

On the Importance of Frailty in Social Science Theory (and other lessons of agent-based modeling)

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Outline

Terminology and Purposes

ABM Checklist

Step 1. Construct The Agents

Step 1b (Afterthought). Formulate Auxiliary Agents

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Simulation Infrastructure

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The Inevitable Follow-Up Questions

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Should we re-design a stable simulation by inserting noise?

Conclusion

What is this Paper/Presentation

- ▶ Survey of issues I have noticed while writing lots of agent-based models in social science
- ▶ Some models to mention here are published in
 - ▶ Johnson (1996). Unraveling in a Variety of Institutional Settings. *Journal of Theoretical Politics*
 - ▶ Brichoux and Johnson (2002). The Power of Commitment in Cooperative Social Action. *Journal of Artificial Societies and Social Simulation*
 - ▶ Huckfeldt, Johnson, Sprague (2004) *Political Disagreement*
 - ▶ Johnson (2002) Agent-Based Modeling: What I Learned From the Artificial Stock Market, *Social Science Computer Review*

I Believe the Open in Open Source

- ▶ Code for these (and many other) projects available at <http://pj.freefaculty.org/Swarm>
- ▶ I use the Swarm Simulation System in Objective-C (don't have axe to grind with other languages or toolkits, though).
- ▶ Paper and slides available:
<http://pj.freefaculty.org/Papers/WCSS2012>

Terminology

agent. An isolated collection of data and routines for gathering, adjusting, and (selectively) revealing data.

agent-based model. A collection of agents (“artificial society”) along with a computer framework required to manage their interaction and to collect information.

Agent-based Modeling is For ...

- ▶ “hypothesis-generating exercise” (Johnson, 1996): what might happen if a collection of separate things are thrown together into a system?
- ▶ Bottom-up explanation of systems (Epstein and Axtell, 1996); self-organization (Bak, 1999).
- ▶ “Something from nothing” models; Emergent properties & artificial life.

Emergent Property: an observation about a system that we might not anticipate from the separate study of its individual components (Holland, 1998; Strogatz, 2003).

Agent-based Modeling is For

- ▶ Seeking understanding, insight, and hope for human problem solving in the general idea that the aggregation of individual-level decisions & behaviors might “solve social problems” (Swarm Intelligence; related, the “Wisdom of Crowds”, Condorcet Jury Theorem)

Varieties of Emergent Properties

- ▶ Are the emergent patterns that “pop out” anticipated by program developers?
- ▶ What if they did anticipate them? Is code “reverse engineered” to produce outcome?
- ▶ Are “emergent properties” sincere, or a deception?
 - ▶ Should we believe researchers who claim have found a bottom-up explanation when they have designed the code for the bottom so as to produce the top?
 - ▶ Worry: slightly different models would produce grossly different answers.
- ▶ Not different from suspicion about statistical researchers to manipulate models to produce “good p values”.

My Best Answer: Separate Kinds of Emergent Behaviors

- ▶ “factoid.” A thing we already noticed, but for which we want a new (micro level) explanation.
 - ▶ Observation: Bee hive temperatures, ant trails, bird flocks
 - ▶ Normative: Swarm Intelligence
- ▶ “revelation.” An unrecognized pattern (probably because theory did not tell us where to look).
power-laws, self-organized criticality, fractals.
- ▶ “hunch.” suspected, possibly hoped for pattern that the designer hopes to produce, and then confirm in nature.

How to Deal with Hunches

- ▶ Concern: Programmers manipulate design of individual pieces to “reverse engineer” an emergent property
 - ▶ Perhaps intellectually dubious.
- ▶ Programmer’s defense: emphasize the simplicity of agent-design (“nothing” in “something for nothing”).

Suggestion: Develop An Ensemble of Models

- ▶ One way to reclaim legitimacy: Reverse engineer several models
 - ▶ to produce different emergent properties! That's interesting, like a “controlled classroom study”.
 - ▶ may need to write code for an opposing theory so as to demonstrate it has unacceptable emergent properties
 - ▶ produce same emergent properties: That's interesting too! Details are not important, that all models within a family produce same answer.

The Ideal Modeling Experience

- ▶ A scientist has
 - ▶ exhaustive mathematical characterization of the agents that are being studied.
 - ▶ clear plan to guide a model for the over-time interaction of agents.
 - ▶ clear idea of data collection plan.
- ▶ The programmers “implement” that idea to “see what happens”.

The Real Modeling Experience

The Ideal Modeling Experience is not Usual.

- ▶ Researchers usually lack “exhaustive mathematical characterization”.
- ▶ They hope to elaborate their ideas by writing out a computer model.
- ▶ Models always require more detail than researchers expect.

Thus, I propose a “checklist”, a list of items to structure the dialogue of project development

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Variables Internal to Each Agent

The “instance variables” inside each agent.

1. Permanent (unchangeable) characteristics of the agent (which may differentiate that agent from other agents)
2. Individual variables that may change as the simulation progresses. This includes
 - ▶ “private” and
 - ▶ externally observable states (substantively, we might call these behaviors).
3. Class variables, simultaneously recognized by sub-groups of agents.

Helps to know if user expects these to be integers, real-valued, character, etc.

More Instance Variables

- ▶ ABM intrinsically dynamic (contrast “static” social science models).
- ▶ Thus: Agents Need To Remember Their Experiences.
We need:

1. Instance variables to record everything the agent “remembers” about
 - 1.1 itself: the values of its instance variables as they existed in the past
 - 1.2 other agents, either as individual agents or as summarized by recollections about groups or regions
 - 1.3 the “world”
2. An algorithm to initialize the agent’s recollections. The agent-based model begins at time 0. What to agents remember about the time before that?

Methods: Actions the Agents are able to “Do”

- ▶ Method: similar to function, but tied to a type of object (see the Apple Objective-C manual, Apple Computing, 2009).
- ▶ Messaging concept. Especially in frameworks based on SmallTalk or Objective-C, we think of agents sending and responding to messages that ask for and send information.

What's in the Step Method?

- ▶ step is a common name for the actions an agent will carry out when it is told to do so.
 - ▶ “book-keeping”
 - ▶ growth, recognition, update, etc.
 - ▶ move: requires update of internal variables and “self-registration” with space
 - ▶ change externally visible instance variables

Information Revelation

- ▶ How is agent's information made available
 - ▶ to other agents, and
 - ▶ to “us” as we monitor?
- ▶ “object-oriented design philosophy”: Keep agent information inside the agent, unless explicit decision is made to expose it.
- ▶ There is no difference between an “attitude” and a “behavior,” so far as ABM is concerned. These are just variables that exist within an agent and may (or may not) be visible to others.

Substantively Based Design Questions

1. Does the agent's existence in the model directly expose instance variables to access by other agents?
Can other agents simply “look” at at one agent and instantly “know” the values of some of its instance variables?
2. Is the revelation of private information a focal point in the model? If so, think hard on the problem of private preferences and public declarations.
 - ▶ Example: political protest. Action focuses on each individual agent's willingness to take a risky action that depends on perceived agreement with others (Granovetter and Soong, 1988; Brichoux, 2002).

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Auxiliary agents

- ▶ Substantively-unimportant, at least in mind of substantive researcher.
- ▶ auxiliary agents glue the other agents together.
 - ▶ A “specialist” links buyers and sellers in a stock market
 - ▶ An electioneer in a voting model counts votes
- ▶ Record keepers monitor the simulation. In Swarm, we call that the “observer swarm.”

Unexpected Profusion of Auxiliary Agents

Mixed Electoral Systems model Herron and Johnson (2005).

- ▶ District and National level political parties and candidates compete for votes
 - ▶ Voters are simple,
 - ▶ they choose a party to join
 - ▶ attend a caucus with other people who join that party and nominate local candidates
 - ▶ vote in the general election.
 - ▶ Vandidates are simple; declare policy proposals to the voters within a district.

Auxiliary agents abound

- ▶ National parties must exist (and simulation needs infrastructure to “split” and “join” parties). The national party decides whether to pay for a single member campaigns in the separate districts.
- ▶ Each district needs a political party object to convene caucuses
- ▶ Electioneer: District-level agents that keep a list of eligible voters, candidates, and candidate promises. Holds the elections and then report the results to the national offices, for which another auxiliary agent must be created.
- ▶ I'd guess that about 30% of the computer programming effort was dedicated to the substantively important actors, the voters and the candidates, and about 70% of the effort is in the auxiliary agents.

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Where Do Agents “Live”?

1. Checkerboard: one agent per cell.
 - 1.1 Has been a dominant approach in cellular automata (Conway, Schelling, Axelrod).
 - 1.2 von Neumann or Moore neighborhoods? (Can agents “look up” or “look sideways” or “look diagonally”).
 - 1.3 Cells need not be squares, could be or triangles, or hexagons, or what? (Exposes weakness of approach, IMHO. Setup is just a trivial, easy, way to generate “neighborhoods”)
 - 1.4 Do we insist that only one agent can be “in” a cell at one time?
2. Place the agents at points in a Euclidean plane.
3. Collect the agents in an unordered “list”. (Santa FE ASM)

Do Agents Interact?

1. Do agents “find each other?” How? Do agents ask the environment “who is there?”
2. When they meet, how do agents exchange information? Agents may reveal information, but also must update own records.
3. How “aware” are agents? Are agents oblivious to each other, or to aggregates?
 - ▶ Oblivious to aggregates: “something for nothing” models: bee hives, ant trails, public opinion, Schelling segregation
 - ▶ Oblivious to other individual agents: Agents don’t interact dyadically, they only focus on aggregates. Examples: Santa Fe ASM, El Farol, “minority games”.

Why Emphasis on Interaction: Avoid “social telepathy”

- ▶ Models should avoid “social telepathy” (Erbring and Young, 1979), the assumption that agents are magically able to know about each other
- ▶ Need a plausible method through which agents “find out” information they use.
- ▶ Troublesome example: the Social Impact Model (Latane, 1996), each agent somehow knows the opinions of all the other agents in the whole society.

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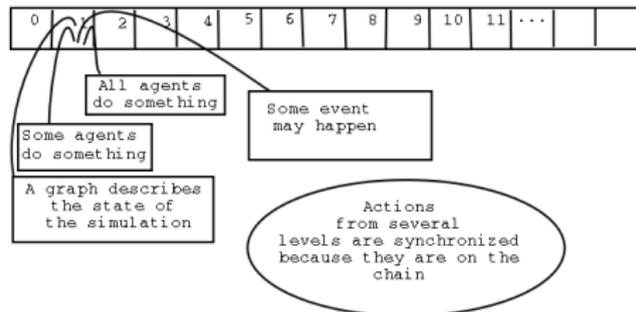
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What is time?

- ▶ Einstein quip, “The only reason for time is so that everything doesn’t happen at once.”
- ▶ In agent-based model, the separate existences of agents must be brought into an objective temporal sequence!
- ▶ In the Swarm Simulation System, the schedule is like an infinitely long conveyor belt.



Scheduling. Huh?

- ▶ Does each agent “step” every time period?
- ▶ Pick one agent at random to step every time?
- ▶ Shuffle the list of agents at each time step and step through?

Synchronous or Asynchronous Updating

- ▶ Synchronous: we believe that the world is a “frozen snapshot” in the mind of each agent when the agents are deciding what to do
 - ▶ Easier to code
 - ▶ Perhaps not so “realistic.”
- ▶ Asynchronous: re-consider information revelation assumptions.
 - ▶ Think harder on “how soon” the others will learn of an agent’s action.
- ▶ Swarm proposed an early version of “dynamic scheduling,” the agents can “add future actions” to the schedule (see Mousetrap).

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Aggregate and Individual Information

Table: Record Keeping

		Record keeper	
		internal	external
Data to be collected	individualistic	door-to-door opinion survey	involuntary DNA collection
	aggregated	unemployment rate, social entropy	Relocate individual data into a communal record system

Limited Record Collectors

- ▶ In “reality,” there is no one that automatically knows everything about us at any given instant (so far as we know ...).
- ▶ Thus OO philosophy & realism indicate we should think of record keepers as agents that gather data.
- ▶ Disadvantage: Code runs slowly! Usually faster to automatically expose and tabulate agent information.

Record-Keeper: Speed versus Verisimilitude Compromise

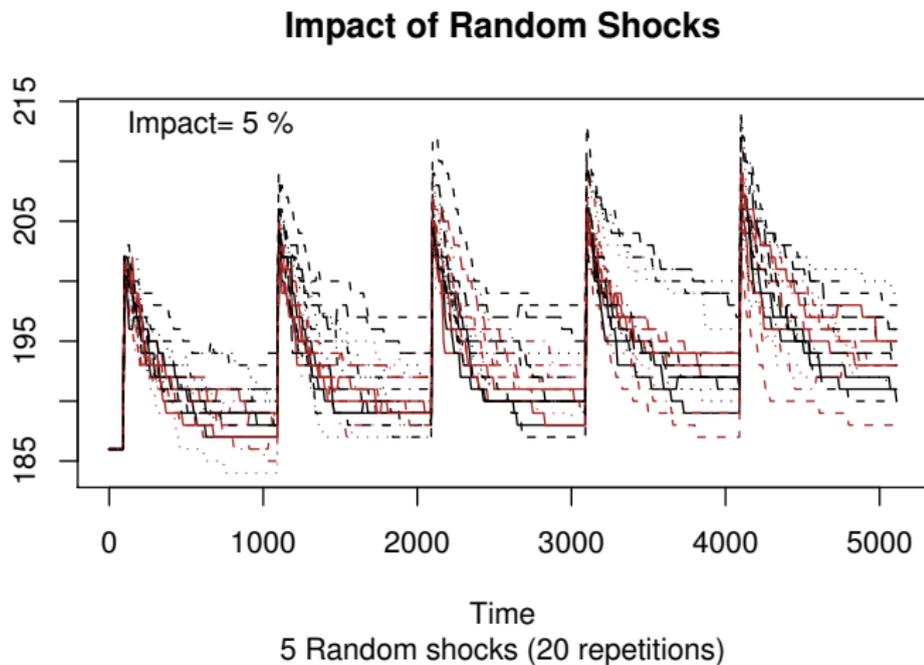
- ▶ *Political Disagreement* (Huckfeldt et al., 2004).
 - ▶ Individual agents who have their own opinions and personal records
 - ▶ To accelerate record keeping, we don't survey the agents. Agents automatically report opinion changes to a centralized, highly visible, aggregate tabulation.
- ▶ Protest “grid” evaluation (Brichoux, 2002).
 - ▶ Each agent wonders, “how many others around me are protesting?”
 - ▶ Calculations very slow if each agent conducts own survey at every time point.
 - ▶ Workable solution: an optimized, centralized record keeping system that agents
 - ▶ report their discontent to, and
 - ▶ use for reports on conditions

Serialization: The Most Fun in Record-Keeping

- ▶ Serialization: Save complete state of each agent in model.
- ▶ Re-start the simulation and subject it to
 - ▶ external shocks or
 - ▶ algorithmic variations.

Examples From Political Disagreement

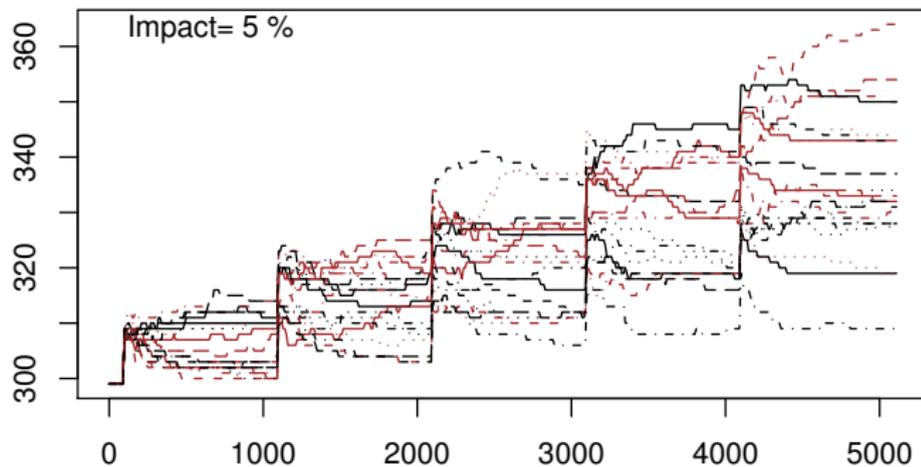
Number of agents that hold opinion 0 on issue 3



Examples From Political Disagreement

Number of agents that hold opinion 2 on issue 4

Impact of Random Shocks



Time

5 Random shocks (20 repetitions)

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Every Theory Has a Belt

Traditional philosophy of science

A theory has two parts.

1. Substantively important concepts, relationships, processes
2. *Other stuff*: a band of auxiliary decisions to
 - ▶ make the substantively important pieces fit with each other and
 - ▶ tie the substantively important pieces to empirical referents.

Auxiliary Decisions Everywhere

- ▶ Auxiliary decisions (ad hoc other stuff) are not unique to ABM
- ▶ But I believe flexibility of ABM allows for them to play a more distracting, more serious role.
- ▶ Models that pile on *other stuff* are preparing for their own rejection.

I suggest:

- ▶ strive to transfer decisions from the auxiliary belt into the core of the theory. Reduce number of “irrelevant decisions.”

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The n^m Model Problem

- ▶ Proliferation of conditional design decisions weighs us down an impossibly complicated series of model designs (1993).
- ▶ Consider one decision with 3 (apparently indifferent, equally irrelevant) design choices. Write 3 models to see if they differ.
 - ▶ If they don't differ, pick any one
 - ▶ If they do differ, then the other stuff is having an effect, but it shouldn't, and we are in real trouble.
- ▶ Confront another decision with 3 indifferent (auxiliary) specification decisions. Work out models for each of them, and see if they interact with the others. That is, work on 3^2 models.

n^m examples

Substantive purpose: study social influence.

1. Put agents one-per-cell on checker board (Latane, 1996; Axelrod, 1997a). There's a big chunk of *other stuff*.
2. Proliferation of *other stuff*
 - 2.1 Moore or von Neumann neighborhoods?
 - 2.2 Can agents "reach over immediate neighbors to interact with others?"
 - 2.3 Should we down-weight interactions with further away agents? If so, by what metric?
 - 2.4 What about the edge of the grid? Do we need a torus?
3. How do the agents learn about conditions in neighboring cells?
4. What about scheduling? Do they all decide at once against a common snapshot of society? Recall sharp exchange in asynchronous updating in the spatial prisoner's dilemma (Huberman and Glance, 1993; 

Ad Hoc Afterthought

- ▶ Its an afterthought. It is an artifact of our conceptualization of the way that agents are connected together.
- ▶ In the best case scenario: re-think our theory.
 - ▶ If the computer model's development is dominated by seemingly irrelevant decisions, perhaps the problem is in the theory itself. The theory is lacking in details, and that void is filled by computer implementation, not substantive research ideas.
- ▶ In *Political Disagreement*, we were plagued by a series of these problems because we started with “agents smeared evenly on a square grid”. The most complete explanation is, as always, in the source code itself:
<http://pj.freefaculty.org/Swarm/MySwarmCode/OpinionFormation>.

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2 Manageable Situations

1. The substance dictates that time ends after T steps.
 - ▶ American Idol model.
 - ▶ Baseball/Football game model.
 - ▶ Daily election campaign model.
2. The computer model “grinds” to a halt—a fixed pattern that does not change
 - ▶ schelling segregation
 - ▶ opinion models
 - ▶ some organizational formation models

Unmanageable Situations

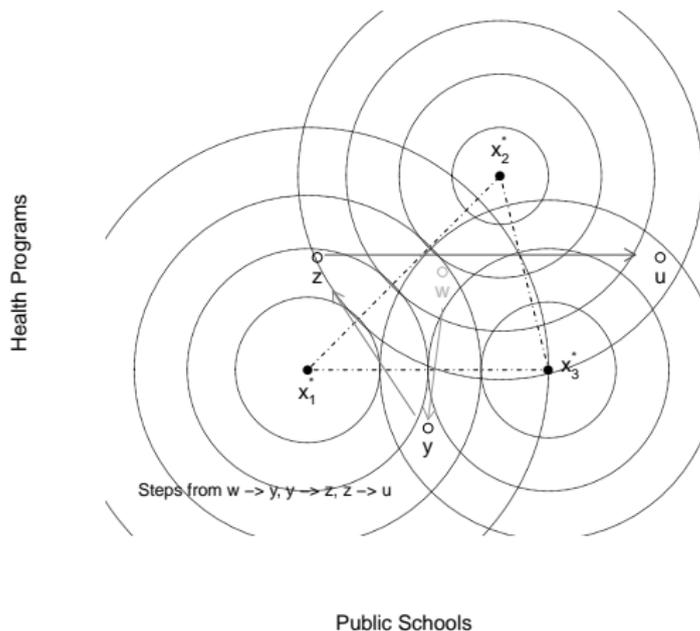
- ▶ Model has no substantive stopping point
- ▶ Agents never lock into a constant mode of behavior
- ▶ We shouldn't quit too soon. Right?

How to summarize that to the reader?

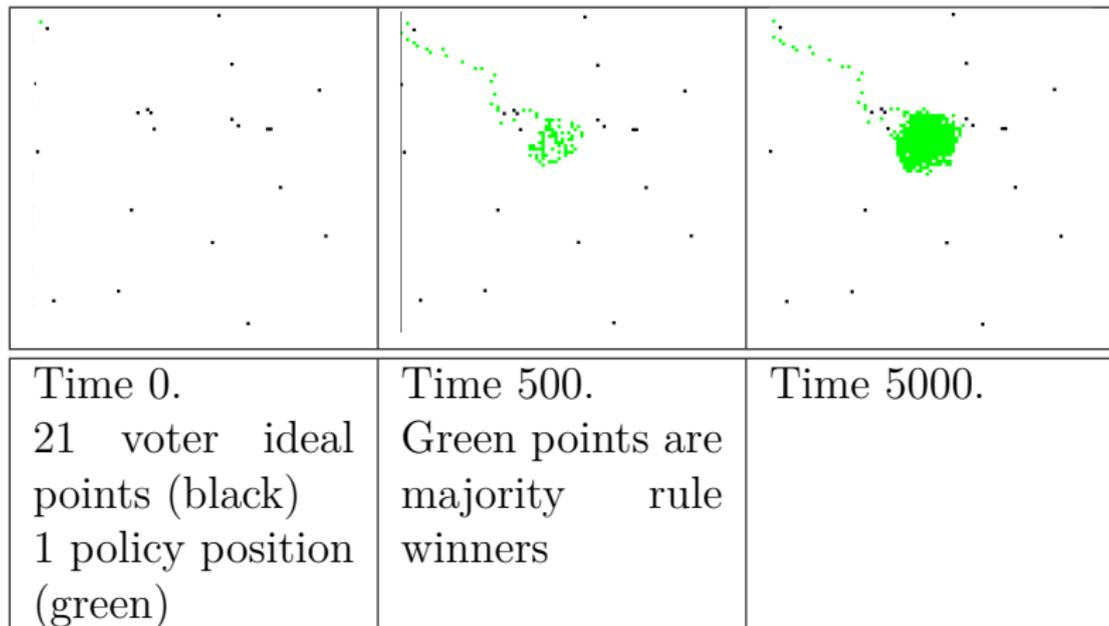
- ▶ If models stabilize to a distribution of outcomes, could treat like Bayesian MCMC simulations.
 - ▶ Bee hives.
- ▶ Some models—the really exciting ones that make us love complex systems in the first place—do not stabilize to a pattern.
 - ▶ Exciting evolution of a system toward the “edge of chaos”!
 - ▶ Constant change and adjustment among clever agents competing in a minority game or a stock market

Social Choice Sidenote

- ▶ Spatial voting model is unstable (McKelvey, 1976).



Simulation Suggest Stabilization Toward a Distribution



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What If It Grinds To a Halt?

And we don't think it ought to?

- ▶ We've faithfully coded our agents & interactions
- ▶ We expected the model would “go on forever”, but it doesn't.
- ▶ What should we do then?
- ▶ Temptation:
 - ▶ Throw in some ad hoc random error to make the model “realistic.”
 - ▶ OK, but ... lets try to find a substantive “inside the model” justification.
 - ▶ That is, don't make the auxiliary belt bigger if not necessary.

This Comes Up in Statistics.

- ▶ Data will generally reject model 1:

$$y_i = \beta_0 + \beta_1 x_i$$

- ▶ Data prefers a model that injects some *other stuff* (that is very pleasantly behaved, with an expected value of 0 and fixed variance):

$$y_i = \beta_0 + \beta_1 x_i + e_i.$$

- ▶ Some models develop increasingly fine grained assumptions about the e_i
- ▶ We want a more “substantively integrated” sort of randomness. Some interpretations of “Random effects models” have that quality
 - ▶ Individual frailty—day to day changes in taste and preference—may become a central part of our theory.
 - ▶ Contextual effects: unmeasured effects on population sub-groups (clusters) (Goldstein, 1995; Pinheiro and Bates, 2000).

Insertion of randomness in game theory

- ▶ Suppose game equilibrium analysis points to one outcome that seems unrealistic (empirically, normatively)
- ▶ Trembling hand perfect analysis. Some apparently wrong equilibrium points can be ruled out if we insist that each agent has a small chance of choosing (unintentionally) the wrong action (Selten, 1975).
- ▶ Quantal response equilibrium approach to game theory (McKelvey and Palfrey, 1988). Models imposing some intrinsic randomness are more consistent are more readily integrated with empirical data (Signorino, 1999).
- ▶ How to Substantively justify that randomness so that it can be inside our core, and not in the auxiliary belt? One answer might be incomplete information (Harsanyi).

Plausible randomness in agent-based models.

- ▶ Do you accept the “agent based complex systems” philosophy?

A complex system is a collection of autonomous agents who are only loosely linked together. These systems “appear” random because there are uncountably many unorganized interactions that may slightly affect the instance variables (“deterministic randomness”).

- ▶ Inserting agent-level detail & ad hoc randomness goes against our core philosophy (IMHO) because it obscures our view of the randomness we expect to find (via our core philosophy).
 - ▶ we are urged to “keep it simple, stupid” (Axelrod, 1997b).

How To Justify Additional Randomness

If model “grinds to halt,” then agents lack enough detail

- ▶ Problem solving by local search with simulated annealing is substantively plausible. Agents look in immediate vicinity for a solution to trouble, but may make “random leap” to some other position.
- ▶ Static theories do not translate to dynamic context unless we introduce random changes in agent taste and behavior
- ▶ Examples
 - ▶ Protest model hinges on number of continuous time points during which an agent is willing to protest, and random appearance of “other things to do” is one way to address that (Brichoux, 2002).

The Take-Away Message

- ▶ The substantive researchers who want “us” (agent-based modelers) to write agent-based models for “them” don’t understand what “we” need
- ▶ “We” can help the situation by making it clear to them that creating an agent based model is not like “running a regression”. Much more detailed information can be supplied.
- ▶ The simultaneous development of the substantive theory and the computer model is very useful and also very dangerous
 - ▶ Useful: empty spaces in theories are exposed by formalism of computer code
 - ▶ Danger: ABM coders can “throw in” standard frameworks to make model development easier, but also makes models less useful.
- ▶ I WISH we would try to enrich our theories, rather than quibble over ad hoc differences in model

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