The semexamples: R point of view

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Outline

The Repository

- CRMDA created an archive of SEM working examples.
- To the extent possible, the same models are estimated with
    - Mplus
    - R (R Core Team, 2017)
    - Stata
- The repository can be browsed/downloaded: `https://gitlab.crmda.ku.edu/crmda/semexample`
- The data folder includes the information that is imported into each of the 3 programs.

Runable examples

1. Exploratory Factor Analysis (EFA)
2. Confirmatory Factor Analysis (CFA)
3. Measurement Invariance (Multi-group CFA)
4. Multiple Regression
5. Structural Equation Model (SEM)
6. Latent Growth Curve (LGC)
7. Modeling strategy changes for Ordinal Data

Explatory Factor Analysis

- The Excelsior Springs data.
- We obtained data from job seekers (job_placement.csv)
- Six test score variables that we think could measure latent variables
- Work to do
  1. Identify a factor structure
  2. Determine how our latent variables relate to demographic variables

Excelsior Spring Job Corps center

Items collected from job seekers using the Excelsior Spring Job Corps center

- Variables

  id: Subject identification number

  wjcalc: Subject's score on the WJ calculation subtest. (Numeric)

  wjspl: Subject's score on the WJ spelling subtest. (Numeric)

  wratcalc: Subject's score on the WRAT calculation subtest. (Numeric)

  wratspl: Subject's score on the WRAT spelling subtest. (Numeric)

  waiscalc: Subject's score on the WAIS arithmetic calculations subtest. (Numeric)

  waisspl: Subject's score on the WAIS spelling subtest. (Numeric)

  edlevel: What is the highest level of education completed by the subject? (Ordinal)

  newschl: Did the subject ever change high schools? (Binary: 1=Yes, 0=No)

Excelsior Spring Job Corps center ...

suspend: Has the subject ever been suspended from high school? (Binary: 1=Yes, 0=No)

expelled: Has the subject ever been expelled from high school? (Binary: 1=Yes, 0=No)

haveld: Has the subject been diagnosed with a learning disorder? (Binary: 1=Yes, 0=No)

female: Gender (Binary: 1=Female, 0=Male)

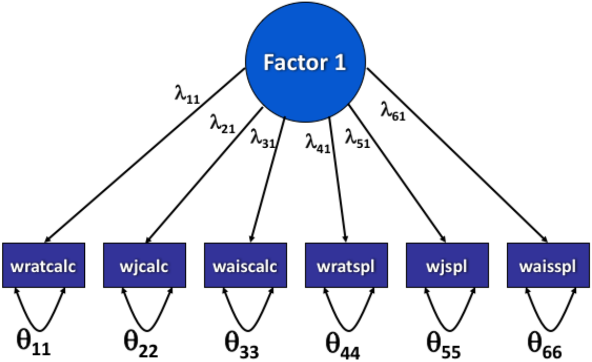
age: Age in years (Numeric)

- Missing data are coded as "99999"

The main idea of EFA

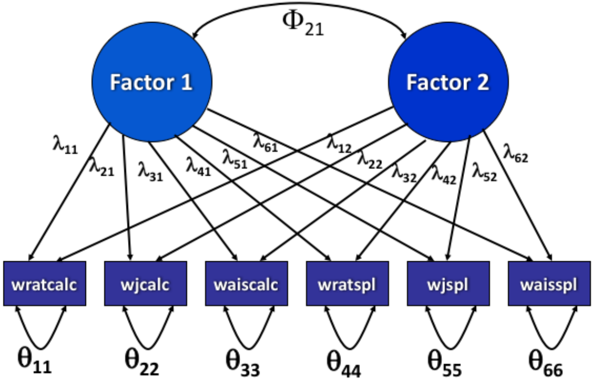
- How many latent variables underlie subtest scores (6 items)?
  - Will decide by running various models and comparing the results.
- EFA lets us specify how many factors will be estimated.
- Then it will estimate a loading for *each and every* observed variable on *each* factor.
- Since the 1970s, EFA has been regarded with suspicion by many because it seems to be "atheoretic data-grubbing".
- Nevertheless, we persist in showing you how to do it.

One Factor



Factor 1
λ_{11}
λ_{21}
λ_{31}
λ_{41}
λ_{51}
λ_{61}
wratcalc
wjcalc
waiscalc
wratspl
wjspl
waisspl
θ_{11}
θ_{22}
θ_{33}
θ_{44}
θ_{55}
θ_{66}

Two Factors



Φ21
Factor 1
Factor 2
λ11
λ21
λ31
λ41
λ51
λ61
λ12
λ22
λ32
λ42
λ52
λ62
wratcalc
wjcalc
waiscalc
wratspl
wjspl
waisspl
θ11
θ22
θ33
θ44
θ55
θ66

R code and results

```
head(scalescores)
```

```
  wjcalc wjspl wratspl wratcalc waiscalc waisspl
1     29    51      47       49       12      45
2     31    54      57       49       21      47
3     23    45      38       40        9      36
4     31    34      38       48       14      38
5     31    31      30       52       15      38
6     27    42      40       42       12      41
```

```
output1 <- factanal(scalescores, 1, rotation =
    "varimax")
output1
```

R code and results ...

```
Call:
factanal(x = scalescores, factors = 1, rotation = "varimax")

Uniquenesses:
  wjcalc    wjspl  wratspl wratcalc waiscalc  waisspl
   0.728    0.093    0.108    0.695    0.749    0.116

Loadings:
         Factor1
wjcalc   0.522
wjspl    0.953
wratspl  0.945
wratcalc 0.552
waiscalc 0.501
waisspl  0.940

               Factor1
SS loadings      3.511
Proportion Var   0.585

Test of the hypothesis that 1 factor is sufficient.
The chi square statistic is 461.38 on 9 degrees of freedom.
The p-value is 1.06e-93
```

R code and results ...

```
output2 <- factanal(scalescores, 2, rotation =
    "varimax")
output2
```

```
Call:
factanal(x = scalescores, factors = 2, rotation = "varimax")

Uniquenesses:
  wjcalc    wjspl  wratspl wratcalc waiscalc  waisspl
   0.184    0.089    0.107    0.096    0.477    0.112

Loadings:
         Factor1 Factor2
wjcalc   0.230   0.873
wjspl    0.907   0.298
wratspl  0.894   0.306
wratcalc 0.248   0.918
waiscalc 0.281   0.667
waisspl  0.896   0.293

               Factor1 Factor2
SS loadings      2.617   2.318
Proportion Var   0.436   0.386
```

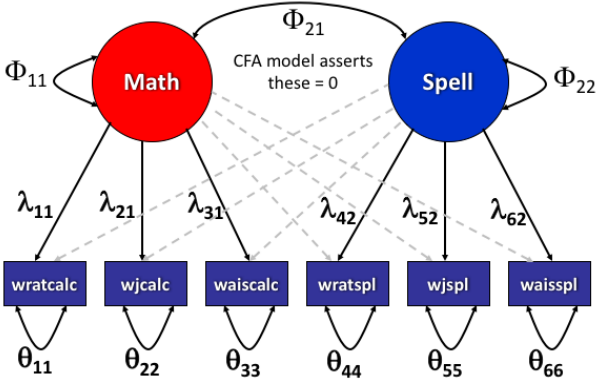
R code and results ...

```
Cumulative Var   0.436   0.823

Test of the hypothesis that 2 factors are sufficient.
The chi square statistic is 3.8 on 4 degrees of freedom.
The p-value is 0.434
```

Confirmatory Factor Analysis

- EFA allowed connections between each factor and each indicator
- In CFA, we restrict–for theoretical reasons–linkages between factors and indicators
- Math is indicated by:
  - wratcalc, wjcalc, waiscalc
- Spelling is indicated by:
  - wratspl, wjspl, waisspl

Two Factor CFA



R code and results

```
library(lavaan)
CFAmodel <- "
   MATH =~ wratcalc + wjcalc + waiscalc
   SPELL =~ wratspl + wjspl + waisspl
 "
testOutput <- cfa(model = CFAmodel, data = dat,
    std.lv = TRUE, missing = "fiml")
summary(testOutput, fit.measures = TRUE)
```

```
lavaan 0.6-3 ended normally after 48 iterations

  Optimization method                           NLMINB
  Number of free parameters                         19

  Number of observations                           322
  Number of missing patterns                         4

  Estimator                                         ML
  Model Fit Test Statistic                       9.540
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.299
```

R code and results ...

```
Model test baseline model:

  Minimum Function Test Statistic              1882.335
  Degrees of freedom                                 15
  P-value                                         0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                     0.999
  Tucker-Lewis Index (TLI)                        0.998

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)               -5127.830
  Loglikelihood unrestricted model (H1)       -5123.060

  Number of free parameters                          19
  Akaike (AIC)                                10293.661
  Bayesian (BIC)                              10365.377
  Sample-size adjusted Bayesian (BIC)         10305.112

Root Mean Square Error of Approximation:

  RMSEA                                           0.024
  90 Percent Confidence Interval          0.000   0.073
```

R code and results ...

```
  P-value RMSEA <= 0.05                          0.761

Standardized Root Mean Square Residual:

  SRMR                                           0.024

Parameter Estimates:

  Information                                 Observed
  Observed information based on                Hessian
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratcalc          6.041    0.276   21.921    0.000
    wjcalc            4.144    0.203   20.370    0.000
    waiscalc          2.410    0.165   14.636    0.000
  SPELL =~
    wratspl           6.532    0.288   22.645    0.000
    wjspl             6.809    0.296   23.025    0.000
    waisspl           6.354    0.283   22.463    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~~
```

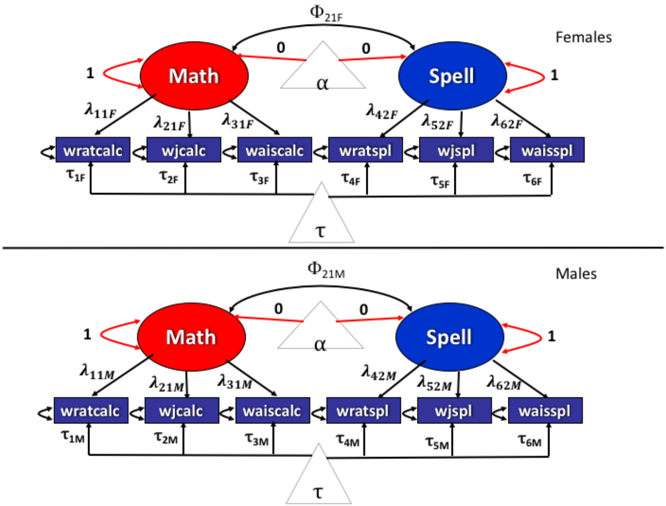
R code and results ...

```
    SPELL              0.553    0.042   13.191    0.000

Intercepts:
                    Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc          38.922    0.355  109.514    0.000
   .wjcalc            23.812    0.255   93.297    0.000
   .waiscalc          11.022    0.186   59.230    0.000
   .wratspl           36.484    0.385   94.751    0.000
   .wjspl             41.674    0.398  104.808    0.000
   .waisspl           37.163    0.376   98.788    0.000
    MATH               0.000
    SPELL              0.000

Variances:
                    Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc           4.179    1.014    4.122    0.000
   .wjcalc             3.769    0.537    7.017    0.000
   .waiscalc           5.304    0.457   11.596    0.000
   .wratspl            5.053    0.612    8.256    0.000
   .wjspl              4.541    0.616    7.369    0.000
   .waisspl            5.156    0.599    8.605    0.000
    MATH               1.000
    SPELL              1.000
```

Multiple Group CFA

- Compare males and females on Math and Spelling

Two group model



Goal: Simplify separate parameter sets

- If the two genders are given completely separate models, there is not much hope of achieving an analytical purpose, which is to compare differences in the latent variable across genders.

Model comparison process

1. Fit the model with all measurement parameters free to vary between groups
2. Fit a model in which measurement parameters are assumed to be the same
3. Conduct a "$\Delta\chi^2$" test to find out if restricting some parameters to be equal between groups caused the model to fit the data poorly.

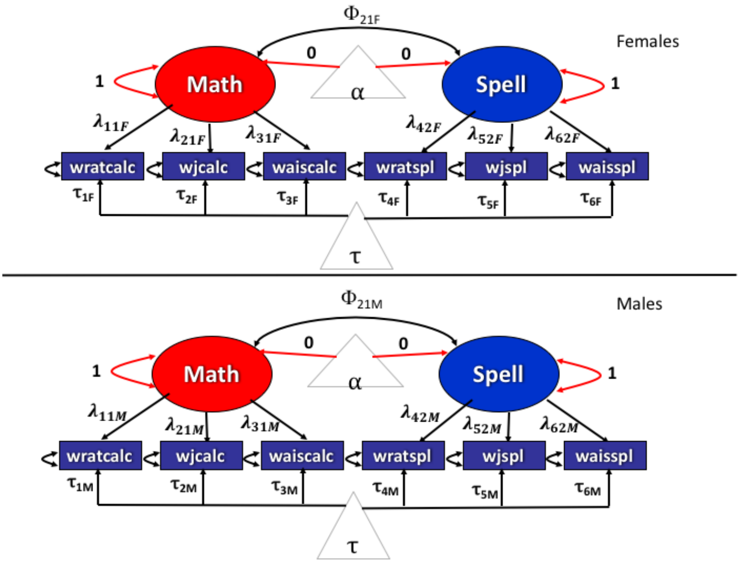
Configural, Metric, Scalar

- Three stages of comparison (models successively drop coefficient differences for the 2 genders)

1. Configural invariance (same model)
2. Metric invariance (factor loadings same for both genders)
3. Scalar invariance (item intercepts are also the same)

Outline

Configural Invariance Model



R code and results

```
configModel <- "
   MATH =~ wratcalc + wjcalc + waiscalc
   SPELL =~ wratspl + wjspl + waisspl
 "
configOutput <- cfa(model = configModel, data =
    dat, group = "female", std.lv = TRUE, missing
    = "fiml")
summary(configOutput)
```

```
lavaan 0.6-3 ended normally after 66 iterations

  Optimization method                           NLMINB
  Number of free parameters                         38

  Number of observations per group
  1                                                101
  0                                                221
  Number of missing patterns per group
  1                                                  4
  0                                                  2
```

R code and results ...

```
  Estimator                                         ML
  Model Fit Test Statistic                      19.215
  Degrees of freedom                                16
  P-value (Chi-square)                           0.258

Chi-square for each group:

  1                                             11.555
  0                                              7.660

Parameter Estimates:

  Information                                 Observed
  Observed information based on                Hessian
  Standard Errors                             Standard


Group 1 [1]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratcalc          6.545    0.496   13.187    0.000
    wjcalc            4.215    0.366   11.530    0.000
    waiscalc          2.290    0.276    8.306    0.000
  SPELL =~
```

R code and results ...

```
    wratspl           7.010    0.547   12.817    0.000
    wjspl             6.833    0.518   13.182    0.000
    waisspl           6.638    0.520   12.763    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~~
    SPELL             0.634    0.064    9.914    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc         38.267    0.666   57.464    0.000
   .wjcalc           23.603    0.465   50.781    0.000
   .waiscalc         10.266    0.316   32.528    0.000
   .wratspl          37.171    0.735   50.590    0.000
   .wjspl            42.337    0.705   60.073    0.000
   .waisspl          38.030    0.697   54.579    0.000
    MATH              0.000
    SPELL             0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc          1.957    1.621    1.208    0.227
   .wjcalc            3.960    0.848    4.671    0.000
   .waiscalc          4.765    0.714    6.678    0.000
   .wratspl           5.307    1.105    4.804    0.000
```

R code and results ...

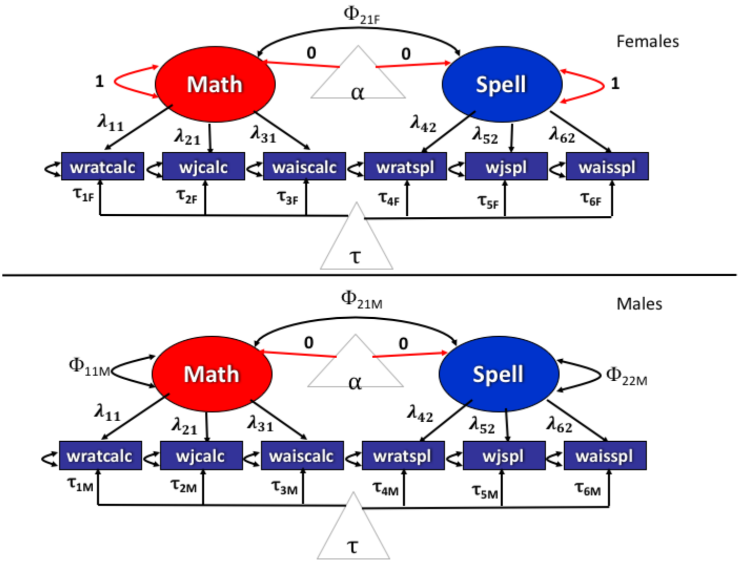
```
    .wjspl          3.474    0.910    3.818    0.000
    .waisspl        4.900    1.007    4.865    0.000
     MATH           1.000
     SPELL          1.000


Group 2 [0]:

Latent Variables:
                  Estimate  Std.Err  z-value  P(>|z|)
   MATH =~
     wratcalc       5.764    0.330   17.492    0.000
     wjcalc         4.123    0.244   16.926    0.000
     waiscalc       2.446    0.200   12.212    0.000
   SPELL =~
     wratspl        6.278    0.337   18.646    0.000
     wjspl          6.788    0.359   18.929    0.000
     waisspl        6.177    0.335   18.441    0.000

Covariances:
                  Estimate  Std.Err  z-value  P(>|z|)
   MATH ~~
     SPELL          0.520    0.053    9.762    0.000

Intercepts:
                  Estimate  Std.Err  z-value  P(>|z|)
```

R code and results ...

```
   .wratcalc         39.222    0.417   93.962    0.000
   .wjcalc           23.910    0.305   78.420    0.000
   .waiscalc         11.367    0.226   50.320    0.000
   .wratspl          36.172    0.448   80.762    0.000
   .wjspl            41.371    0.480   86.157    0.000
   .waisspl          36.767    0.444   82.899    0.000
    MATH              0.000
    SPELL             0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc          5.280    1.232    4.285    0.000
   .wjcalc            3.549    0.664    5.342    0.000
   .waiscalc          5.269    0.557    9.463    0.000
   .wratspl           4.915    0.726    6.772    0.000
   .wjspl             4.881    0.795    6.138    0.000
   .waisspl           5.285    0.736    7.176    0.000
    MATH              1.000
    SPELL             1.000
```

Outline

Metric Invariance Model



R code and results

```
metricModel <- "
   MATH =~ wratcalc + wjcalc + waiscalc
   SPELL =~ wratspl + wjspl + waisspl
   MATH ~~ c(1, NA)*MATH
   SPELL ~~ c(1, NA)*SPELL
 "
metricOutput <- cfa(model = metricModel, data =
   dat, std.lv = TRUE,
                    group = "female",
                       group.equal = "loadings",
                    missing = "fiml")
summary(metricOutput)
```

R code and results ...

```
lavaan 0.6-3 ended normally after 71 iterations

  Optimization method                           NLMINB
  Number of free parameters                         40
  Number of equality constraints                     6

  Number of observations per group
  1                                                101
  0                                                221
  Number of missing patterns per group
  1                                                  4
  0                                                  2

  Estimator                                         ML
  Model Fit Test Statistic                      25.871
  Degrees of freedom                                20
  P-value (Chi-square)                           0.170

Chi-square for each group:

  1                                             15.581
  0                                             10.290

Parameter Estimates:

```

R code and results ...

```
  Information                                  Observed
  Observed information based on                 Hessian
  Standard Errors                              Standard


Group 1 [1]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratclc (.p1.)    6.359    0.490   12.972    0.000
    wjcalc  (.p2.)    4.354    0.342   12.729    0.000
    waisclc (.p3.)    2.502    0.229   10.909    0.000
  SPELL =~
    wratspl (.p4.)    6.751    0.508   13.295    0.000
    wjspl   (.p5.)    7.020    0.516   13.592    0.000
    waisspl (.p6.)    6.568    0.493   13.326    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~~
    SPELL             0.641    0.063   10.164    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc         38.267    0.656   58.319    0.000
```

R code and results ...

```
    .wjcalc          23.602    0.474   49.832    0.000
    .waiscalc        10.263    0.330   31.085    0.000
    .wratspl         37.172    0.713   52.136    0.000
    .wjspl           42.337    0.721   58.719    0.000
    .waisspl         38.031    0.691   55.052    0.000
     MATH             0.000
     SPELL            0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
     MATH             1.000
     SPELL            1.000
    .wratcalc         3.047    1.413    2.157    0.031
    .wjcalc           3.603    0.778    4.632    0.000
    .waiscalc         4.700    0.714    6.586    0.000
    .wratspl          5.686    1.123    5.061    0.000
    .wjspl            3.230    0.929    3.478    0.001
    .waisspl          4.995    1.023    4.883    0.000


Group 2 [0]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratclc (.p1.)    6.359    0.490   12.972    0.000
```

R code and results ...

```
    wjcalc  (.p2.)    4.354    0.342   12.729    0.000
    waisclc (.p3.)    2.502    0.229   10.909    0.000
  SPELL =~
    wratspl (.p4.)    6.751    0.508   13.295    0.000
    wjspl   (.p5.)    7.020    0.516   13.592    0.000
    waisspl (.p6.)    6.568    0.493   13.326    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~~
    SPELL             0.456    0.089    5.103    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wratcalc         39.222    0.422   92.880    0.000
   .wjcalc           23.910    0.302   79.192    0.000
   .waiscalc         11.367    0.221   51.462    0.000
   .wratspl          36.172    0.454   79.673    0.000
   .wjspl            41.371    0.472   87.637    0.000
   .waisspl          36.767    0.446   82.522    0.000
    MATH              0.000
    SPELL             0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    MATH              0.860    0.156    5.501    0.000
```

R code and results ...

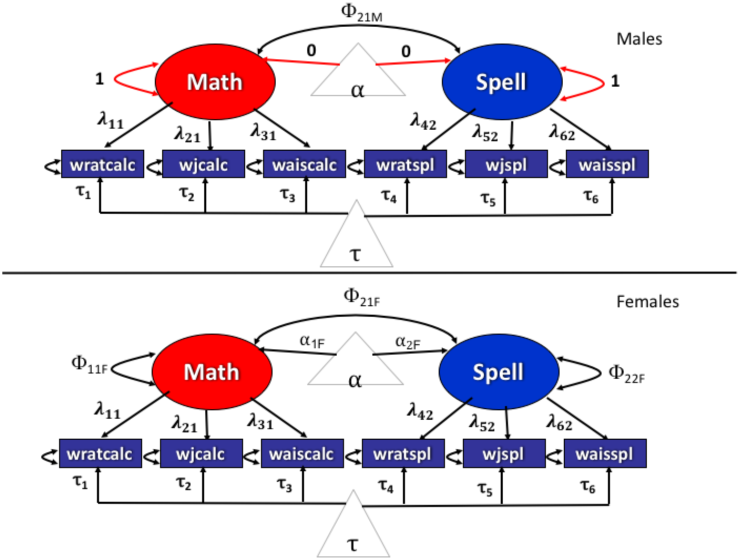
```
    SPELL              0.895     0.158     5.676     0.000
   .wratcalc           4.619     1.205     3.834     0.000
   .wjcalc             3.835     0.654     5.864     0.000
   .waiscalc           5.370     0.564     9.525     0.000
   .wratspl            4.764     0.721     6.607     0.000
   .wjspl              5.154     0.790     6.526     0.000
   .waisspl            5.233     0.732     7.153     0.000
```

```
anova(configOutput, metricOutput)
```

```
Chi Square Difference Test

             Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
configOutput 16 10301 10444 19.215
metricOutput 20 10300 10428 25.871     6.6558       4     0.1552
```

Outline

Scalar Invariance Model



R code and results

```
scalarModel <- "
   MATH =~ wratcalc + wjcalc + waiscalc
   SPELL =~ wratspl + wjspl + waisspl
   MATH ~~ c(1, NA)*MATH
   SPELL ~~ c(1, NA)*SPELL
   MATH ~ c(0, NA)*1
   SPELL ~ c(0, NA)*1
 "
scalarOutput <- cfa(model = scalarModel, data =
   dat, std.lv = TRUE,
                     group = "female",
                     group.equal = c("loadings",
                        "intercepts"),
                     missing = "fiml")
summary(scalarOutput)
```

R code and results ...

```
lavaan 0.6-3 ended normally after 57 iterations

  Optimization method                           NLMINB
  Number of free parameters                         42
  Number of equality constraints                    12

  Number of observations per group
  1                                                101
  0                                                221
  Number of missing patterns per group
  1                                                  4
  0                                                  2

  Estimator                                         ML
  Model Fit Test Statistic                      36.362
  Degrees of freedom                                24
  P-value (Chi-square)                           0.051

Chi-square for each group:

  1                                             22.546
  0                                             13.815

Parameter Estimates:

```

R code and results ...

```
  Information                                    Observed
  Observed information based on                   Hessian
  Standard Errors                                Standard


Group 1 [1]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratclc (.p1.)    6.381    0.491   12.994    0.000
    wjcalc  (.p2.)    4.340    0.342   12.699    0.000
    waisclc (.p3.)    2.529    0.232   10.895    0.000
  SPELL =~
    wratspl (.p4.)    6.751    0.508   13.299    0.000
    wjspl   (.p5.)    7.015    0.516   13.591    0.000
    waisspl (.p6.)    6.578    0.494   13.329    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~~
    SPELL             0.640    0.063   10.128    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
    MATH              0.000
```

R code and results ...

```
    SPELL              0.000
   .wratclc  (.18.)   38.273    0.653   58.628    0.000
   .wjcalc   (.19.)   23.374    0.454   51.465    0.000
   .waisclc  (.20.)   10.751    0.289   37.239    0.000
   .wratspl  (.21.)   37.211    0.692   53.771    0.000
   .wjspl    (.22.)   42.411    0.713   59.443    0.000
   .waisspl  (.23.)   37.869    0.675   56.144    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    MATH               1.000
    SPELL              1.000
   .wratcalc           2.841    1.424    1.995    0.046
   .wjcalc             3.743    0.793    4.723    0.000
   .waiscalc           4.977    0.766    6.496    0.000
   .wratspl            5.681    1.123    5.056    0.000
   .wjspl              3.237    0.930    3.481    0.000
   .waisspl            5.035    1.032    4.880    0.000


Group 2 [0]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratclc  (.p1.)    6.381    0.491   12.994    0.000
```

R code and results ...

```
    wjcalc  (.p2.)    4.340    0.342   12.699    0.000
    waisclc (.p3.)    2.529    0.232   10.895    0.000
  SPELL =~
    wratspl (.p4.)    6.751    0.508   13.299    0.000
    wjspl   (.p5.)    7.015    0.516   13.591    0.000
    waisspl (.p6.)    6.578    0.494   13.329    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~~
    SPELL             0.454    0.089    5.104    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
    MATH              0.148    0.121    1.220    0.222
    SPELL            -0.156    0.121   -1.293    0.196
   .wratclc (.18.)   38.273    0.653   58.628    0.000
   .wjcalc  (.19.)   23.374    0.454   51.465    0.000
   .waisclc (.20.)   10.751    0.289   37.239    0.000
   .wratspl (.21.)   37.211    0.692   53.771    0.000
   .wjspl   (.22.)   42.411    0.713   59.443    0.000
   .waisspl (.23.)   37.869    0.675   56.144    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    MATH              0.857    0.156    5.502    0.000
```

R code and results ...

```
    SPELL                 0.894    0.158    5.676    0.000
   .wratcalc              4.518    1.204    3.753    0.000
   .wjcalc                3.911    0.655    5.970    0.000
   .waiscalc              5.431    0.576    9.435    0.000
   .wratspl               4.761    0.721    6.603    0.000
   .wjspl                 5.173    0.791    6.541    0.000
   .waisspl               5.235    0.734    7.134    0.000
```

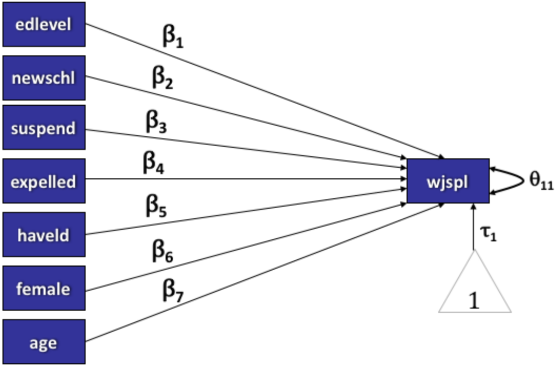
```
anova(metricOutput, scalarOutput)
```

```
Chi Square Difference Test

             Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
metricOutput 20 10300 10428 25.871
scalarOutput 24 10302 10415 36.361     10.491       4    0.03293 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Regression

- The SEM framework includes the ordinary linear regression on observed variables as a special case
- SEM can fit several linear regressions at once
    - possibly allowing correlation between the error terms in the 2 regressions (Seemingly Unrelated Regressions, SUR).

Multiple predictors



R code and results

```
MLRModel <- "
 wjspl ~ edlevel + newschl + suspend + expelled +
    haveld + female + age
 wjspl ~ 1
 "
output <- sem(model = MLRModel, data = dat)
summary(output)
```

```
lavaan 0.6-3 ended normally after 58 iterations

  Optimization method                           NLMINB
  Number of free parameters                          9

                                                  Used       Total
  Number of observations                           313         322

  Estimator                                         ML
  Model Fit Test Statistic                       0.000
  Degrees of freedom                                 0

Parameter Estimates:
```

R code and results ...

```
  Information                                      Expected
  Information saturated (h1) model               Structured
  Standard Errors                                  Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  wjspl ~
    edlevel           1.162    0.324    3.584    0.000
    newschl           0.063    0.747    0.085    0.932
    suspend          -0.052    0.773   -0.067    0.947
    expelled         -2.758    1.098   -2.510    0.012
    haveld           -6.974    0.987   -7.063    0.000
    female            0.720    0.792    0.909    0.363
    age               0.412    0.207    1.994    0.046

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wjspl            21.953    4.414    4.974    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .wjspl            39.904    3.190   12.510    0.000
```

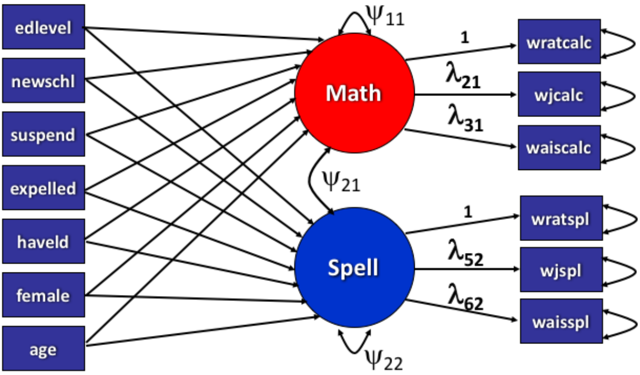
Regression

- Estimate regression coefficients that link together latent (unmeasured) variables.
- This is an example of a LISREL model (measurement and structural component)

Math and Spelling

- Continues with previous data
- Research question: are variations in the latent factors “Math” and “Spell” predictable using the variables
  - edlevel
  - newschl
  - suspend
  - expelled
  - haveld
  - female
  - age

SEM


edlevel
newschl
suspend
expelled
haveld
female
age
Math
Spell
ψ_{11}
ψ_{21}
ψ_{22}
1
λ_{21}
λ_{31}
1
λ_{52}
λ_{62}
wratcalc
wjcalc
waiscalc
wratspl
wjspl
waisspl

R code and results

```
SEMModel <- "
   MATH =~ wratcalc + wjcalc + waiscalc
   SPELL =~ wratspl + wjspl + waisspl
   MATH ~ edlevel + newschl + suspend + expelled
      + haveld + female + age
   SPELL ~ edlevel + newschl + suspend + expelled
      + haveld + female + age
   MATH ~~ SPELL
 "
output <- sem(model = SEMModel, data = dat,
   missing = "fiml")
summary(output)
```

R code and results ...

```
lavaan 0.6-3 ended normally after 198 iterations

  Optimization method                           NLMINB
  Number of free parameters                         33

                                                  Used       Total
  Number of observations                           313         322
  Number of missing patterns                         4

  Estimator                                         ML
  Model Fit Test Statistic                      49.780
  Degrees of freedom                                36
  P-value (Chi-square)                           0.063

Parameter Estimates:

  Information                                 Observed
  Observed information based on                Hessian
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH =~
    wratcalc          1.000
    wjcalc            0.684    0.027   25.116    0.000
```

R code and results ...

```
    waiscalc          0.403    0.024   16.552    0.000
  SPELL =~
    wratspl           1.000
    wjspl             1.038    0.029   35.933    0.000
    waisspl           0.967    0.028   34.549    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  MATH ~
    edlevel           1.720    0.282    6.097    0.000
    newschl           0.535    0.649    0.824    0.410
    suspend          -1.727    0.677   -2.550    0.011
    expelled         -0.524    0.954   -0.549    0.583
    haveld           -1.495    0.858   -1.743    0.081
    female           -0.757    0.689   -1.098    0.272
    age               0.617    0.179    3.441    0.001
  SPELL ~
    edlevel           1.169    0.302    3.874    0.000
    newschl          -0.212    0.694   -0.305    0.760
    suspend          -0.111    0.718   -0.155    0.877
    expelled         -2.540    1.021   -2.488    0.013
    haveld           -6.631    0.923   -7.182    0.000
    female            0.843    0.736    1.145    0.252
    age               0.349    0.192    1.819    0.069

Covariances:
```

R code and results ...

```
                      Estimate  Std.Err  z-value  P(>|z|)
  .MATH ~~
    .SPELL              15.401    2.048    7.520    0.000

Intercepts:
                      Estimate  Std.Err  z-value  P(>|z|)
    .wratcalc            8.718    3.847    2.266    0.023
    .wjcalc              3.169    2.710    1.169    0.242
    .waiscalc           -1.141    1.686   -0.677    0.498
    .wratspl            17.949    4.113    4.364    0.000
    .wjspl              22.456    4.267    5.263    0.000
    .waisspl            19.289    3.978    4.849    0.000
    .MATH                0.000
    .SPELL               0.000

Variances:
                      Estimate  Std.Err  z-value  P(>|z|)
    .wratcalc            4.035    0.966    4.175    0.000
    .wjcalc              3.876    0.528    7.335    0.000
    .waiscalc            5.221    0.457   11.426    0.000
    .wratspl             4.985    0.612    8.151    0.000
    .wjspl               4.751    0.626    7.587    0.000
    .waisspl             4.960    0.591    8.385    0.000
    .MATH               27.561    2.600   10.601    0.000
    .SPELL              32.786    2.961   11.074    0.000
```

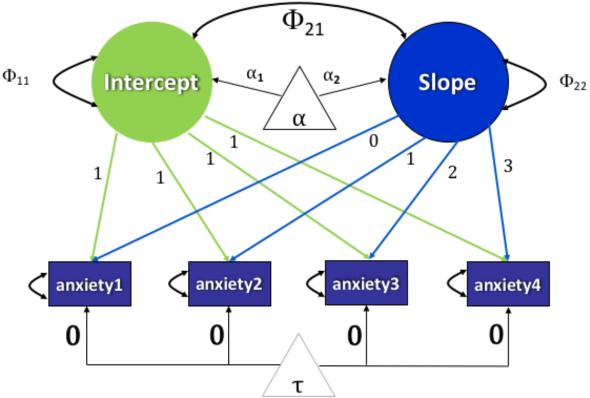
Latent Growth Curve Modeling

- Repeated measurements allow us to think about change in a latent variable over time
- If we can predictably measure the latent variable, then we can assess its linkage to other outcomes (both observed and latent).
- However, in this example we are mostly interested in the possibility that
  - some people are more anxious than others at the outset of the study, and they remain more (or less) anxious across time
  - as time passes, some people become progressively more and more (or less and less) anxious

Measurements

- The data set is named “anxiety.dat”
- participants anxiety level at four different time points were measured
- Do anxiety levels change over time?
- Initial anxiety (intercept) and change in anxiety (slope) can be modeled as latent variables.

Latent Growth Curve Graph



R code and results

```
dat <- read.table("data/anxiety.dat", header = F)
names(dat) <- c("a1", "a2", "a3", "a4")
model <- "
    intercept =~ 1*a1 + 1*a2 + 1*a3 + 1*a4
    slope =~ 0*a1 + 1*a2 + 2*a3 + 3*a4
    a1 ~ 0*1
    a2 ~ 0*1
    a3 ~ 0*1
    a4 ~ 0*1
    intercept ~ 1
    slope ~ 1
 "
output <- sem(model, data = dat)
summary(output)
```

R code and results ...

```
lavaan 0.6-3 ended normally after 45 iterations

  Optimization method                           NLMINB
  Number of free parameters                          9

  Number of observations                           485

  Estimator                                         ML
  Model Fit Test Statistic                      27.288
  Degrees of freedom                                 5
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  intercept =~
    a1                1.000
    a2                1.000
    a3                1.000
    a4                1.000
```

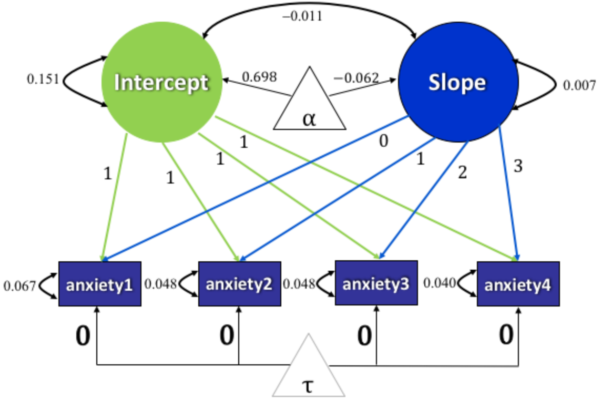
R code and results ...

```
  slope =~
    a1                0.000
    a2                1.000
    a3                2.000
    a4                3.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  intercept ~~
    slope            -0.011    0.003   -3.472    0.001

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .a1                0.000
   .a2                0.000
   .a3                0.000
   .a4                0.000
    intercept         0.698    0.020   35.050    0.000
    slope            -0.062    0.006  -10.513    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .a1                0.067    0.007    8.910    0.000
   .a2                0.048    0.004   11.119    0.000
   .a3                0.048    0.004   11.383    0.000
   .a4                0.040    0.006    6.658    0.000
```

R code and results ...

```
    intercept          0.151    0.013   11.871    0.000
    slope              0.007    0.001    4.790    0.000
```

Latent Growth Results



SEM Examples

- More examples will be created in future

References

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria.

Session

```
sessionInfo()
```

```
R version 3.6.0 (2019-04-26)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 19.04

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/atlas/libblas.so.3.10.3
LAPACK: /usr/lib/x86_64-linux-gnu/atlas/liblapack.so.3.10.3

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
     LC_TIME=en_US.UTF-8
 [4] LC_COLLATE=en_US.UTF-8     LC_MONETARY=en_US.UTF-8
     LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C                  LC_ADDRESS=C
[10] LC_TELEPHONE=C             LC_MEASUREMENT=en_US.UTF-8
    LC_IDENTIFICATION=C

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
[1] lavaan_0.6-3
```

Session ...

```
loaded via a namespace (and not attached):
[1] MASS_7.3-51.4  compiler_3.6.0 tools_3.6.0    mnormt_1.5-5
    pbivnorm_0.6.0 stats4_3.6.0
```