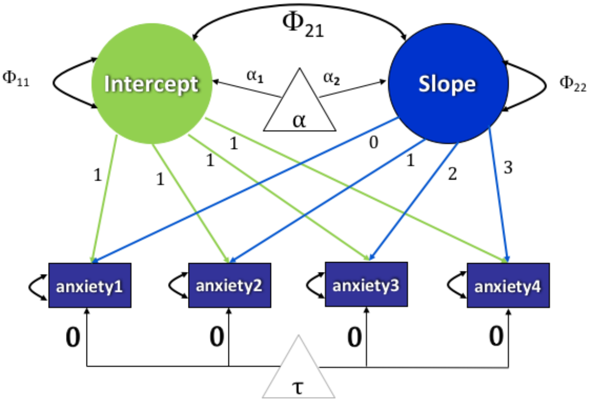
```
Copyright (c) 1998-2015 Muthen & Muthen
```

Outline

Anxiety data

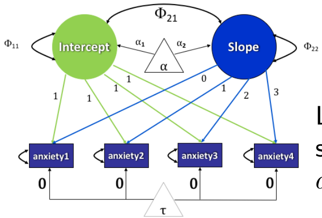
- "anxiety.dat" includes anxiety measures at 4 time points named `anxiety1`, `anxiety2`, `anxiety3`, `anxiety4`
- Some people (eg current CRMDA director) find the graphic representation of the Latent Growth Curve Model to be quite confusing and difficult to understand. You've been warned.

LGCM Diagram



LGCM Diagram

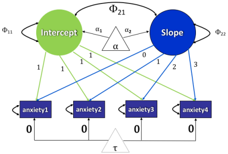
This amounts to a "random effects" multilevel model



- time, is coded 0, 1, 2, 3. Time values are written on the graph as if they were "slope coefficients".
  - In the Mplus code, this is "LINEAR" latent variable

LGCM is based on idea of "wide" data, predicts for separate outcomes $anxiety_1$, $anxiety_2$, $anxiety_3$, $anxiety_4$

LGCM Diagram



- LGCM: Think of intercept and slope as random effects, where the center points are α_1 and α_2.
- MLM view: Effect of time passing for case i on anxiety at time t

$$anxiety_{it} = intercept_i + slope_i \cdot time + error_{it}$$

- MLM typically asserts $intercept_i \sim N(\alpha_1, \sigma^2_{b0})$ and $slope_i \sim N(\alpha_2, \sigma^2_{b1})$, using the LGCM notation

LGCM Syntax

```
TITLE:
Example 6 - Linear Latent Growth Curve Model with
   4 Time Points
DATA:
FILE IS "../../data/anxiety.dat";
VARIABLE:
NAMES ARE anxiety1 anxiety2 anxiety3 anxiety4;
MODEL:
INTCEPT BY anxiety1@1 anxiety2@1 anxiety3@1
   anxiety4@1;
LINEAR BY anxiety1@0 anxiety2@1 anxiety3@2
   anxiety4@3;
anxiety1-anxiety4;
[anxiety1-anxiety4@0];
INTCEPT LINEAR;
[INTCEPT LINEAR];
```

LGCM Syntax ...

```
INTCEPT WITH LINEAR;
```

LGCM Results

View Online: lgc-01.out

Inline Display:

```
Mplus VERSION 7.4 (Linux)
MUTHEN & MUTHEN
05/28/2018   9:50 PM

INPUT INSTRUCTIONS


  TITLE:
    Example 6 - Linear Latent Growth Curve Model with 4 Time Points

  DATA:
    FILE IS "../../data/anxiety.dat";

  VARIABLE:
    NAMES ARE
    anxiety1 anxiety2 anxiety3 anxiety4;

  MODEL:
    INTCEPT BY anxiety1@1 anxiety2@1 anxiety3@1 anxiety4@1;
    LINEAR BY anxiety1@0 anxiety2@1 anxiety3@2 anxiety4@3;
```

LGCM Results ...

```
  anxiety1-anxiety4;
  [anxiety1-anxiety4@0];

  INTCEPT LINEAR;
  [INTCEPT LINEAR];
  INTCEPT WITH LINEAR;



INPUT READING TERMINATED NORMALLY




Example 6 - Linear Latent Growth Curve Model with 4 Time Points

SUMMARY OF ANALYSIS

Number of groups                                                 1
Number of observations                                         485

Number of dependent variables                                    4
Number of independent variables                                  0
Number of continuous latent variables                            2

Observed dependent variables
```

LGCM Results ...

```
  Continuous
   ANXIETY1    ANXIETY2    ANXIETY3    ANXIETY4

Continuous latent variables
   INTCEPT     LINEAR


Estimator                                                  ML
Information matrix                                   OBSERVED
Maximum number of iterations                             1000
Convergence criterion                               0.500D-04
Maximum number of steepest descent iterations              20

Input data file(s)
  ../../data/anxiety.dat

Input data format  FREE



THE MODEL ESTIMATION TERMINATED NORMALLY



MODEL FIT INFORMATION
```

LGCM Results ...

```
Number of Free Parameters                        9

Loglikelihood

         H0 Value                         -567.499
         H1 Value                         -553.855

Information Criteria

         Akaike (AIC)                     1152.997
         Bayesian (BIC)                   1190.654
         Sample-Size Adjusted BIC         1162.089
           (n* = (n + 2) / 24)

Chi-Square Test of Model Fit

         Value                              27.288
         Degrees of Freedom                      5
         P-Value                            0.0001

RMSEA (Root Mean Square Error Of Approximation)

         Estimate                            0.096
         90 Percent C.I.                     0.063  0.133
         Probability RMSEA <= .05            0.013
```

LGCM Results ...

```
CFI/TLI

          CFI                                0.981
          TLI                                0.977

Chi-Square Test of Model Fit for the Baseline Model

          Value                           1182.102
          Degrees of Freedom                     6
          P-Value                           0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                              0.061



MODEL RESULTS

                                                 Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

 INTCEPT  BY
    ANXIETY1           1.000      0.000    999.000    999.000
    ANXIETY2           1.000      0.000    999.000    999.000
```

LGCM Results ...

```
   ANXIETY3          1.000      0.000    999.000    999.000
   ANXIETY4          1.000      0.000    999.000    999.000

LINEAR   BY
   ANXIETY1          0.000      0.000    999.000    999.000
   ANXIETY2          1.000      0.000    999.000    999.000
   ANXIETY3          2.000      0.000    999.000    999.000
   ANXIETY4          3.000      0.000    999.000    999.000

INTCEPT  WITH
   LINEAR           -0.011      0.003     -3.485      0.000

Means
   INTCEPT           0.698      0.020     34.866      0.000
   LINEAR           -0.062      0.006    -10.405      0.000

Intercepts
   ANXIETY1          0.000      0.000    999.000    999.000
   ANXIETY2          0.000      0.000    999.000    999.000
   ANXIETY3          0.000      0.000    999.000    999.000
   ANXIETY4          0.000      0.000    999.000    999.000

Variances
   INTCEPT           0.151      0.013     11.996      0.000
   LINEAR            0.007      0.001      4.667      0.000
```

LGCM Results ...

```
 Residual Variances
    ANXIETY1           0.067      0.008      8.675      0.000
    ANXIETY2           0.048      0.004     10.965      0.000
    ANXIETY3           0.048      0.004     11.180      0.000
    ANXIETY4           0.040      0.006      6.545      0.000


QUALITY OF NUMERICAL RESULTS

     Condition Number for the Information Matrix              0.150E-02
       (ratio of smallest to largest eigenvalue)


     Beginning Time:  21:50:25
        Ending Time:  21:50:25
       Elapsed Time:  00:00:00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```

LGCM Results ...

```
Support: Support@StatModel.com

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```
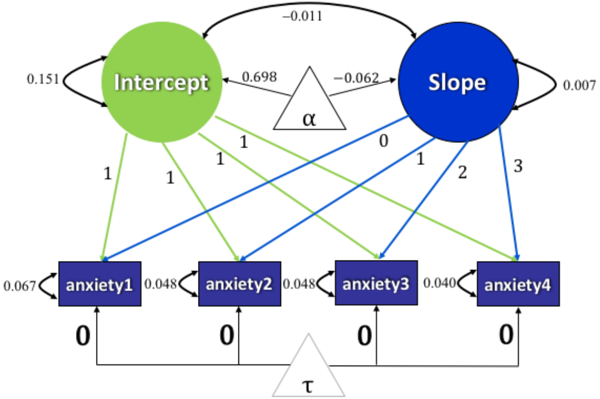

LGCM Diagram w/Estimates

LGCM Diagram w/Estimates ...



Compare with a multi-level model

- R's lme4 package is a state of the art framework for estimating models with random effects
- Because this Mplus example has observed indicators of anxiety, we can estimate the same model with lme4 and compare the results.

R data import

```
ddir <- "data"
anxiety <- read.table(file.path(ddir,
    "anxiety.dat"), sep = "\t", header = FALSE,
 col.names = paste0("anxiety", 1:4))
## rockchalk::summarize(anxiety)
```

- The data must be reshaped, to one long column for anxiety.

```
anxiety2 <- reshape(anxiety,
    varying=c("anxiety1", "anxiety2", "anxiety3",
    "anxiety4"),
 v.names = "anxiety", direction = "long")
anxiety2 <- anxiety2[order(anxiety2$id,
    anxiety2$time), ]
head(anxiety2, 15)
```

R data import ...

```
     time  anxiety id
1.1     1 1.714286  1
1.2     2 1.500000  1
1.3     3 1.714286  1
1.4     4 1.928571  1
2.1     1 1.642857  2
2.2     2 1.642857  2
2.3     3 1.214286  2
2.4     4 1.928571  2
3.1     1 1.642857  3
3.2     2 2.000000  3
3.3     3 1.928571  3
3.4     4 1.857143  3
4.1     1 1.428571  4
4.2     2 1.857143  4
4.3     3 1.928571  4
```

R data import ...

```
library(lme4)
## set time to begin at 0 to match the Mplus
   calculations
anxiety2$time <- anxiety2$time -1
m1 <- lmer(anxiety ~ time + (time | id), data =
   anxiety2)
summary(m1)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: anxiety ~ time + (time | id)
   Data: anxiety2

REML criterion at convergence: 1161.2

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.0428 -0.5191 -0.0757  0.4761  3.2378

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 id       (Intercept) 0.154950 0.39364
```

R data import ...

```
          time          0.007544 0.08686  -0.39
 Residual               0.049801 0.22316
Number of obs: 1940, groups:  id, 485

Fixed effects:
             Estimate Std. Error t value
(Intercept)  0.702800   0.019783   35.53
time        -0.065068   0.006008  -10.83

Correlation of Fixed Effects:
     (Intr)
time -0.489
```

R data import ...

- Estimated coefficients are very similar

	Mplus LGCM	lme4
Intercept Variance	0.151	0.1549
Slope Variance	0.007	0.0075
Correlation Intercept/Slope	*	-0.39
* Mplus reports covariance, not correlation, as	-0.011	
Intercept Mean	0.698	0.702
Time (Mean of random slope)	-0.062	-0.0650
Anxiety Residual variance time 1	0.067	
time 2	0.048	
time 3	0.048	
time 4	0.050	+
lme4 did not separately estimate variances		0.0498

R data import ...

Why do I need Mplus, if I can estimate that with lme4?

- lme4 works because the outcome variable is observed, not latent.
- If the outcome variable were latent, with indicators, then there's almost no hope we could code this up with lme4
- lme4 has not (yet, maybe ever) implemented convenient method to allow residual variance to differ by time point.

Why did I wish to try lme4?

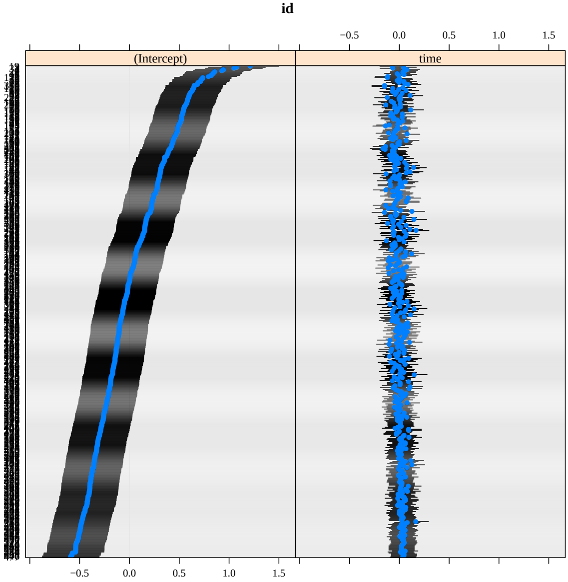
- The model is more understandable for me to think of this as data in the long format
- It is "just" a time regression with intercepts and slopes that vary among people

$$anxiety_{it} = \beta_{oi} + \beta_{1i} \cdot time_t + \varepsilon_{it}$$

$$\beta_{0i} \sim N(0, \sigma^2_{\beta_0})\, and\, \beta_{1i} \sim N(0, \sigma^2_{\beta_1})$$

- lme4 (R more broadly) offers many convenient tools to visualize the output.

R data import ...



Outline

We will keep making examples

- It is difficult to keep all of these details straight, but we are dedicated to try
- We should have full working input and output from Mplus (Muthén & Muthén, 2017) , R (R Core Team, 2017), and Stata

References

Muthén, L. K. & Muthén, B. O. (2017). *Mplus User's Guide*. Los Angeles, CA: Muthén & Muthén, 8th edition.

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria.

Session

```
sessionInfo()
```

```
R version 3.6.0 (2019-04-26)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 19.04

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/atlas/libblas.so.3.10.3
LAPACK: /usr/lib/x86_64-linux-gnu/atlas/liblapack.so.3.10.3

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
     LC_TIME=en_US.UTF-8
 [4] LC_COLLATE=en_US.UTF-8     LC_MONETARY=en_US.UTF-8
     LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C                  LC_ADDRESS=C
[10] LC_TELEPHONE=C             LC_MEASUREMENT=en_US.UTF-8
    LC_IDENTIFICATION=C

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
[1] lattice_0.20-38 lme4_1.1-21     Matrix_1.2-17
```

Session ...

```
loaded via a namespace (and not attached):
 [1] minqa_1.2.4     MASS_7.3-51.4   compiler_3.6.0 tools_3.6.0
      Rcpp_1.0.1      splines_3.6.0
 [7] nlme_3.1-140    grid_3.6.0      nloptr_1.2.1   boot_1.3-22
```