The effects of entanglement between gravity and matter on the motion of localized massive particles

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Abstract

We derive an effective equation of motion for a pointlike particle in the framework of quantum gravity from simple basic assumptions [1]. The geodesic motion of a classical particle can be deduced by coupling a classical field theory to general relativity. We use a similar method to obtain an effective equation of motion, starting from an abstract quantum gravity description. We find that entanglement between gravity and matter leads to modifications of the geodesic trajectory, mainly because of nonzero overlap terms between gravity-matter coherent states. Lastly, we discuss a possible violation of the weak equivalence principle due to the nongeodesic motion. We show that in the Newtonian limit, the acceleration of the particle depends on its mass and the inertial and gravitational masses are not equal.

Derivation of the geodesic equation from general relativity

Single pole approximation:

$$T^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau\, B^{\mu\nu}(\tau)\frac{\delta^{(4)}(x-z(\tau))}{\sqrt{-g}} \qquad (1)$$

Assuming the local Poincaré invariance for *both* $S_G[g]$ and $S_M[g,\phi]$:

$$\nabla_\nu T^{\mu\nu} = 0 \qquad (2)$$

Substituting (1) into (2), we obtain the geodesic equation, with $u^\lambda \nabla_\lambda \equiv \nabla$, $u^\mu \equiv \frac{dz^\mu(\tau)}{d\tau}$ and $u^\mu u_\mu \equiv -1$ (Mathisson and Papapetrou [2,3]; see also [4]):

$$\nabla u^\mu = 0$$

Quantising gravity

Given the fundamental gravitational degrees of freedom $\hat{g}$ and $\hat{\pi}_g$:

$$\Delta\hat{g}\Delta\hat{\pi}_g \geq \frac{\hbar}{2} \qquad \Delta\hat{\phi}\Delta\hat{\pi}_\phi \geq \frac{\hbar}{2}$$

$$|\Psi\rangle = |g\rangle \otimes |\phi\rangle$$

$|g\rangle$ and $|\phi\rangle$ – coherent states of gravity and matter.

$$g_{\mu\nu} \equiv \langle\Psi|\hat{g}_{\mu\nu}|\Psi\rangle \qquad T_{\mu\nu} \equiv \langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle$$

Deviation of the geodesic motion due to entanglement

$$|\boldsymbol{\Psi}\rangle = \kappa|\Psi\rangle + \eta|\Psi^\perp\rangle$$

Orthogonal state. $|\kappa|^2 = 1-\eta^2$, with η small

"Entangled" metric and stress-energy:
$\boldsymbol{g}_{\mu\nu} = \langle\boldsymbol{\Psi}|\hat{g}_{\mu\nu}|\boldsymbol{\Psi}\rangle$
$\boldsymbol{T}_{\mu\nu} = \langle\boldsymbol{\Psi}|\hat{T}_{\mu\nu}|\boldsymbol{\Psi}\rangle$

$$\boldsymbol{g}_{\mu\nu} = g_{\mu\nu} + \eta\, h_{\mu\nu} + \mathcal{O}(\eta^2)$$
$$\boldsymbol{T}_{\mu\nu} = T_{\mu\nu} + \eta\, t_{\mu\nu} + \mathcal{O}(\eta^2)$$

$$h_{\mu\nu} = 2\,\mathrm{Re}\left(\kappa\langle\Psi^\perp|\hat{g}_{\mu\nu}|\Psi\rangle\right) + \mathcal{O}(\eta)$$
$$t_{\mu\nu} = 2\,\mathrm{Re}\left(\kappa\langle\Psi^\perp|\hat{T}_{\mu\nu}|\Psi\rangle\right) + \mathcal{O}(\eta)$$

From the quantum analogue of (2), $\boldsymbol{\nabla}_\nu \boldsymbol{T}^{\mu\nu} = 0$, we get:

An effective mass parameter $m(\tau)$,

$$(B+\eta\bar{B})u^\mu u^\nu \equiv m(\tau)u^\mu u^\nu$$

$$t^{\mu\nu} = \int_{\mathcal{C}} d\tau\, \bar{B}^{\mu\nu}(\tau)\frac{\delta^{(4)}(x-z(\tau))}{\sqrt{-g}}$$

The proper time evolution of the mass parameter,

$$\nabla m = \eta m u^\sigma \left(u^\nu u_\lambda F^\lambda{}_{\nu\sigma} - F^\nu{}_{\nu\sigma}\right)\,,$$

The effective equation of motion of the particle (modified geodesic trajectory),

$$\nabla u^\mu + \eta u^\nu u^\sigma F^\mu_{\perp\nu\sigma} = 0$$

$$F^\mu_{\perp\nu\sigma} = P^\mu_{\perp\lambda} F^\lambda{}_{\nu\sigma}.$$
$$F^\lambda{}_{\nu\sigma} \equiv \nabla_{(\sigma} h^\lambda{}_{\nu)} - \frac{1}{2}\nabla^\lambda h_{\nu\sigma}$$
$$A_{(\sigma\nu)} \equiv \frac{1}{2}(A_{\sigma\nu} + A_{\nu\sigma})$$

Equivalence principle and quantum theory

$$\begin{tikzcd}
\mathrm{QFT}_\eta \arrow[r, "\text{classical limit}"] \arrow[d, dashed, "\text{QSEP}"'] & \mathrm{ECFT}_\eta \arrow[r, "\text{single pole approx.}"] & \mathrm{CM}_\eta \arrow[d, "\text{WEP}"] \\
\mathrm{QG} \arrow[r, "\text{classical limit}"] & \mathrm{ECFT}_g \arrow[r, "\text{single pole approx.}"] & \mathrm{CM}_g
\end{tikzcd}$$

Newtonian limit of the effective equation

The acceleration depends on mass and the inertial and gravitational masses are not equal:

$$m_I\frac{d^2z^k}{d\tau^2} = -m_I\left(1-\frac{1}{3}\eta h^i{}_i\right)\frac{GM}{r^3}z^k - \eta m_I\left[\partial_0 h_{0k} - \frac{1}{2}\partial_k h_{00} - \frac{GM}{r^3}z^j\tilde{h}_{jk}\right]$$

$$h^i{}_i = 2\delta^{ij}\,\mathrm{Re}\left(\kappa\langle\phi|\tilde{\phi}\rangle\frac{\epsilon_G}{\epsilon}\langle g^\perp|\hat{g}_{ij}|g\rangle\right) + \mathcal{O}(\eta)$$

It depends on the overlap of the matter fields, $\langle\phi|\tilde{\phi}\rangle$.

$$\frac{m_G}{m_I} \equiv \left(1-\frac{1}{3}\eta h^i{}_i\right)$$

Conclusions and discussion

- We derived an effective equation of motion within an abstract framework of quantum gravity, in particular in the case where both matter and gravity are in a quantum superposition of macroscopic states. This equation contains a non-geodesic term, giving rise to an effective force acting on the particle, as a consequence of overlap terms.
- We found a violation of the weak equivalence principle relative to the dominant metric $g_{\mu\nu}$, in the context of field theory and deriving a point-particle mechanics as a consequence of this theory, working in a fully general-relativistic regime, and making use of proper relativistic observables (formally quantized).
- It would be interesting to estimate the magnitude of the nongeodesic term to compare different quantum gravity models. We would need to specify the concrete quantum gravity models. Then, use the equation of motion or the equation that relates the gravitational and inertial masses.

References

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