Structural Equation Models

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CENTER FOR RESEARCH METHODS & DATA ANALYSIS
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Outline

Outline

Use affect.rds from summeR-3-2-lm

```
file.copy(from =
    "../summeR-3-2-lm/workingdata/affect.rds", to
    = "workingdata/affect.rds", overwrite = TRUE)
```

```
[1] TRUE
```

```
dat <- readRDS("workingdata/affect.rds")
```

```
head(dat)
```

```
  Agency1 Agency2 Agency3 Intrin1 Intrin2 Intrin3 Extrin1
1  3.5000  4.0000  4.0000  4.0000     4.0       4  1.0000
2  2.5000  3.1667  3.0000  3.2123     2.0       3  1.8333
3  1.8333  2.0000  1.5000  3.0000     3.0       2  1.0000
4  2.7714  3.0602  2.3639  3.1337     4.0       3  1.0774
5  3.1667  3.3333  2.8333  3.5000     4.0       4  1.8333
6  2.3333  2.8333  2.3333  3.0000     2.5       3  3.0588
  Extrin2 Extrin3 PosAFF1 PosAFF2 PosAFF3 NegAFF1 NegAFF2
1  1.0000  1.5000  4.0000     4.0     4.0     1.0  1.0000
2  2.6667  1.8333  3.0000     3.5     2.5     1.5  1.6858
3  1.0000  1.0000  3.0184     2.5     3.0     1.0  1.0000
4  1.1667  1.0000  3.0000     2.5     3.0     2.5  2.5000
5  2.0000  1.8333  3.7804     3.5     3.0     2.5  2.0000
6  2.4125  2.6667  4.0000     3.0     3.0     2.0  1.5000
```

Outline

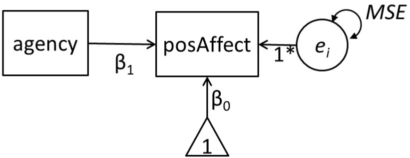
Path: This goes into that

- The regression equation

$$posAffect_i = \beta_0 + \beta_1 agency_i + \epsilon_i, \epsilon \sim N(0, \sigma^2_\epsilon)$$

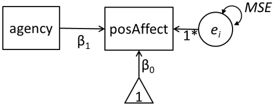
  The parameters to be estimated are β_0 , β_1, and σ^2_ϵ.

- Some people understand this more readily in a path diagram:



MSE = "mean squared error" is an estimate of σ^2_ϵ

Symbols



Rectangles: Observed variables

Triangle: a constant value

Circle: unobserved variable (error term, in this case)

Double arrowed-loop: modern way of saying there's random variance in this thing.

1*: a coefficient is fixed at 1, the error is assumed additive.

β: path coefficients

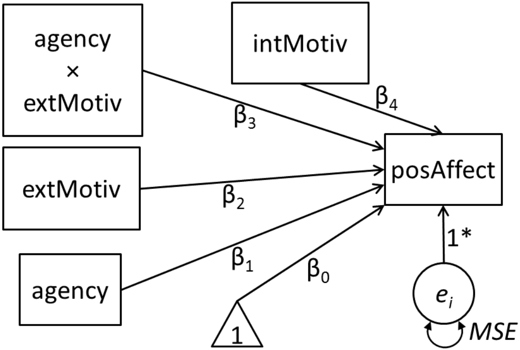
Throw in More Variables

- I think of this:

$$\begin{aligned} posAffect_i &= \beta_0 + \beta_1 agency_i + \beta_2 extMotiv_i \\ &\quad \beta_3 agency_i \times extMotiv + \beta_4 intMotiv_i + \\ &\quad \epsilon_i,\ \epsilon \sim N(0, \sigma^2_\epsilon) \end{aligned}$$

Path Diagram of posAffect Regression

- A Path diagram oriented person thinks of this (might want to quibble about how to draw interaction)



Moderator effects

When we ran the regression model

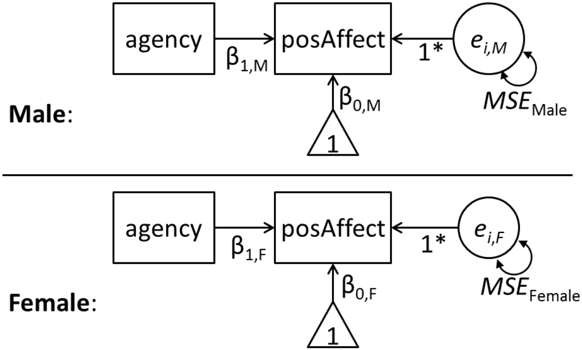
$$posAffect_i = \beta_0 + \beta_1 agency + \beta_2 female_i + \beta_3 agency_i \cdot female_i, \epsilon_i \sim N(0, \sigma_\epsilon^2) \quad (1)$$

we allowed the genders to differ in intercept and slope

- Gender is a "**moderator**" of the agency effect
- But we assumed that the error variance is the same in the 2 groups.
    - That's a testable assumption

In Psychology, they would rather look at the problem in this way

- In the usual path diagram, interactions don't fit well, especially to fully represent the interaction of gender.
- An alternative sketch segregates the 2 genders entirely.



Assume β_j and σ_ϵ^2 differ between groups

If the 2 groups have all different parameters, we might as well think of them as completely separate data exercises.

- Could run one model for males, one for females, or
- fit both within some larger structure that allows us to see if the intercepts, slopes, and error variances are truly different.
- We are (probably) hoping to "simplify" our analysis to a more "parsimonious" model.
- In econometrics, they'd call this a "pooling test"
- In psychology, they call it "measurement invariance testing"

Write out 2 equations

- agency affects intrinsic motivation

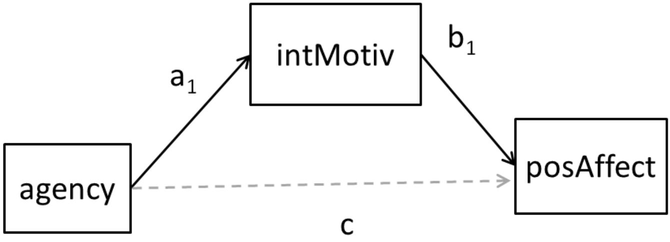
$$intMotiv_i = const1 + a_1 agency_i + \epsilon 1_i \tag{2}$$

- agency affects positiveAffect, but positiveAffect is also affected by intrinsic motivation

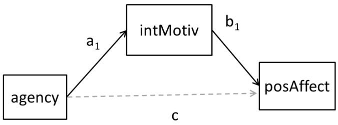
$$posAffect = const2 + b_1 intMotiv + c\, agency_i + \epsilon 2_i \tag{3}$$

- The coefficient "c" represents the direct effect of agency on positive affect

Some people like a picture



Some people like a picture



- The product "$a_1 \cdot b_1$" is the indirect effect of agency. To see that, substitute

$$\begin{aligned} posAffect &= const1 + b_1\{const1 + a_1 agency_i + error_i\} + c\,agency_i + \epsilon 2_i \quad (4 \\ &= \{const2 + b_1 const1\} + \{b_1 \cdot a_1\} agency_i + c\,agency_i + \{\epsilon 1_i + \epsilon 2_i \end{aligned}$$

Some people like a picture ...

- As a result, a regression model of posAffect on agency might be wrong, in the sense that if we estimate

$$posAffect_i = \beta_0 + \beta_1 agency_i + \epsilon_i \quad (5)$$

  then the coefficient we estimate, β_1 includes both the "direct" effect b_1 and the indirect effect $a_1 b_1$
- The age old question is, "does agency's *direct* effect outweigh the *indirect* effect"?
- In the olden days (maybe even today),
  - students will fit 2 separate regression models, and
  - try to test the hypothesis that the product $a_1 b_1$ is different from 0.
- As long as $\epsilon 1_i$ and $\epsilon 2_i$ are uncorrelated, that should be sufficient.
- If they are correlated, then some sort of simultaneous regression is needed
  - in economics, they might use "Seemingly Unrelated Regression" (3SLS)
  - in psychology, they would see this as a structural equation model (SEM)

There is a horrible detail waiting around the corner

- The indirect effect is $a_1 b_1$, a *product*
- It will be zero (really small) if either a_1 or b_1 is zero (really small).
- Thus, the hull hypothesis we would like to test,

$$H_0 : a_1 b_1 = 0$$

  - is a gnarly, horrible thing that amounts to a null statement that "either a_1 or b_1 (or both), is (are) 0."
- After chasing approximations for 20 years, it appears the best answer we can get is 'bootstrapped standard errors'.

Outline

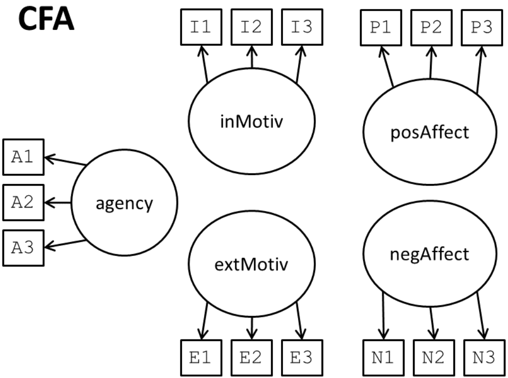
We created scales

$$
\begin{aligned}
agency &= (Agency1 + Agency2 + Agency3)/3 \\
intMotiv &= (Intrin1 + Intrin2 + Intrin3)/3 \\
extMotiv &= (Extrin1 + Extrin2 + Extrin3)/3 \\
posAffect &= (PosAFF1 + PosAFF2 + PosAFF3)/3 \\
negAffect &= (NegAFF1 + NegAFF2 + NegAFF3)/3
\end{aligned}
$$

- And then we ran regressions *as if* those were “real measurements”.

We were Careless about Measurement

- A structural equation modeler sees a need to build a measurement model
- A "Confirmatory Factor Analysis" simultaneously
  1. Checks how the individual variables load on the unmeasured latent "constructs"
  2. Investigates relationships between the latent "constructs"



Outline

Competing packages

- The sem() function in the "car" package was the first SEM fitter for R (R Core Team, 2017)
- lavaan is a complete framework for SEM modeling in the style of Mplus
- OpenMX is a large package recently introduced in CRAN that can estimate SEM and many other matrix-based models
- Various other R packages exist, either as wholesale replacements (lava) or as supplementary tools (semPlot).

SEM software can fit single equation models

- The linear regression estimated with `lm()`, for example

```
lm1 <- lm(posAffect ~ agency + intMotiv +
    extMotiv*agency, data = dat)
coef(lm1)
```

```
  (Intercept)          agency        intMotiv
    2.7505404      -0.1104093       0.2526032
     extMotiv agency:extMotiv
   -0.6521769       0.2150615
```

- SEM software seems to want us to manually create the interaction variable. I manually calculate that interaction:

```
dat$agency.X.extMotiv <- dat$agency * dat$extMotiv
```

then specify a model formula and fit with lavaan's function sem()

SEM software can fit single equation models ...

```
library(lavaan)
sem.fmla <- 'posAffect ~ agency + intMotiv +
    extMotiv + agency.X.extMotiv'
sem1 <- sem(sem.fmla, data = dat, meanstructure =
    TRUE)
```

"meanstructure" = "I want to see the intercept in the output"

SEM output

- The default print output is a little uninspiring

```
sem1
```

```
lavaan (0.5-23.1097) converged normally after  22 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic                0.000
  Degrees of freedom                                 0
  Minimum Function Value               0.0000000000000
```

- The lavaan summary method more helpful

```
summary(sem1)
```

SEM output ...

```
lavaan (0.5-23.1097) converged normally after  22 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic                0.000
  Degrees of freedom                                 0
  Minimum Function Value               0.0000000000000

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    agency           -0.110    0.192   -0.576    0.565
    intMotiv          0.253    0.051    4.992    0.000
    extMotiv         -0.652    0.314   -2.074    0.038
    agency.X.xtMtv    0.215    0.114    1.878    0.060

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         2.751    0.512    5.371    0.000
```

SEM output ...

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         0.339    0.025   13.784    0.000
```

- The lavaan summary method allows customization. If you want the R-square, for example

```
lavaan (0.5-23.1097) converged normally after  22 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic                0.000
  Degrees of freedom                                 0
  Minimum Function Value               0.0000000000000

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
```

SEM output ...

```
  posAffect ~
    agency              -0.110    0.192   -0.576    0.565
    intMotiv             0.253    0.051    4.992    0.000
    extMotiv            -0.652    0.314   -2.074    0.038
    agency.X.xtMtv       0.215    0.114    1.878    0.060
   Std.lv  Std.all

   -0.110   -0.088
    0.253    0.264
   -0.652   -0.492
    0.215    0.571

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         2.751    0.512    5.371    0.000
   Std.lv  Std.all
    2.751    4.361

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         0.339    0.025   13.784    0.000
   Std.lv  Std.all
    0.339    0.851

R-Square:
                   Estimate
```

SEM output ...

```
    posAffect          0.149
```

- I don't know anything about this, but I thought `parTable()` and `parameterEstimates()` were super informative

```
options(width=60)
parameterEstimates(sem1)
```

```
             lhs op              rhs    est    se
1      posAffect  ~           agency -0.110 0.192
2      posAffect  ~         intMotiv  0.253 0.051
3      posAffect  ~         extMotiv -0.652 0.314
4      posAffect  ~ agency.X.extMotiv  0.215 0.114
5      posAffect ~~        posAffect  0.339 0.025
6         agency ~~           agency  0.250 0.000
7         agency ~~         intMotiv  0.140 0.000
8         agency ~~         extMotiv  0.058 0.000
9         agency ~~ agency.X.extMotiv  0.548 0.000
10      intMotiv ~~         intMotiv  0.435 0.000
11      intMotiv ~~         extMotiv -0.012 0.000
12      intMotiv ~~ agency.X.extMotiv  0.184 0.000
```

SEM output ...

```
13            extMotiv ~~            extMotiv  0.226 0.000
14            extMotiv ~~ agency.X.extMotiv  0.698 0.000
15 agency.X.extMotiv ~~ agency.X.extMotiv  2.800 0.000
16           posAffect ~1                    2.751 0.512
17              agency ~1                    2.512 0.000
18            intMotiv ~1                    3.023 0.000
19            extMotiv ~1                    1.645 0.000
20 agency.X.extMotiv ~1                    4.190 0.000
          z pvalue ci.lower ci.upper
1   -0.576  0.565   -0.486    0.265
2    4.992  0.000    0.153    0.352
3   -2.074  0.038   -1.268   -0.036
4    1.878  0.060   -0.009    0.439
5   13.784  0.000    0.290    0.387
6       NA     NA    0.250    0.250
7       NA     NA    0.140    0.140
8       NA     NA    0.058    0.058
9       NA     NA    0.548    0.548
10      NA     NA    0.435    0.435
11      NA     NA   -0.012   -0.012
12      NA     NA    0.184    0.184
13      NA     NA    0.226    0.226
14      NA     NA    0.698    0.698
15      NA     NA    2.800    2.800
16   5.371  0.000    1.747    3.754
17      NA     NA    2.512    2.512
```

SEM output ...

```
18       NA       NA     3.023     3.023
19       NA       NA     1.645     1.645
20       NA       NA     4.190     4.190
```

```
options(options.orig)
```

I want a nice output table

```
library(kutils)
semtable10 <- semTable(sem1, fits = c("cfi",
    "rmsea"), longtable=TRUE, type = "latex",
    print.results = FALSE)
cat(semtable10)
```

	Model			
	Estimate	Std. Err.	z	p
	Regression Slopes			
posAffect				
agency	-0.11	0.19	-0.58	.565
intMotiv	0.25	0.05	4.99	.000
extMotiv	-0.65	0.31	-2.07	.038
agency.X.extMotiv	0.22	0.11	1.88	.060
	Intercepts			
posAffect	2.75	0.51	5.37	.000
agency	2.51^{+}			
intMotiv	3.02^{+}			
extMotiv	1.64^{+}			
agency.X.extMotiv	4.19^{+}			
	Residual Variances			
posAffect	0.34	0.02	13.78	.000
agency	0.25^{+}			
intMotiv	0.44^{+}			
extMotiv	0.23^{+}			
agency.X.extMotiv	2.80^{+}			

		Residual Covariances
agency w/intMotiv	0.14^{+}	
agency w/extMotiv	0.06^{+}	
agency w/agency.X.extMotiv	0.55^{+}	
intMotiv w/extMotiv	-0.01^{+}	
intMotiv w/agency.X.extMotiv	0.18^{+}	
extMotiv w/agency.X.extMotiv	0.70^{+}	
		Fit Indices
CFI	1.00	
RMSEA	0.00	

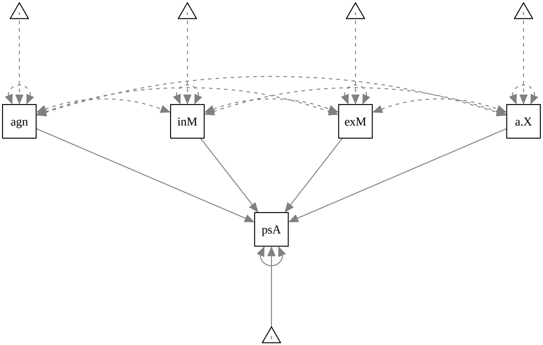
$^{+}$Fixed parameter

Plots

- There is no plot method for a lavaan object.
- But there are other packages devoted to creating graphics for models that involve path diagrams (e.g., SEM and neural networks).
- The path diagram is visualized here with the `semPaths()` function in semPlot.

```
library(semPlot)
semPaths(sem1)
```

Plots ...



Other follow-up functions

- R-style accessor functions for lavaan

```
coef(sem1)
fitted(sem1)
resid(sem1)
anova(sem1)
```

- If you run those things, you will notice some wrinkles.
  - Notice that predicted observations (and residuals) are not 1-per-person
  - `anova()` returns a chi-squared test indicates there are 0 degrees of freedom (that's an SEM concept).

2 fits

- Moderation analysis focuses on the differences between groups. In this case, males and females.
- When one of the predictors in an interaction is categorical (e.g., gender), the sem "measurement invariance" approach would lead us to compare
  - a model in which the coefficients for the two groups are assumed to be entirely different, against
  - a simpler model in which some coefficients might be the same
- Estimate separately

```
group.mod.1 <- 'posAffect ~ c(A, B)*agency'
group.out.1 <- sem(group.mod.1, data = dat,
    meanstructure = TRUE, group = "gender")
summary(group.out.1)
```

2 fits ...

```
lavaan (0.5-23.1097) converged normally after  19 iterations

  Number of observations per group
  male                                          195
  female                                        185

  Estimator                                      ML
  Minimum Function Test Statistic             0.000
  Degrees of freedom                              0
  Minimum Function Value          0.0000000000000

Chi-square for each group:

  male                                        0.000
  female                                      0.000

Parameter Estimates:

  Information                              Expected
  Standard Errors                          Standard


Group 1 [male]:

Regressions:
```

2 fits ...

```
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    agency      (A)   0.452    0.079    5.704    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         1.890    0.203    9.329    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         0.309    0.031    9.874    0.000


Group 2 [female]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    agency      (B)   0.237    0.096    2.477    0.013

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         2.514    0.245   10.241    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
```

2 fits ...

```
    .posAffect           0.419     0.044     9.618     0.000
```

- Now fit same, with the exception that agency coefficient is same in both.

```
group.mod.0 <- 'posAffect ~ c(B, B)*agency'
group.out.0 <- sem(group.mod.0, data = dat,
    meanstructure = TRUE, group = "gender")
summary(group.out.0)
```

```
lavaan (0.5-23.1097) converged normally after  16 iterations

  Number of observations per group
  male                                              195
  female                                            185

  Estimator                                          ML
  Minimum Function Test Statistic                 2.978
  Degrees of freedom                                  1
  P-value (Chi-square)                            0.084

Chi-square for each group:
```

2 fits ...

```
  male                                              1.207
  female                                            1.771

Parameter Estimates:

  Information                                    Expected
  Standard Errors                                Standard


Group 1 [male]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    agency      (B)   0.365    0.061    5.954    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         2.108    0.159   13.285    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         0.311    0.032    9.874    0.000
```

2 fits ...

```
Group 2 [female]:

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    agency    (B)     0.365    0.061    5.954    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         2.192    0.161   13.587    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         0.423    0.044    9.618    0.000
```

- The fit of the more complex model is similar to the less complex model, so there is no evidence of an interaction.

```
anova(group.out.0, group.out.1)
```

2 fits ...

```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
group.out.1   0 1252.4 1276.0 0.0000
group.out.0   1 1253.4 1273.1 2.9781     2.9781       1     0.0844 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

2 fits ...

- The slopes "appear to differ" in the two models.

```
parameterEstimates(group.out.1, ci = FALSE)[ ,
    c(1,2, 3, 4, 5, 6, 7)]
```

```
          lhs op       rhs block group label   est
1   posAffect  ~    agency     1     1     A 0.452
2   posAffect ~~ posAffect     1     1       0.309
3      agency ~~    agency     1     1       0.253
4   posAffect ~1               1     1       1.890
5      agency ~1               1     1       2.508
6   posAffect  ~    agency     2     2     B 0.237
7   posAffect ~~ posAffect     2     2       0.419
8      agency ~~    agency     2     2       0.247
9   posAffect ~1               2     2       2.514
10     agency ~1               2     2       2.516
```

- The agency effect difference between male and female seems to be 0.452 - 0.237 = 0.215, but that's not "statistically significant"
- The effect of agency is .365 if the effects are assumed the same

I want a nice output table

```
library(kutils)
cfaTable10 <- compareLavaan(list("Restricted" =
    group.out.0, "Unrestricted" = group.out.1),
    fitmeas = c("chisq", "df", "rmsea", "cfi"),
    nesting = NULL, type = "latex", file =
    "output/cfaTable10.tex", print.results =
    FALSE)
cat(cfaTable10)
```

	χ^2	df	rmsea	cfi	$\Delta\chi^2$	Δdf	*p*
Unrestricted	0.00	0.00	0.00	1.00	-	-	-
Restricted	2.98	1.00	0.10	0.94	2.978a	1	0.084

a = Restricted vs Unrestricted

linear regression is equivalent

- If outcome is a single measure like posAffect, we can double-check the SEM result with a linear model.
- I'll assume the variances of the error terms are the same in the 2 groups (otherwise I'd have to go on a weighted least squares adventure)
- This allows both slopes and intercepts to differ among groups

```
modlm1 <- lm(posAffect ~ -1 + gender/agency, data
    = dat)
summary(modlm1, signif.stars = FALSE)
```

linear regression is equivalent ...

```
Call:
lm(formula = posAffect ~ -1 + gender/agency, data = dat)

Residuals:
     Min       1Q   Median       3Q      Max
-2.10608 -0.40401  0.02606  0.44460  1.32508

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
gendermale           1.88976    0.22056   8.568 2.75e-16 ***
genderfemale         2.51356    0.22955  10.950  < 2e-16 ***
gendermale:agency    0.45182    0.08624   5.239 2.69e-07 ***
genderfemale:agency  0.23701    0.08950   2.648  0.00843 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6054 on 376 degrees of freedom
Multiple R-squared:  0.963, Adjusted R-squared:  0.9626
F-statistic:  2444 on 4 and 376 DF,  p-value: < 2.2e-16
```

- Keep the same slope, but different intercepts

linear regression is equivalent ...

```
modlm2 <- lm(posAffect ~ -1 + gender + agency,
    data = dat)
summary(modlm2, signif.stars = FALSE)
```

```
Call:
lm(formula = posAffect ~ -1 + gender + agency, data = dat)

Residuals:
     Min       1Q   Median       3Q      Max
-2.10427 -0.39890  0.05395  0.44156  1.35513

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
gendermale    2.14912    0.16207  13.260  < 2e-16 ***
genderfemale  2.23329    0.16290  13.710  < 2e-16 ***
agency        0.34839    0.06226   5.596 4.24e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.607 on 377 degrees of freedom
Multiple R-squared:  0.9627,  Adjusted R-squared:  0.9624
F-statistic:  3241 on 3 and 377 DF,  p-value: < 2.2e-16
```

linear regression is equivalent ...

- Test the difference between the models

```
anova(modlm1, modlm2)
```

```
Analysis of Variance Table

Model 1: posAffect ~ -1 + gender/agency
Model 2: posAffect ~ -1 + gender + agency
  Res.Df    RSS Df Sum of Sq      F  Pr(>F)
1    376 137.82
2    377 138.92 -1    -1.095 2.9873 0.08474 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

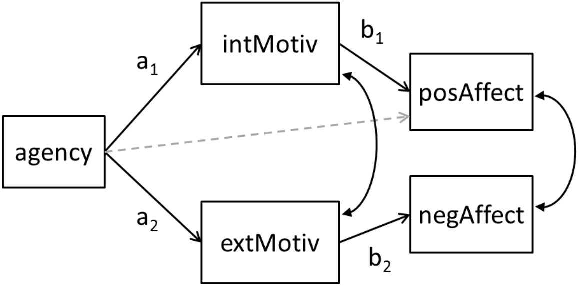
- Note anova defaults to a χ^2 test, and the p-value is identical to SEM output

Use SEM to represent several endogenous variables

- Suppose you had a more complex theory about the association among variables:
  - Positive Affect is affected by Intrinsic Motivation
  - Negative Affect is affected by Extrinsic Motivation
  - Intrinsic and Extrinsic Motivation are both affected by sense of Agency

Path Analysis: Mediation

We are still using the “scale” variables we created as averages



lavaan implementation of mediation model

- We'll need to get a psychologist to explain why the positive and negative affects are modeled separately in this way

```
mediat.mod.0 <- '
 posAffect ~ intMotiv
 negAffect ~ extMotiv
 intMotiv + extMotiv ~ agency
 intMotiv ~~ extMotiv
 '
mediat.out.0 <- sem(mediat.mod.0, data = dat)
summary(mediat.out.0, rsquare = TRUE, fit = TRUE)
```

lavaan implementation of mediation model ...

```
lavaan (0.5-23.1097) converged normally after  23 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic               11.604
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.021

Model test baseline model:

  Minimum Function Test Statistic              182.877
  Degrees of freedom                                10
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.956
  Tucker-Lewis Index (TLI)                       0.890

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1555.935
  Loglikelihood unrestricted model (H1)      -1550.133

```

lavaan implementation of mediation model ...

```
  Number of free parameters                          10
  Akaike (AIC)                                 3131.870
  Bayesian (BIC)                               3171.271
  Sample-size adjusted Bayesian (BIC)          3139.543

Root Mean Square Error of Approximation:

  RMSEA                                           0.071
  90 Percent Confidence Interval          0.025   0.120
  P-value RMSEA <= 0.05                           0.192

Standardized Root Mean Square Residual:

  SRMR                                            0.036

Parameter Estimates:

  Information                                  Expected
  Standard Errors                              Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    intMotiv          0.329    0.046    7.153    0.000
  negAffect ~
    extMotiv          0.245    0.067    3.683    0.000
```

lavaan implementation of mediation model ...

```
  intMotiv ~
    agency           0.558    0.061    9.102    0.000
  extMotiv ~
    agency           0.233    0.047    4.938    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .intMotiv ~~
   .extMotiv        -0.045    0.014   -3.130    0.002
 .posAffect ~~
   .negAffect       -0.026    0.019   -1.401    0.161

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect        0.351    0.025   13.784    0.000
   .negAffect        0.383    0.028   13.784    0.000
   .intMotiv         0.357    0.026   13.784    0.000
   .extMotiv         0.212    0.015   13.784    0.000

R-Square:
                   Estimate
    posAffect        0.118
    negAffect        0.034
    intMotiv         0.179
    extMotiv         0.060
```

lavaan implementation of mediation model ...

```
mediat.mod.0 <- '
 posAffect ~ intMotiv
 negAffect ~ extMotiv
 intMotiv + extMotiv ~ agency
 intMotiv ~~ extMotiv
 '
mediat.out.0 <- sem(mediat.mod.0, data = dat)
summary(mediat.out.0, rsquare = TRUE, fit = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  23 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic               11.604
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.021

Model test baseline model:
```

lavaan implementation of mediation model ...

```
  Minimum Function Test Statistic             182.877
  Degrees of freedom                               10
  P-value                                       0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                   0.956
  Tucker-Lewis Index (TLI)                      0.890

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -1555.935
  Loglikelihood unrestricted model (H1)     -1550.133

  Number of free parameters                        10
  Akaike (AIC)                               3131.870
  Bayesian (BIC)                             3171.271
  Sample-size adjusted Bayesian (BIC)        3139.543

Root Mean Square Error of Approximation:

  RMSEA                                         0.071
  90 Percent Confidence Interval          0.025  0.120
  P-value RMSEA <= 0.05                         0.192

Standardized Root Mean Square Residual:
```

lavaan implementation of mediation model ...

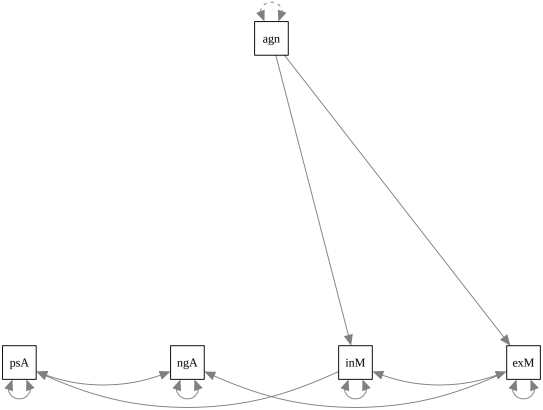
```
  SRMR                                           0.036

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    intMotiv          0.329    0.046    7.153    0.000
  negAffect ~
    extMotiv          0.245    0.067    3.683    0.000
  intMotiv ~
    agency            0.558    0.061    9.102    0.000
  extMotiv ~
    agency            0.233    0.047    4.938    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .intMotiv ~~
   .extMotiv         -0.045    0.014   -3.130    0.002
 .posAffect ~~
   .negAffect        -0.026    0.019   -1.401    0.161
```

lavaan implementation of mediation model ...

```
Variances:
                    Estimate  Std.Err  z-value  P(>|z|)
   .posAffect          0.351    0.025   13.784    0.000
   .negAffect          0.383    0.028   13.784    0.000
   .intMotiv           0.357    0.026   13.784    0.000
   .extMotiv           0.212    0.015   13.784    0.000

R-Square:
                    Estimate
    posAffect          0.118
    negAffect          0.034
    intMotiv           0.179
    extMotiv           0.060
```

Trouble with semPaths

```
semPaths(mediat.out.0)
```



Should Agency have a direct effect on positive affect?

- The regression paths are significant, but so is our model fit statistic.
  - in SEM, a significant fit statistic is a "bad thing", it means our model is not doing well as the "saturated" model.
- Perhaps a sense of Agency still has a direct effect on Positive Affect. Note we insert "+ agency" in the posAffect equation.

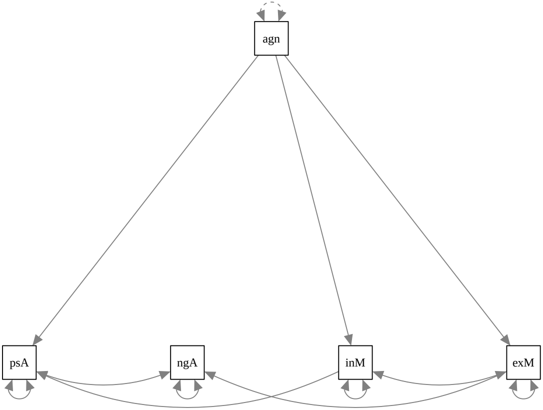
```
mediat.mod.1 <- '
 posAffect ~ intMotiv + agency
 negAffect ~ extMotiv
 intMotiv + extMotiv ~ agency
 intMotiv ~~ extMotiv
 '
mediat.out.1 <- sem(mediat.mod.1, data = dat)
```

Should Agency have a direct effect on positive affect? ...

```
mediat.mod.1 <- '
 posAffect ~ intMotiv + agency
 negAffect ~ extMotiv
 intMotiv + extMotiv ~ agency
 intMotiv ~~ extMotiv
 '
mediat.out.1 <- sem(mediat.mod.1, data = dat)
```

Visualize Whirrled Peas II

```
semPaths(mediat.out.1)
```



Compare the fit of the models with and without the direct effect

- Are we wise to include the direct effect?
- It appears there is a statistically significant difference between the models in the anova() output.

```
anova(mediat.out.0, mediat.out.1)
```

```
Chi Square Difference Test

             Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
mediat.out.1  3 3124.9 3168.3  2.6647
mediat.out.0  4 3131.9 3171.3 11.6040     8.9393       1   0.002791 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(mediat.out.0, mediat.out.1)
```

Compare the fit of the models with and without the direct effect ...

```
Chi Square Difference Test

               Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
mediat.out.1  3 3124.9 3168.3  2.6647
mediat.out.0  4 3131.9 3171.3 11.6040     8.9393       1   0.002791 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The small p value indicates there is probably a difference

Review the summary of the output

- The fit improvement may be reflected in the R^2 for Positive Affect (increased by 2%).

```
summary(mediat.out.1, rsquare = TRUE, fit = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  26 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic                2.665
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.446

Model test baseline model:

  Minimum Function Test Statistic              182.877
  Degrees of freedom                                10
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
```

Review the summary of the output ...

```
  Tucker-Lewis Index (TLI)                       1.006

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1551.465
  Loglikelihood unrestricted model (H1)      -1550.133

  Number of free parameters                         11
  Akaike (AIC)                                3124.930
  Bayesian (BIC)                              3168.272
  Sample-size adjusted Bayesian (BIC)         3133.371

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.083
  P-value RMSEA <= 0.05                          0.758

Standardized Root Mean Square Residual:

  SRMR                                           0.017

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

Review the summary of the output ...

```

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    intMotiv          0.265    0.050    5.280    0.000
    agency            0.199    0.066    3.011    0.003
  negAffect ~
    extMotiv          0.242    0.067    3.624    0.000
  intMotiv ~
    agency            0.558    0.061    9.102    0.000
  extMotiv ~
    agency            0.233    0.047    4.938    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .intMotiv ~~
   .extMotiv         -0.045    0.014   -3.130    0.002
 .posAffect ~~
   .negAffect        -0.024    0.019   -1.277    0.202

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .posAffect         0.343    0.025   13.784    0.000
   .negAffect         0.383    0.028   13.784    0.000
   .intMotiv          0.357    0.026   13.784    0.000
   .extMotiv          0.212    0.015   13.784    0.000
```

Review the summary of the output ...

```
R-Square:
                   Estimate
    posAffect         0.139
    negAffect         0.033
    intMotiv          0.179
    extMotiv          0.060
```

```
summary(mediat.out.1, rsquare = TRUE, fit = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  26 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic                2.665
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.446

Model test baseline model:

  Minimum Function Test Statistic              182.877
  Degrees of freedom                                10
  P-value                                        0.000
```

Review the summary of the output ...

```

User model versus baseline model:

  Comparative Fit Index (CFI)                   1.000
  Tucker-Lewis Index (TLI)                      1.006

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -1551.465
  Loglikelihood unrestricted model (H1)     -1550.133

  Number of free parameters                        11
  Akaike (AIC)                               3124.930
  Bayesian (BIC)                             3168.272
  Sample-size adjusted Bayesian (BIC)        3133.371

Root Mean Square Error of Approximation:

  RMSEA                                         0.000
  90 Percent Confidence Interval          0.000  0.083
  P-value RMSEA <= 0.05                         0.758

Standardized Root Mean Square Residual:

  SRMR                                          0.017

```

Review the summary of the output ...

```
Parameter Estimates:

  Information                                     Expected
  Standard Errors                                 Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  posAffect ~
    intMotiv          0.265    0.050    5.280    0.000
    agency            0.199    0.066    3.011    0.003
  negAffect ~
    extMotiv          0.242    0.067    3.624    0.000
  intMotiv ~
    agency            0.558    0.061    9.102    0.000
  extMotiv ~
    agency            0.233    0.047    4.938    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .intMotiv ~~
   .extMotiv         -0.045    0.014   -3.130    0.002
 .posAffect ~~
   .negAffect        -0.024    0.019   -1.277    0.202

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
```

Review the summary of the output ...

```
   .posAffect          0.343    0.025   13.784    0.000
   .negAffect          0.383    0.028   13.784    0.000
   .intMotiv           0.357    0.026   13.784    0.000
   .extMotiv           0.212    0.015   13.784    0.000

R-Square:
                    Estimate
    posAffect          0.139
    negAffect          0.033
    intMotiv           0.179
    extMotiv           0.060
```

Hypothesis test for the indirect effect

- For testing the significance of indirect paths in mediation models, the major advantage of SEM over OLS regression is that all parameters can be estimated in a single model.
- Most SEM software packages include bootstrapping options, which is the preferred method for testing indirect paths because products of parameters are not normally distributed.

```
Nboot <- 500
mediat.mod.1 <- '
 posAffect ~ b1*intMotiv + agency
 negAffect ~ b2*extMotiv
 intMotiv ~ a1*agency
 extMotiv ~ a2*agency
 intMotiv ~~ extMotiv
 ## define mediation parameters (indirect effects)
 ind1 := a1 * b1
 ind2 := a2 * b2
```

Hypothesis test for the indirect effect ...

```
 '
mediat.out.1 <- sem(mediat.mod.1, data = dat,
                    se = "boot", bootstrap =
                       Nboot)
```

```
Nboot <- 500
mediat.mod.1 <- '
 posAffect ~ b1*intMotiv + agency
 negAffect ~ b2*extMotiv
 intMotiv ~ a1*agency
 extMotiv ~ a2*agency
 intMotiv ~~ extMotiv
 ## define mediation parameters (indirect effects)
 ind1 := a1 * b1
 ind2 := a2 * b2
 '
mediat.out.1 <- sem(mediat.mod.1, data = dat,
```

Hypothesis test for the indirect effect ...

```
                    se = "boot", bootstrap =
                       Nboot)
```

- This takes a couple of minutes. We can adjust the “Nboot” parameter for testing. In analysis for a report, Nboot should be a large value, such as 1000. Here, the bootstrap estimates are requested, with 500 iterations

```
parameterEstimates(mediat.out.1, boot.ci.type =
   "bca.simple")
```

Hypothesis test for the indirect effect ...

```
         lhs op      rhs label    est    se      z pvalue ci.lower
    ci.upper
1   posAffect ~  intMotiv    b1  0.265 0.051  5.219  0.000    0.150
   0.360
2   posAffect ~    agency        0.199 0.073  2.738  0.006    0.059
   0.338
3   negAffect ~  extMotiv    b2  0.242 0.079  3.060  0.002    0.097
   0.397
4    intMotiv ~    agency    a1  0.558 0.067  8.277  0.000    0.443
   0.713
5    extMotiv ~    agency    a2  0.233 0.051  4.534  0.000    0.139
   0.334
6    intMotiv ~~  extMotiv      -0.045 0.013 -3.523  0.000   -0.071
   -0.019
7   posAffect ~~ posAffect       0.343 0.026 13.128  0.000    0.295
   0.395
8   negAffect ~~ negAffect       0.383 0.039  9.722  0.000    0.313
   0.464
9    intMotiv ~~  intMotiv       0.357 0.024 14.781  0.000    0.310
   0.404
10   extMotiv ~~  extMotiv       0.212 0.018 11.854  0.000    0.181
   0.249
11 posAffect ~~ negAffect      -0.024 0.022 -1.091  0.275   -0.068
   0.019
```

Hypothesis test for the indirect effect ...

```
12     agency ~~     agency          0.250  0.000      NA      NA     0.250
     0.250
13       ind1 :=      a1*b1   ind1   0.148  0.029   5.062   0.000     0.097
     0.216
14       ind2 :=      a2*b2   ind2   0.056  0.023   2.493   0.013     0.021
     0.110
```

```
parameterEstimates(mediat.out.1, boot.ci.type =
    "bca.simple")
```

```
         lhs op       rhs label    est    se       z pvalue ci.lower
             ci.upper
1  posAffect  ~  intMotiv    b1  0.265 0.051   5.219  0.000    0.150
     0.360
2  posAffect  ~    agency        0.199 0.073   2.738  0.006    0.059
     0.338
3  negAffect  ~  extMotiv    b2  0.242 0.079   3.060  0.002    0.097
     0.397
4   intMotiv  ~    agency    a1  0.558 0.067   8.277  0.000    0.443
     0.713
5   extMotiv  ~    agency    a2  0.233 0.051   4.534  0.000    0.139
     0.334
```

Hypothesis test for the indirect effect ...

```
6   intMotiv ~~  extMotiv        -0.045 0.013 -3.523  0.000   -0.071
     -0.019
7  posAffect ~~ posAffect         0.343 0.026 13.128  0.000    0.295
     0.395
8  negAffect ~~ negAffect         0.383 0.039  9.722  0.000    0.313
     0.464
9   intMotiv ~~  intMotiv         0.357 0.024 14.781  0.000    0.310
     0.404
10  extMotiv ~~  extMotiv         0.212 0.018 11.854  0.000    0.181
     0.249
11 posAffect ~~ negAffect        -0.024 0.022 -1.091  0.275   -0.068
     0.019
12    agency ~~    agency         0.250 0.000     NA     NA    0.250
     0.250
13      ind1 :=     a1*b1  ind1  0.148 0.029  5.062  0.000    0.097
     0.216
14      ind2 :=     a2*b2  ind2  0.056 0.023  2.493  0.013    0.021
     0.110
```

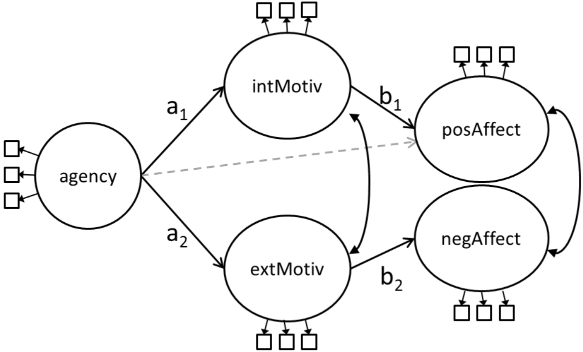
Outline

Regression relationships among latent variables

- This is the SEM in its full glory.
- Assume
  1. there are "latent variables" (unmeasured personal traits)
  2. assume that there is a multivariate normal relationship among those traits
  3. the observed scores are a reflection of each individual's latent variables.
- Instead of blinding you with mathematical wizardry, I better show a picture.

The Big Picture

The circles are unmeasured variables.



- The SEM integrates
  - "measurement error" analysis with
  - analysis of the underlying relationships.

The Big Picture ...

- Regression models that we used before assume predictors are measured without error.
  - If there is random error in predictors, slope coefficients are “attenuated”– underestimated.
  - Hence, we miss relationships because of measurement error.
  - However, to be honest, the mean-scale score regressions we have discussed so far are quite good in comparison to the usual regression in the social science literature, where just a single indicator is typically available for each construct.

Quick Jargon review

Indicators: The observed scale items

Latent variables: (aka "factors", "latent constructs" or "common factors"): unobserved variables thought to be the things we are truly interested in. We'd really like to study the relationship among them, but we are unable to do so.

factor loadings: the coefficients which indicate how tightly an indicator is linked to the latent variable.

Confirmatory Factor Analysis

- We are focused on the measurement properties
- We estimate loadings for all of the indicators.
- We don't worry about the relationship among the latent variables yet.
  - But we don't ignore them.
  - The opposite is true. CFA models freely estimate all correlations among latent variables, without imposing any further constraints on their relationships.
- These are also called "measurement models" because they provide details about how strongly each item is related to the latent construct.

Confirmatory Factor Analysis

- The lavaan model has the style
  latent_construct =˜ indicator1 + indicator2 + indicator3
- The function "cfa()" is used.
  - The only argument worth mentioning is "std.lv = TRUE".
  - The underlying unmeasured constructs are to be scaled so that the variance of each one is equal to 1.0. (standardized)

```
cfa.mod <- '
 ## factor loadings
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 '
cfa.out <- cfa(cfa.mod, data = dat, std.lv = TRUE)
```

Confirmatory Factor Analysis ...

```
cfa.mod <- '
 ## factor loadings
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 '
cfa.out <- cfa(cfa.mod, data = dat, std.lv = TRUE)
```

- So far, it does not seem dangerous. Wait till the output vectors start flying on the next slide.

CFA Output

```
summary(cfa.out, standardized = TRUE, fit = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  59 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              106.847
  Degrees of freedom                                80
  P-value (Chi-square)                           0.024

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.990

Loglikelihood and Information Criteria:

```

CFA Output ...

```
  Loglikelihood user model (H0)              -3699.110
  Loglikelihood unrestricted model (H1)      -3645.686

  Number of free parameters                         40
  Akaike (AIC)                                7478.219
  Bayesian (BIC)                              7635.826
  Sample-size adjusted Bayesian (BIC)         7508.914

Root Mean Square Error of Approximation:

  RMSEA                                          0.030
  90 Percent Confidence Interval          0.011  0.044
  P-value RMSEA <= 0.05                          0.994

Standardized Root Mean Square Residual:

  SRMR                                           0.031

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
```

CFA Output ...

```
    Agency1          0.466    0.021   22.560    0.000    0.466    0.905
    Agency2          0.492    0.021   23.774    0.000    0.492    0.934
    Agency3          0.497    0.022   22.815    0.000    0.497    0.911
  Intrinsic =~
    Intrin1          0.541    0.039   13.960    0.000    0.541    0.704
    Intrin2          0.581    0.043   13.489    0.000    0.581    0.683
    Intrin3          0.615    0.038   16.201    0.000    0.615    0.799
  Extrinsic =~
    Extrin1          0.388    0.022   17.606    0.000    0.388    0.808
    Extrin2          0.456    0.027   17.168    0.000    0.456    0.792
    Extrin3          0.471    0.026   18.100    0.000    0.471    0.826
  Positive =~
    PosAFF1          0.569    0.028   20.132    0.000    0.569    0.853
    PosAFF2          0.603    0.029   21.150    0.000    0.603    0.882
    PosAFF3          0.632    0.029   21.648    0.000    0.632    0.895
  Negative =~
    NegAFF1          0.634    0.029   21.670    0.000    0.634    0.889
    NegAFF2          0.585    0.027   21.457    0.000    0.585    0.883
    NegAFF3          0.598    0.026   22.805    0.000    0.598    0.918

Covariances:
                  Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency ~~
    Intrinsic        0.507    0.047   10.782    0.000    0.507    0.507
    Extrinsic        0.270    0.054    5.028    0.000    0.270    0.270
    Positive         0.302    0.051    5.950    0.000    0.302    0.302
```

CFA Output ...

```
     Negative          0.017    0.055    0.312    0.755    0.017    0.017
   Intrinsic ~~
     Extrinsic        -0.028    0.063   -0.441    0.659   -0.028   -0.028
     Positive          0.414    0.052    7.918    0.000    0.414    0.414
     Negative          0.021    0.060    0.341    0.733    0.021    0.021
   Extrinsic ~~
     Positive         -0.029    0.058   -0.503    0.615   -0.029   -0.029
     Negative          0.209    0.056    3.764    0.000    0.209    0.209
   Positive ~~
     Negative         -0.071    0.056   -1.274    0.203   -0.071   -0.071

Variances:
                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    .Agency1           0.048    0.005    9.649    0.000    0.048    0.181
    .Agency2           0.036    0.005    7.604    0.000    0.036    0.129
    .Agency3           0.051    0.005    9.278    0.000    0.051    0.170
    .Intrin1           0.298    0.029   10.204    0.000    0.298    0.505
    .Intrin2           0.386    0.036   10.620    0.000    0.386    0.533
    .Intrin3           0.214    0.029    7.474    0.000    0.214    0.361
    .Extrin1           0.080    0.009    8.924    0.000    0.080    0.347
    .Extrin2           0.123    0.013    9.465    0.000    0.123    0.372
    .Extrin3           0.103    0.012    8.264    0.000    0.103    0.318
    .PosAFF1           0.121    0.012    9.921    0.000    0.121    0.272
    .PosAFF2           0.104    0.012    8.615    0.000    0.104    0.222
    .PosAFF3           0.099    0.013    7.870    0.000    0.099    0.198
    .NegAFF1           0.107    0.012    9.202    0.000    0.107    0.210
```

CFA Output ...

```
 .NegAFF2          0.096    0.010    9.498    0.000    0.096    0.220
 .NegAFF3          0.067    0.009    7.369    0.000    0.067    0.158
  Agency           1.000                               1.000    1.000
  Intrinsic        1.000                               1.000    1.000
  Extrinsic        1.000                               1.000    1.000
  Positive         1.000                               1.000    1.000
  Negative         1.000                               1.000    1.000
```

```
summary(cfa.out, standardized = TRUE, fit = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  59 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              106.847
  Degrees of freedom                                80
  P-value (Chi-square)                           0.024

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000
```

CFA Output ...

```
User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.990

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3699.110
  Loglikelihood unrestricted model (H1)      -3645.686

  Number of free parameters                         40
  Akaike (AIC)                                7478.219
  Bayesian (BIC)                              7635.826
  Sample-size adjusted Bayesian (BIC)         7508.914

Root Mean Square Error of Approximation:

  RMSEA                                          0.030
  90 Percent Confidence Interval          0.011  0.044
  P-value RMSEA <= 0.05                          0.994

Standardized Root Mean Square Residual:

  SRMR                                           0.031
```

CFA Output ...

```
Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.466    0.021   22.560    0.000    0.466    0.905
    Agency2           0.492    0.021   23.774    0.000    0.492    0.934
    Agency3           0.497    0.022   22.815    0.000    0.497    0.911
  Intrinsic =~
    Intrin1           0.541    0.039   13.960    0.000    0.541    0.704
    Intrin2           0.581    0.043   13.489    0.000    0.581    0.683
    Intrin3           0.615    0.038   16.201    0.000    0.615    0.799
  Extrinsic =~
    Extrin1           0.388    0.022   17.606    0.000    0.388    0.808
    Extrin2           0.456    0.027   17.168    0.000    0.456    0.792
    Extrin3           0.471    0.026   18.100    0.000    0.471    0.826
  Positive =~
    PosAFF1           0.569    0.028   20.132    0.000    0.569    0.853
    PosAFF2           0.603    0.029   21.150    0.000    0.603    0.882
    PosAFF3           0.632    0.029   21.648    0.000    0.632    0.895
  Negative =~
    NegAFF1           0.634    0.029   21.670    0.000    0.634    0.889
    NegAFF2           0.585    0.027   21.457    0.000    0.585    0.883
```

CFA Output ...

```
    NegAFF3          0.598    0.026   22.805    0.000    0.598    0.918

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency ~~
    Intrinsic        0.507    0.047   10.782    0.000    0.507    0.507
    Extrinsic        0.270    0.054    5.028    0.000    0.270    0.270
    Positive         0.302    0.051    5.950    0.000    0.302    0.302
    Negative         0.017    0.055    0.312    0.755    0.017    0.017
  Intrinsic ~~
    Extrinsic       -0.028    0.063   -0.441    0.659   -0.028   -0.028
    Positive         0.414    0.052    7.918    0.000    0.414    0.414
    Negative         0.021    0.060    0.341    0.733    0.021    0.021
  Extrinsic ~~
    Positive        -0.029    0.058   -0.503    0.615   -0.029   -0.029
    Negative         0.209    0.056    3.764    0.000    0.209    0.209
  Positive ~~
    Negative        -0.071    0.056   -1.274    0.203   -0.071   -0.071

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1          0.048    0.005    9.649    0.000    0.048    0.181
   .Agency2          0.036    0.005    7.604    0.000    0.036    0.129
   .Agency3          0.051    0.005    9.278    0.000    0.051    0.170
   .Intrin1          0.298    0.029   10.204    0.000    0.298    0.505
   .Intrin2          0.386    0.036   10.620    0.000    0.386    0.533
```

CFA Output ...

.Intrin3	0.214	0.029	7.474	0.000	0.214	0.361
.Extrin1	0.080	0.009	8.924	0.000	0.080	0.347
.Extrin2	0.123	0.013	9.465	0.000	0.123	0.372
.Extrin3	0.103	0.012	8.264	0.000	0.103	0.318
.PosAFF1	0.121	0.012	9.921	0.000	0.121	0.272
.PosAFF2	0.104	0.012	8.615	0.000	0.104	0.222
.PosAFF3	0.099	0.013	7.870	0.000	0.099	0.198
.NegAFF1	0.107	0.012	9.202	0.000	0.107	0.210
.NegAFF2	0.096	0.010	9.498	0.000	0.096	0.220
.NegAFF3	0.067	0.009	7.369	0.000	0.067	0.158
Agency	1.000				1.000	1.000
Intrinsic	1.000				1.000	1.000
Extrinsic	1.000				1.000	1.000
Positive	1.000				1.000	1.000
Negative	1.000				1.000	1.000

CFA Commentary

- The model appears to fit well,
  - all factor loadings are significant, and the
  - standardized factor loadings indicate strong correlations between indicators and constructs.
- However, we did not take into account the relationships among the latent variables.
  - We don't have "Agency" as predictor of "Positive" affect.

Structural Regressions among Latent Constructs

- Now that we have a strong measurement model, we can specify a structure among the latent variables (instead of free correlations).
- Because the CFA was freely allowed to estimate correlations among the latent variables, this step can be viewed as a "restriction" of allowed paths.

```
mediat.mod.2 <- '
 ## measurement model declares the latent
    constructs
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 ## structural model represents relationships
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
```

Structural Regressions among Latent Constructs ...

```
 Intrinsic ~ a1*Agency
 Extrinsic ~ a2*Agency
 Intrinsic ~~ Extrinsic
 ## define mediation parameters (indirect effects)
 ind1 := a1 * b1
 ind2 := a2 * b2
 '
```

```
mediat.mod.2 <- '
 ## measurement model declares the latent constructs
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 ## structural model represents relationships
```

Structural Regressions among Latent Constructs ...

```
Positive ~ b1*Intrinsic + Agency
Negative ~ b2*Extrinsic
Intrinsic ~ a1*Agency
Extrinsic ~ a2*Agency
Intrinsic ~~ Extrinsic
## define mediation parameters (indirect effects)
ind1 := a1 * b1
ind2 := a2 * b2
'
```

```
mediat.out.2 <- sem(mediat.mod.2, data = dat,
                    std.lv = TRUE, se = "boot",
                       bootstrap = Nboot)
summary(mediat.out.2, standardized = TRUE, fit =
   TRUE)
```

Structural Regressions among Latent Constructs ...

```
lavaan (0.5-23.1097) converged normally after  62 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.991

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3700.262
  Loglikelihood unrestricted model (H1)      -3645.686

```

Structural Regressions among Latent Constructs ...

```
  Number of free parameters                          37
  Akaike (AIC)                                 7474.524
  Bayesian (BIC)                               7620.310
  Sample-size adjusted Bayesian (BIC)          7502.917

Root Mean Square Error of Approximation:

  RMSEA                                           0.029
  90 Percent Confidence Interval          0.010  0.043
  P-value RMSEA <= 0.05                           0.996

Standardized Root Mean Square Residual:

  SRMR                                            0.034

Parameter Estimates:

  Information                                  Observed
  Standard Errors                             Bootstrap
  Number of requested bootstrap draws               500
  Number of successful bootstrap draws              500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.466    0.021   22.302    0.000    0.466    0.905
```

Structural Regressions among Latent Constructs ...

```
    Agency2            0.492    0.021   23.227    0.000    0.492    0.933
    Agency3            0.497    0.024   20.574    0.000    0.497    0.912
  Intrinsic =~
    Intrin1            0.466    0.035   13.489    0.000    0.540    0.703
    Intrin2            0.500    0.037   13.502    0.000    0.580    0.682
    Intrin3            0.531    0.032   16.397    0.000    0.616    0.800
  Extrinsic =~
    Extrin1            0.373    0.022   16.779    0.000    0.388    0.808
    Extrin2            0.440    0.028   15.945    0.000    0.457    0.793
    Extrin3            0.454    0.029   15.466    0.000    0.471    0.827
  Positive =~
    PosAFF1            0.514    0.025   20.161    0.000    0.569    0.854
    PosAFF2            0.545    0.029   18.693    0.000    0.604    0.882
    PosAFF3            0.570    0.027   21.232    0.000    0.632    0.895
  Negative =~
    NegAFF1            0.620    0.038   16.174    0.000    0.633    0.889
    NegAFF2            0.573    0.042   13.579    0.000    0.585    0.883
    NegAFF3            0.586    0.035   16.865    0.000    0.598    0.918

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)     0.343    0.075    4.557    0.000    0.359    0.359
    Agency             0.129    0.071    1.824    0.068    0.116    0.116
  Negative ~
    Extrinsic (b2)     0.199    0.069    2.902    0.004    0.202    0.202
```

Structural Regressions among Latent Constructs ...

```
  Intrinsic ~
    Agency     (a1)    0.588    0.080    7.315    0.000    0.507    0.507
  Extrinsic ~
    Agency     (a2)    0.278    0.064    4.328    0.000    0.268    0.268

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.062   -3.235    0.001   -0.202   -0.202
 .Positive ~~
   .Negative         -0.076    0.070   -1.091    0.275   -0.076   -0.076

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.006    8.464    0.000    0.048    0.181
   .Agency2           0.036    0.005    7.928    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.976    0.000    0.050    0.169
   .Intrin1           0.299    0.032    9.299    0.000    0.299    0.506
   .Intrin2           0.387    0.038   10.217    0.000    0.387    0.535
   .Intrin3           0.213    0.035    6.099    0.000    0.213    0.359
   .Extrin1           0.080    0.011    7.217    0.000    0.080    0.348
   .Extrin2           0.123    0.016    7.492    0.000    0.123    0.372
   .Extrin3           0.103    0.017    5.948    0.000    0.103    0.317
   .PosAFF1           0.121    0.016    7.750    0.000    0.121    0.271
   .PosAFF2           0.104    0.015    6.798    0.000    0.104    0.222
   .PosAFF3           0.099    0.014    6.879    0.000    0.099    0.199
```

Structural Regressions among Latent Constructs ...

```
   .NegAFF1           0.107    0.016    6.789    0.000    0.107    0.210
   .NegAFF2           0.096    0.013    7.540    0.000    0.096    0.219
   .NegAFF3           0.067    0.012    5.555    0.000    0.067    0.158
    Agency            1.000                               1.000    1.000
   .Intrinsic         1.000                               0.743    0.743
   .Extrinsic         1.000                               0.928    0.928
   .Positive          1.000                               0.815    0.815
   .Negative          1.000                               0.959    0.959

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ind1              0.202    0.046    4.348    0.000    0.182    0.182
    ind2              0.055    0.023    2.436    0.015    0.054    0.054
```

```
mediat.out.2 <- sem(mediat.mod.2, data = dat,
                    std.lv = TRUE, se = "boot",
                       bootstrap = Nboot)
summary(mediat.out.2, standardized = TRUE, fit =
   TRUE)
```

Structural Regressions among Latent Constructs ...

```
lavaan (0.5-23.1097) converged normally after  62 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.991

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3700.262
  Loglikelihood unrestricted model (H1)      -3645.686
```

Structural Regressions among Latent Constructs ...

```
  Number of free parameters                         37
  Akaike (AIC)                                7474.524
  Bayesian (BIC)                              7620.310
  Sample-size adjusted Bayesian (BIC)         7502.917

Root Mean Square Error of Approximation:

  RMSEA                                          0.029
  90 Percent Confidence Interval          0.010  0.043
  P-value RMSEA <= 0.05                          0.996

Standardized Root Mean Square Residual:

  SRMR                                           0.034

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws              500
  Number of successful bootstrap draws             500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.466    0.021   22.302    0.000    0.466    0.905
```

Structural Regressions among Latent Constructs ...

```
    Agency2            0.492    0.021   23.227    0.000    0.492    0.933
    Agency3            0.497    0.024   20.574    0.000    0.497    0.912
  Intrinsic =~
    Intrin1            0.466    0.035   13.489    0.000    0.540    0.703
    Intrin2            0.500    0.037   13.502    0.000    0.580    0.682
    Intrin3            0.531    0.032   16.397    0.000    0.616    0.800
  Extrinsic =~
    Extrin1            0.373    0.022   16.779    0.000    0.388    0.808
    Extrin2            0.440    0.028   15.945    0.000    0.457    0.793
    Extrin3            0.454    0.029   15.466    0.000    0.471    0.827
  Positive =~
    PosAFF1            0.514    0.025   20.161    0.000    0.569    0.854
    PosAFF2            0.545    0.029   18.693    0.000    0.604    0.882
    PosAFF3            0.570    0.027   21.232    0.000    0.632    0.895
  Negative =~
    NegAFF1            0.620    0.038   16.174    0.000    0.633    0.889
    NegAFF2            0.573    0.042   13.579    0.000    0.585    0.883
    NegAFF3            0.586    0.035   16.865    0.000    0.598    0.918

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)     0.343    0.075    4.557    0.000    0.359    0.359
    Agency             0.129    0.071    1.824    0.068    0.116    0.116
  Negative ~
    Extrinsic (b2)     0.199    0.069    2.902    0.004    0.202    0.202
```

Structural Regressions among Latent Constructs ...

```
  Intrinsic ~
    Agency      (a1)    0.588    0.080    7.315    0.000    0.507    0.507
  Extrinsic ~
    Agency      (a2)    0.278    0.064    4.328    0.000    0.268    0.268

Covariances:
                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic         -0.202    0.062   -3.235    0.001   -0.202   -0.202
 .Positive ~~
   .Negative          -0.076    0.070   -1.091    0.275   -0.076   -0.076

Variances:
                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1            0.048    0.006    8.464    0.000    0.048    0.181
   .Agency2            0.036    0.005    7.928    0.000    0.036    0.129
   .Agency3            0.050    0.006    8.976    0.000    0.050    0.169
   .Intrin1            0.299    0.032    9.299    0.000    0.299    0.506
   .Intrin2            0.387    0.038   10.217    0.000    0.387    0.535
   .Intrin3            0.213    0.035    6.099    0.000    0.213    0.359
   .Extrin1            0.080    0.011    7.217    0.000    0.080    0.348
   .Extrin2            0.123    0.016    7.492    0.000    0.123    0.372
   .Extrin3            0.103    0.017    5.948    0.000    0.103    0.317
   .PosAFF1            0.121    0.016    7.750    0.000    0.121    0.271
   .PosAFF2            0.104    0.015    6.798    0.000    0.104    0.222
   .PosAFF3            0.099    0.014    6.879    0.000    0.099    0.199
```

Structural Regressions among Latent Constructs ...

```
    .NegAFF1          0.107    0.016    6.789    0.000    0.107    0.210
    .NegAFF2          0.096    0.013    7.540    0.000    0.096    0.219
    .NegAFF3          0.067    0.012    5.555    0.000    0.067    0.158
     Agency           1.000                               1.000    1.000
    .Intrinsic        1.000                               0.743    0.743
    .Extrinsic        1.000                               0.928    0.928
    .Positive         1.000                               0.815    0.815
    .Negative         1.000                               0.959    0.959

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
     ind1             0.202    0.046    4.348    0.000    0.182    0.182
     ind2             0.055    0.023    2.436    0.015    0.054    0.054
```

- The mediation model with latent variables estimates fewer parameters than the CFA, but it fits just as well as the CFA.

```
anova(cfa.out, mediat.out.2)
```

Structural Regressions among Latent Constructs ...

```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
cfa.out       80 7478.2 7635.8 106.85
mediat.out.2 83 7474.5 7620.3 109.15     2.3051       3     0.5116
```

```
anova(cfa.out, mediat.out.2)
```

```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
cfa.out       80 7478.2 7635.8 106.85
mediat.out.2 83 7474.5 7620.3 109.15     2.3051       3     0.5116
```

- Compared to the model with observed scale means, the model with latent variables has greater effect size for the indirect effect of Agency on Positive Affect via Internal Motivation. Internal Motivation indicators have the most measurement error (lowest standardized factor loadings), so it is not surprising that this indirect path was attenuated most by measurement error.

Structural Regressions among Latent Constructs ...

```
summary(mediat.out.2, standardized = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  62 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws              500
  Number of successful bootstrap draws             500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.466    0.021   22.302    0.000    0.466    0.905
    Agency2           0.492    0.021   23.227    0.000    0.492    0.933
    Agency3           0.497    0.024   20.574    0.000    0.497    0.912
```

Structural Regressions among Latent Constructs ...

```
  Intrinsic =~
    Intrin1           0.466    0.035   13.489    0.000    0.540    0.703
    Intrin2           0.500    0.037   13.502    0.000    0.580    0.682
    Intrin3           0.531    0.032   16.397    0.000    0.616    0.800
  Extrinsic =~
    Extrin1           0.373    0.022   16.779    0.000    0.388    0.808
    Extrin2           0.440    0.028   15.945    0.000    0.457    0.793
    Extrin3           0.454    0.029   15.466    0.000    0.471    0.827
  Positive =~
    PosAFF1           0.514    0.025   20.161    0.000    0.569    0.854
    PosAFF2           0.545    0.029   18.693    0.000    0.604    0.882
    PosAFF3           0.570    0.027   21.232    0.000    0.632    0.895
  Negative =~
    NegAFF1           0.620    0.038   16.174    0.000    0.633    0.889
    NegAFF2           0.573    0.042   13.579    0.000    0.585    0.883
    NegAFF3           0.586    0.035   16.865    0.000    0.598    0.918

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)    0.343    0.075    4.557    0.000    0.359    0.359
    Agency            0.129    0.071    1.824    0.068    0.116    0.116
  Negative ~
    Extrinsic (b2)    0.199    0.069    2.902    0.004    0.202    0.202
  Intrinsic ~
    Agency    (a1)    0.588    0.080    7.315    0.000    0.507    0.507
```

Structural Regressions among Latent Constructs ...

```
  Extrinsic ~
    Agency      (a2)    0.278    0.064    4.328    0.000    0.268    0.268

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.062   -3.235    0.001   -0.202   -0.202
 .Positive ~~
   .Negative         -0.076    0.070   -1.091    0.275   -0.076   -0.076

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.006    8.464    0.000    0.048    0.181
   .Agency2           0.036    0.005    7.928    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.976    0.000    0.050    0.169
   .Intrin1           0.299    0.032    9.299    0.000    0.299    0.506
   .Intrin2           0.387    0.038   10.217    0.000    0.387    0.535
   .Intrin3           0.213    0.035    6.099    0.000    0.213    0.359
   .Extrin1           0.080    0.011    7.217    0.000    0.080    0.348
   .Extrin2           0.123    0.016    7.492    0.000    0.123    0.372
   .Extrin3           0.103    0.017    5.948    0.000    0.103    0.317
   .PosAFF1           0.121    0.016    7.750    0.000    0.121    0.271
   .PosAFF2           0.104    0.015    6.798    0.000    0.104    0.222
   .PosAFF3           0.099    0.014    6.879    0.000    0.099    0.199
   .NegAFF1           0.107    0.016    6.789    0.000    0.107    0.210
   .NegAFF2           0.096    0.013    7.540    0.000    0.096    0.219
```

Structural Regressions among Latent Constructs ...

```
   .NegAFF3          0.067    0.012    5.555    0.000    0.067    0.158
    Agency           1.000                               1.000    1.000
   .Intrinsic        1.000                               0.743    0.743
   .Extrinsic        1.000                               0.928    0.928
   .Positive         1.000                               0.815    0.815
   .Negative         1.000                               0.959    0.959

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ind1             0.202    0.046    4.348    0.000    0.182    0.182
    ind2             0.055    0.023    2.436    0.015    0.054    0.054
```

```
parameterEstimates(mediat.out.1, standardized =
    TRUE, ci =
    FALSE)[which(parameterEstimates(mediat.out.1)[,"lhs'
    %in% c("ind1", "ind2")), ]
```

```
    lhs op    rhs label   est    se     z pvalue std.lv std.all std.nox
13 ind1 := a1*b1  ind1 0.148 0.029 5.062  0.000  0.148   0.117   0.234
14 ind2 := a2*b2  ind2 0.056 0.023 2.493  0.013  0.056   0.045   0.090
```

Structural Regressions among Latent Constructs ...

```
summary(mediat.out.2, standardized = TRUE)
```

```
lavaan (0.5-23.1097) converged normally after  62 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              109.152
  Degrees of freedom                                83
  P-value (Chi-square)                           0.029

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws              500
  Number of successful bootstrap draws             500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.466    0.021   22.302    0.000    0.466    0.905
    Agency2           0.492    0.021   23.227    0.000    0.492    0.933
    Agency3           0.497    0.024   20.574    0.000    0.497    0.912
```

Structural Regressions among Latent Constructs ...

```
  Intrinsic =~
    Intrin1            0.466    0.035   13.489    0.000    0.540    0.703
    Intrin2            0.500    0.037   13.502    0.000    0.580    0.682
    Intrin3            0.531    0.032   16.397    0.000    0.616    0.800
  Extrinsic =~
    Extrin1            0.373    0.022   16.779    0.000    0.388    0.808
    Extrin2            0.440    0.028   15.945    0.000    0.457    0.793
    Extrin3            0.454    0.029   15.466    0.000    0.471    0.827
  Positive =~
    PosAFF1            0.514    0.025   20.161    0.000    0.569    0.854
    PosAFF2            0.545    0.029   18.693    0.000    0.604    0.882
    PosAFF3            0.570    0.027   21.232    0.000    0.632    0.895
  Negative =~
    NegAFF1            0.620    0.038   16.174    0.000    0.633    0.889
    NegAFF2            0.573    0.042   13.579    0.000    0.585    0.883
    NegAFF3            0.586    0.035   16.865    0.000    0.598    0.918

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)     0.343    0.075    4.557    0.000    0.359    0.359
    Agency             0.129    0.071    1.824    0.068    0.116    0.116
  Negative ~
    Extrinsic (b2)     0.199    0.069    2.902    0.004    0.202    0.202
  Intrinsic ~
    Agency     (a1)    0.588    0.080    7.315    0.000    0.507    0.507
```

Structural Regressions among Latent Constructs ...

```
  Extrinsic ~
    Agency     (a2)    0.278    0.064    4.328    0.000    0.268    0.268

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.062   -3.235    0.001   -0.202   -0.202
 .Positive ~~
   .Negative         -0.076    0.070   -1.091    0.275   -0.076   -0.076

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.006    8.464    0.000    0.048    0.181
   .Agency2           0.036    0.005    7.928    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.976    0.000    0.050    0.169
   .Intrin1           0.299    0.032    9.299    0.000    0.299    0.506
   .Intrin2           0.387    0.038   10.217    0.000    0.387    0.535
   .Intrin3           0.213    0.035    6.099    0.000    0.213    0.359
   .Extrin1           0.080    0.011    7.217    0.000    0.080    0.348
   .Extrin2           0.123    0.016    7.492    0.000    0.123    0.372
   .Extrin3           0.103    0.017    5.948    0.000    0.103    0.317
   .PosAFF1           0.121    0.016    7.750    0.000    0.121    0.271
   .PosAFF2           0.104    0.015    6.798    0.000    0.104    0.222
   .PosAFF3           0.099    0.014    6.879    0.000    0.099    0.199
   .NegAFF1           0.107    0.016    6.789    0.000    0.107    0.210
   .NegAFF2           0.096    0.013    7.540    0.000    0.096    0.219
```

Structural Regressions among Latent Constructs ...

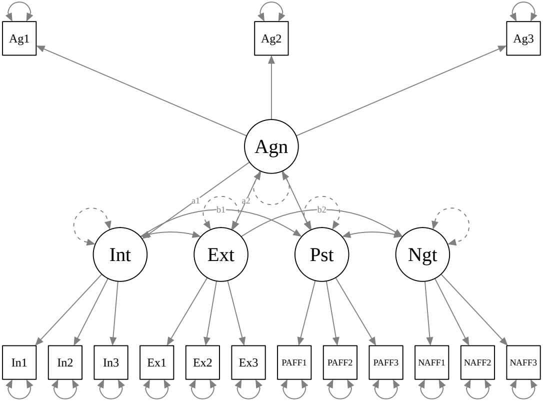
```
   .NegAFF3           0.067    0.012    5.555    0.000    0.067    0.158
    Agency            1.000                               1.000    1.000
   .Intrinsic         1.000                               0.743    0.743
   .Extrinsic         1.000                               0.928    0.928
   .Positive          1.000                               0.815    0.815
   .Negative          1.000                               0.959    0.959

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ind1              0.202    0.046    4.348    0.000    0.182    0.182
    ind2              0.055    0.023    2.436    0.015    0.054    0.054
```

```
parameterEstimates(mediat.out.1, standardized =
    TRUE, ci =
    FALSE)[which(parameterEstimates(mediat.out.1)[,"lhs"
    %in% c("ind1", "ind2")), ]
```

```
   lhs op    rhs label   est    se     z pvalue std.lv std.all std.nox
13 ind1 :=  a1*b1  ind1 0.148 0.029 5.062  0.000  0.148   0.117   0.234
14 ind2 :=  a2*b2  ind2 0.056 0.023 2.493  0.013  0.056   0.045   0.090
```

I was afraid to see what the plotter would do

```
semPaths(mediat.out.2)
```



Check if ind1 and ind2 are signficantly different

- Did you wonder if ind1 is significantly different from ind2?
- The assumption that they are the same is another restriction we can put on the model.
- We re-fit, and then run the anova test.

```
mediat.mod.3 <- '
 ## measurement model
 Agency =~ Agency1 + Agency2 + Agency3
 Intrinsic =~ Intrin1 + Intrin2 + Intrin3
 Extrinsic =~ Extrin1 + Extrin2 + Extrin3
 Positive =~ PosAFF1 + PosAFF2 + PosAFF3
 Negative =~ NegAFF1 + NegAFF2 + NegAFF3
 ## structural model
 Positive ~ b1*Intrinsic + Agency
 Negative ~ b2*Extrinsic
 Intrinsic ~ a1*Agency
```

Check if ind1 and ind2 are signficantly different ...

```
  Extrinsic ~ a2*Agency
  Intrinsic ~~ Extrinsic
  ## define mediation parameters (indirect effects)
  ind1 := a1 * b1
  ind2 := a2 * b2
  ind1 == ind2
  '
```

```
mediat.mod.3 <- '
  ## measurement model
  Agency =~ Agency1 + Agency2 + Agency3
  Intrinsic =~ Intrin1 + Intrin2 + Intrin3
  Extrinsic =~ Extrin1 + Extrin2 + Extrin3
  Positive =~ PosAFF1 + PosAFF2 + PosAFF3
  Negative =~ NegAFF1 + NegAFF2 + NegAFF3
  ## structural model
  Positive ~ b1*Intrinsic + Agency
```

Check if ind1 and ind2 are signficantly different ...

```
  Negative ~ b2*Extrinsic
  Intrinsic ~ a1*Agency
  Extrinsic ~ a2*Agency
  Intrinsic ~~ Extrinsic
  ## define mediation parameters (indirect effects)
  ind1 := a1 * b1
  ind2 := a2 * b2
  ind1 == ind2
  '
```

```
mediat.out.3 <- sem(mediat.mod.3, data = dat,
                    std.lv = TRUE, se = "boot",
                       bootstrap = Nboot)
summary(mediat.out.3, standardized = TRUE, fit =
   TRUE)
```

Check if ind1 and ind2 are signficantly different ...

```
lavaan (0.5-23.1097) converged normally after 480 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              118.955
  Degrees of freedom                                84
  P-value (Chi-square)                           0.007

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.990
  Tucker-Lewis Index (TLI)                       0.988

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3705.163
  Loglikelihood unrestricted model (H1)      -3645.686
```

Check if ind1 and ind2 are signficantly different ...

```
  Number of free parameters                           36
  Akaike (AIC)                                  7482.326
  Bayesian (BIC)                                7624.172
  Sample-size adjusted Bayesian (BIC)           7509.951

Root Mean Square Error of Approximation:

  RMSEA                                            0.033
  90 Percent Confidence Interval          0.018   0.046
  P-value RMSEA <= 0.05                            0.986

Standardized Root Mean Square Residual:

  SRMR                                             0.051

Parameter Estimates:

  Information                                   Observed
  Standard Errors                              Bootstrap
  Number of requested bootstrap draws                500
  Number of successful bootstrap draws               500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.467    0.021   22.738    0.000    0.467    0.905
```

Check if ind1 and ind2 are signficantly different ...

```
    Agency2           0.491    0.020   24.518    0.000    0.491    0.933
    Agency3           0.497    0.022   22.271    0.000    0.497    0.911
  Intrinsic =~
    Intrin1           0.468    0.041   11.435    0.000    0.528    0.693
    Intrin2           0.508    0.036   14.071    0.000    0.573    0.679
    Intrin3           0.540    0.032   16.825    0.000    0.609    0.801
  Extrinsic =~
    Extrin1           0.373    0.023   15.931    0.000    0.395    0.813
    Extrin2           0.439    0.029   15.095    0.000    0.465    0.798
    Extrin3           0.452    0.029   15.606    0.000    0.478    0.829
  Positive =~
    PosAFF1           0.530    0.024   21.669    0.000    0.564    0.853
    PosAFF2           0.562    0.028   19.847    0.000    0.598    0.881
    PosAFF3           0.587    0.028   21.244    0.000    0.624    0.892
  Negative =~
    NegAFF1           0.614    0.038   15.994    0.000    0.639    0.890
    NegAFF2           0.567    0.039   14.484    0.000    0.590    0.885
    NegAFF3           0.579    0.037   15.774    0.000    0.603    0.919

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)    0.183    0.078    2.347    0.019    0.194    0.194
    Agency            0.218    0.077    2.831    0.005    0.205    0.205
  Negative ~
    Extrinsic (b2)    0.275    0.056    4.961    0.000    0.280    0.280
```

Check if ind1 and ind2 are signficantly different ...

```
  Intrinsic ~
    Agency     (a1)    0.523    0.104    5.041    0.000    0.463    0.463
  Extrinsic ~
    Agency     (a2)    0.348    0.064    5.462    0.000    0.328    0.328

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.064   -3.163    0.002   -0.202   -0.202
 .Positive ~~
   .Negative         -0.063    0.070   -0.909    0.363   -0.063   -0.063

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.006    8.495    0.000    0.048    0.181
   .Agency2           0.036    0.005    7.550    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.839    0.000    0.050    0.170
   .Intrin1           0.301    0.033    9.066    0.000    0.301    0.519
   .Intrin2           0.383    0.039    9.800    0.000    0.383    0.539
   .Intrin3           0.208    0.039    5.373    0.000    0.208    0.359
   .Extrin1           0.080    0.010    7.633    0.000    0.080    0.339
   .Extrin2           0.123    0.016    7.500    0.000    0.123    0.363
   .Extrin3           0.104    0.017    6.180    0.000    0.104    0.312
   .PosAFF1           0.120    0.016    7.510    0.000    0.120    0.273
   .PosAFF2           0.103    0.016    6.352    0.000    0.103    0.224
   .PosAFF3           0.100    0.014    7.035    0.000    0.100    0.205
```

Check if ind1 and ind2 are signficantly different ...

```
   .NegAFF1          0.107    0.016    6.657    0.000    0.107    0.207
   .NegAFF2          0.096    0.013    7.209    0.000    0.096    0.216
   .NegAFF3          0.067    0.011    6.245    0.000    0.067    0.156
    Agency           1.000                               1.000    1.000
   .Intrinsic        1.000                               0.785    0.785
   .Extrinsic        1.000                               0.892    0.892
   .Positive         1.000                               0.883    0.883
   .Negative         1.000                               0.922    0.922

Defined Parameters:
                  Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ind1             0.096    0.026    3.708    0.000    0.090    0.090
    ind2             0.096    0.026    3.708    0.000    0.092    0.092

Constraints:
                                             |Slack|
    ind1 - (ind2)                              0.000
```

```
mediat.out.3 <- sem(mediat.mod.3, data = dat,
                    std.lv = TRUE, se = "boot",
                       bootstrap = Nboot)
summary(mediat.out.3, standardized = TRUE, fit =
   TRUE)
```

Check if ind1 and ind2 are signficantly different ...

```
lavaan (0.5-23.1097) converged normally after 480 iterations

  Number of observations                           380

  Estimator                                         ML
  Minimum Function Test Statistic              118.955
  Degrees of freedom                                84
  P-value (Chi-square)                           0.007

Model test baseline model:

  Minimum Function Test Statistic             3749.411
  Degrees of freedom                               105
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.990
  Tucker-Lewis Index (TLI)                       0.988

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3705.163
  Loglikelihood unrestricted model (H1)      -3645.686
```

Check if ind1 and ind2 are signficantly different ...

```
  Number of free parameters                          36
  Akaike (AIC)                                 7482.326
  Bayesian (BIC)                               7624.172
  Sample-size adjusted Bayesian (BIC)          7509.951

Root Mean Square Error of Approximation:

  RMSEA                                           0.033
  90 Percent Confidence Interval          0.018  0.046
  P-value RMSEA <= 0.05                           0.986

Standardized Root Mean Square Residual:

  SRMR                                            0.051

Parameter Estimates:

  Information                                  Observed
  Standard Errors                             Bootstrap
  Number of requested bootstrap draws               500
  Number of successful bootstrap draws              500

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Agency =~
    Agency1           0.467    0.021   22.738    0.000    0.467    0.905
```

Check if ind1 and ind2 are signficantly different ...

```
    Agency2           0.491    0.020   24.518    0.000    0.491    0.933
    Agency3           0.497    0.022   22.271    0.000    0.497    0.911
  Intrinsic =~
    Intrin1           0.468    0.041   11.435    0.000    0.528    0.693
    Intrin2           0.508    0.036   14.071    0.000    0.573    0.679
    Intrin3           0.540    0.032   16.825    0.000    0.609    0.801
  Extrinsic =~
    Extrin1           0.373    0.023   15.931    0.000    0.395    0.813
    Extrin2           0.439    0.029   15.095    0.000    0.465    0.798
    Extrin3           0.452    0.029   15.606    0.000    0.478    0.829
  Positive =~
    PosAFF1           0.530    0.024   21.669    0.000    0.564    0.853
    PosAFF2           0.562    0.028   19.847    0.000    0.598    0.881
    PosAFF3           0.587    0.028   21.244    0.000    0.624    0.892
  Negative =~
    NegAFF1           0.614    0.038   15.994    0.000    0.639    0.890
    NegAFF2           0.567    0.039   14.484    0.000    0.590    0.885
    NegAFF3           0.579    0.037   15.774    0.000    0.603    0.919

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Positive ~
    Intrinsic (b1)    0.183    0.078    2.347    0.019    0.194    0.194
    Agency            0.218    0.077    2.831    0.005    0.205    0.205
  Negative ~
    Extrinsic (b2)    0.275    0.056    4.961    0.000    0.280    0.280
```

Check if ind1 and ind2 are signficantly different ...

```
  Intrinsic ~
    Agency    (a1)    0.523    0.104    5.041    0.000    0.463    0.463
  Extrinsic ~
    Agency    (a2)    0.348    0.064    5.462    0.000    0.328    0.328

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .Intrinsic ~~
   .Extrinsic        -0.202    0.064   -3.163    0.002   -0.202   -0.202
 .Positive ~~
   .Negative         -0.063    0.070   -0.909    0.363   -0.063   -0.063

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .Agency1           0.048    0.006    8.495    0.000    0.048    0.181
   .Agency2           0.036    0.005    7.550    0.000    0.036    0.129
   .Agency3           0.050    0.006    8.839    0.000    0.050    0.170
   .Intrin1           0.301    0.033    9.066    0.000    0.301    0.519
   .Intrin2           0.383    0.039    9.800    0.000    0.383    0.539
   .Intrin3           0.208    0.039    5.373    0.000    0.208    0.359
   .Extrin1           0.080    0.010    7.633    0.000    0.080    0.339
   .Extrin2           0.123    0.016    7.500    0.000    0.123    0.363
   .Extrin3           0.104    0.017    6.180    0.000    0.104    0.312
   .PosAFF1           0.120    0.016    7.510    0.000    0.120    0.273
   .PosAFF2           0.103    0.016    6.352    0.000    0.103    0.224
   .PosAFF3           0.100    0.014    7.035    0.000    0.100    0.205
```

Check if ind1 and ind2 are signficantly different ...

```
   .NegAFF1           0.107    0.016    6.657    0.000    0.107    0.207
   .NegAFF2           0.096    0.013    7.209    0.000    0.096    0.216
   .NegAFF3           0.067    0.011    6.245    0.000    0.067    0.156
    Agency            1.000                               1.000    1.000
   .Intrinsic         1.000                               0.785    0.785
   .Extrinsic         1.000                               0.892    0.892
   .Positive          1.000                               0.883    0.883
   .Negative          1.000                               0.922    0.922

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    ind1              0.096    0.026    3.708    0.000    0.090    0.090
    ind2              0.096    0.026    3.708    0.000    0.092    0.092

Constraints:
                                               |Slack|
    ind1 - (ind2)                                0.000
```

```
anova(mediat.out.3, mediat.out.2)
```

Check if ind1 and ind2 are signficantly different ...

```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
mediat.out.2 83 7474.5 7620.3 109.15
mediat.out.3 84 7482.3 7624.2 118.95     9.8021       1   0.001743 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(mediat.out.3, mediat.out.2)
```

```
Chi Square Difference Test

              Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
mediat.out.2 83 7474.5 7620.3 109.15
mediat.out.3 84 7482.3 7624.2 118.95     9.8021       1   0.001743 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Outline

Structural Equation Modeling

- In Psychology, SEM has been an area of tremendous growth since 1980.
- SEM is being absorbed slowly into other fields
- The "gold standard" software for SEM modeling is Mplus, although lavaan has succeeded in "matching" side-by-side many of the calculations.

References

R Core Team (2017). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria.

Session

```
sessionInfo()
```

```
R version 3.4.4 (2018-03-15)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 18.04 LTS

Matrix products: default
BLAS: /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.7.1
LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.7.1

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
      LC_TIME=en_US.UTF-8
 [4] LC_COLLATE=en_US.UTF-8     LC_MONETARY=en_US.UTF-8
      LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C                  LC_ADDRESS=C
[10] LC_TELEPHONE=C             LC_MEASUREMENT=en_US.UTF-8
    LC_IDENTIFICATION=C

attached base packages:
[1] stats     graphics  grDevices utils     datasets  base

other attached packages:
[1] semPlot_1.1        kutils_1.44        lavaan_0.5-23.1097
```

Session ...

```
loaded via a namespace (and not attached):
 [1] splines_3.4.4       ellipse_0.4.1       gtools_3.5.0
     network_1.13.0
 [5] Formula_1.2-2       semTools_0.4-14     BDgraph_2.44
     stats4_3.4.4
 [9] latticeExtra_0.6-28 d3Network_0.5.2.1   lisrelToR_0.1.4
     pbivnorm_0.6.0
[13] pillar_1.1.0        backports_1.1.2     lattice_0.20-35
    quantreg_5.35
[17] quadprog_1.5-5      digest_0.6.15       RColorBrewer_1.1-2
    checkmate_1.8.5
[21] ggm_2.3             minqa_1.2.4         colorspace_1.3-2
    htmltools_0.3.6
[25] Matrix_1.2-14       plyr_1.8.4          psych_1.7.8
    XML_3.98-1.9
[29] pkgconfig_2.0.1     SparseM_1.77        xtable_1.8-2
    corpcor_1.6.9
[33] scales_0.5.0        whisker_0.3-2       glasso_1.8
    sna_2.4
[37] jpeg_0.1-8          openxlsx_4.0.17     fdrtool_1.2.15
    lme4_1.1-17
[41] MatrixModels_0.4-1  huge_1.2.7          arm_1.9-3
    htmlTable_1.11.2
[45] tibble_1.4.2        rockchalk_1.8.111   mgcv_1.8-23
    car_2.1-6
```

Session ...

```
[49] ggplot2_2.2.1        nnet_7.3-12          lazyeval_0.2.1
     pbkrtest_0.4-7
[53] mnormt_1.5-5         statnet.common_4.0.0 survival_2.41-3
     magrittr_1.5
[57] methods_3.4.4        nlme_3.1-137         MASS_7.3-49
     foreign_0.8-69
[61] OpenMx_2.8.3         tools_3.4.4          data.table_1.10.4-3
     stringr_1.2.0
[65] munsell_0.4.3        cluster_2.0.6        compiler_3.4.4
     sem_3.1-9
[69] rlang_0.1.6          grid_3.4.4           nloptr_1.0.4
     rstudioapi_0.7
[73] rjson_0.2.15         htmlwidgets_1.0      igraph_1.1.2
     base64enc_0.1-3
[77] boot_1.3-20          mi_1.0               gtable_0.2.0
     abind_1.4-5
[81] reshape2_1.4.3       qgraph_1.4.4         gridExtra_2.3
     knitr_1.19
[85] Hmisc_4.1-1          stringi_1.2.2        matrixcalc_1.0-3
     parallel_3.4.4
[89] Rcpp_0.12.15         rpart_4.1-13         acepack_1.4.1
     png_0.1-7
[93] coda_0.19-1
```