MixedElection simulation model description

2005

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The MixedElection model is an agent-based model that represents the formation of political parties and the practice of elections in a parliamentary system. The agent-based modeling approach strives to represent the substantively significant actors as self-contained objects that gather and receive information. In this case, each citizen is represented by an object that is faced with decisions about which party to join and how to vote in an election that offers a choice of parties in a proportional election as well as a choice of individual candidates in a single-member district election. There are also objects that represent the district-level electioneers who keep the rolls and collect votes from the citizens, political parties that nominate candidates and allocate them across districts, and the parliament itself, which receives election returns from the electioneers and decides how many seats are allocated to each party.

The code for the MixedElection model uses the libraries provided by the Swarm Simulation System (http://www.swarm.org), a free and open programming toolkit which is available under the GNU Greater Public License (this model requires Swarm version 2.150 or better).

In this writeup, I'm referring to objects by "human style" index numbers, $1, 2, \ldots, N$. In the code, and on-screen presentations, it uses the C style index that begins at 0 and counts up to $N - 1$. This creates presentational hassles with graphs that we should ignore as long as we possibly can.

Districts

Districts are numbered $k = 1, \ldots, K$.

One "electioneer" is created for each district. That agent:

1. keeps a record of all voters who exist in the district
2. notifies voters that it is time to register for political parties
3. collects party registrations, maintains list of voters registered with each party
4. notifies voters of the list of SMD candidates for the district
5. notifies voters when it is time to vote (simultaneously in PR and SMD election)
6. collects SMD and PR votes from voters (voters retrieve party information from national source)

Voters

There are N_k voters in district k. Suppose for simplicity at the outset that $N_k = N \forall k \in \{1, \ldots K\}$, but this can be easily generalized.

The voter has 2 decisions to make in each election. One vote is cast for a party in the PR contest, the other for an SMD candidate.

Euclidean Preferences.

The policy space is one dimensional, $X \in [0, 100]$. This assumption can be (relatively easily) relaxed with some recoding.

The voter's preferences are represented by utility function $U_i = (x - x_i^*)^2$. When offered a choice, the voter always chooses the alternative closest to its personal ideal point, x_i^*.

The ideal points of voters in district k are drawn from a distribution:

$$x_i^* \sim Beta(\alpha_k, \beta_k)$$

Voters in district k are drawn from a Beta distribution that is characterized by 2 district-level parameters, (α_k, β_k). In the range of parameters that is employed, the Beta distribution is unimodal. As illustrated in Figure 1, this allows one to assign ideal point distributions that are centered and symmetric, or leaning to one edge and asymmetric. Detailed comments on the design of the voter ideal point distribution is in Appendix 1.

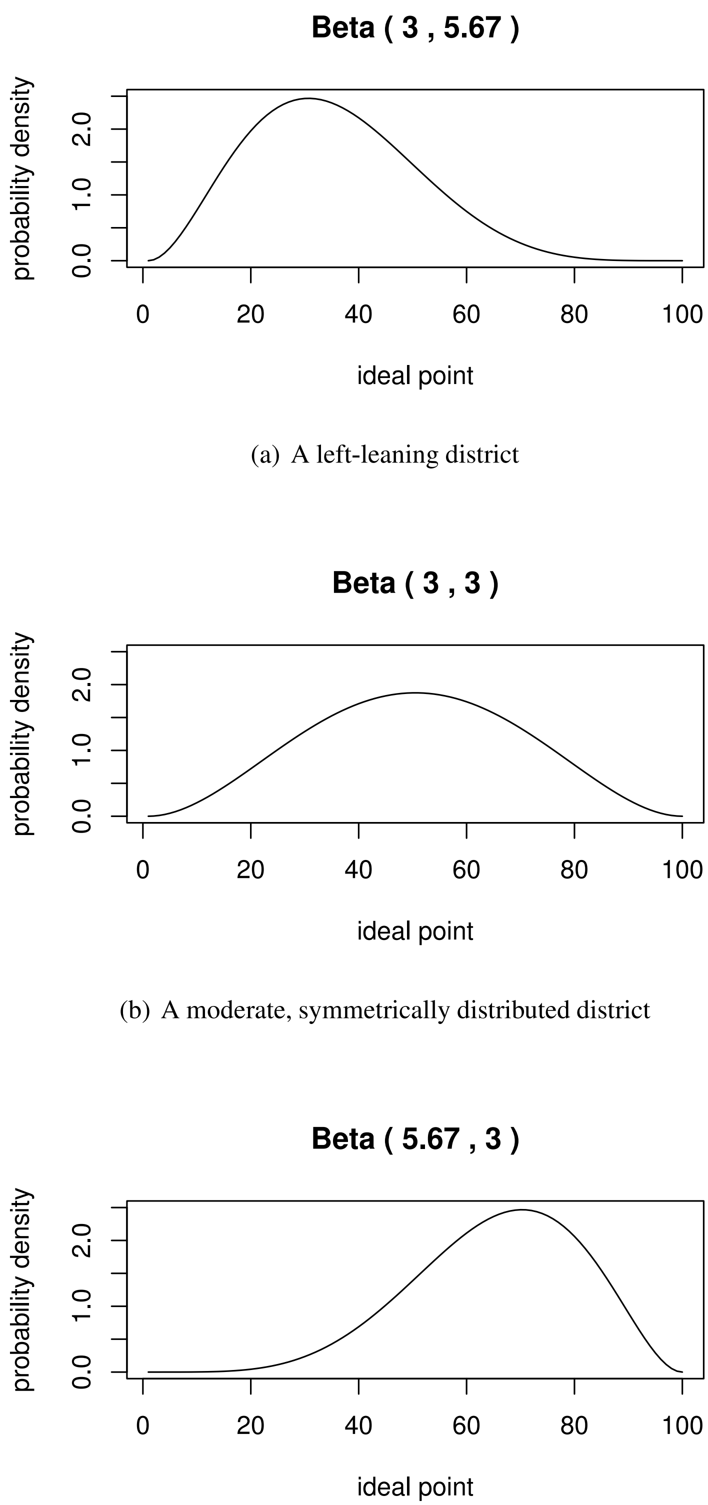
The district level parameters are chosen so that districts may differ from each other in systematic ways. We parametrize districts by their modes, the "peak" values in their probability distributions. The implementation requires some "fancy footwork" to make sure the distributions maintain the correct properties–bounded by 0 and 100, reflective of our "substantive" motivation of having districts that can range from far left leaning to far right leaning.

Partisan Types.

There are 3 types of voters

1. Pure Partisan: (candidate vote follows party choice)
   (a) voter chooses party that has position closest to its ideal, then also votes for that party's candidate in the SMD competition.
   (b) If the party does not offer a candidate in the SMD competition, the voter will choose among SMD candidates by Euclidean distance.
2. Independent Partisan (choose party and candidate separately)
   (a) vote for closest party in PR
   (b) vote for most attractive candidate (regardless of party) in SMD
3. Reverse Partisan (party follows candidate vote)

Figure 1: Ideal Point Distributions



(a) A left-leaning district

(b) A moderate, symmetrically distributed district

(c) A right-leaning district

(a) vote for favorite candidate in SMD

(b) vote for favorite candidate's party in PR

The composition of the electorate is determined at run-time. It is important to note that voters of partisan type 1 and 3 are observationally equivalent–they never split-tickets. Only partisan type 2 voters can offer behavioral evidence of their type in the form of a split ticket vote.

Parties

At the outset, there is an exogenously given number of political parties, indexed by $1, \ldots, P$. The model has been recently redesigned so that the graphical interface can be used to create new parties at indicated positions, kill off particular parties, or merge 2 parties.

The intial parties are "bootstrapped" according to the following algorithm. This is intended to represent the idea that political organizations are founded by people who have certain organizational qualities and entrepreneurial ambitions that are distributed in a way that is uncorrelated with political ideology. That is, people of any ideological bent might attempt to found a political party.

1. Randomly select (equal likelihood) P voters from the entire nation. The ideal points of those voters serve as the "founding" policy positions of the political parties.

2. The position of each party is broadcast to the voters, who then choose their favorite political party and they register (in their districts) with the party of their choice.

3. Each party "collects up" its membership rolls from the district electioneers.

4. Each party is able to afford to run a certain number (see below in point 6) of candidates in the SMD contests. Suppose that number is $S_p$for party $p \in \{1, \ldots, P\}$. The party prioritizes among districts when allocating candidates. The districts are ranked according to the number of registered party members, and the S_p districts with the highest number of members are expected to offer candidates.

5. For each district in which a party intends to run an SMD candidate, the party selects a candidate whose policy offering is the median of registered party members in that district.

6. The number of candidates that a party, S_p, is allowed to enter in SMD contests reflects the mass membership of the parties. The following procedure is used.

   (a) Calculate the smallest number of candidates that a party p can be allowed to run:

   $$minCandidates_p = N * \frac{membership\,of\,party\,p}{membership\,of\,largest\,party}$$

   (b) Select the number of candidates allowed to p by choosing a random integer in $\{minCandidates_p, \ldots, numberOfDistricts\}$.

The interpretation is that the largest party always gets to run candidates in all districts. The other parties are allowed fewer candidates, but the number is a variable between two extremes. This generates some potentially "unwanted randomness" in the results that will frustrate replication and the focus on other parameters. To make that unwanted randomness disappear, some changes in the code can be contemplated.

Parliament

There is a parliament which has the duty of calculating the allocation of seats in the PR phase of the electoral process. The parliament object polls the electioneers in the districts to collect both the distribution of PR votes among parties and the SMD winners and the SMD vote distributions.

The PR calculation follows this algorithm.

1. Calculate threshold, exclude votes for parties which fall below the threshold
2. Calculate the PR seat quota on the basis of the valid votes
3. Apply the greatest remainder algorithm

Output

In September 2004, the model was re-designed to facilitate batch runs and data collection. This followed the model design paradigm recently advocated in the updated Swarm tutorial exercises called *simpleBatchBug*. The output class now drives the on-screen display when the model is run interactively. It also controls the data output process for models that are run in batch mode, non interactively.

For each "run" of the model–a startup according to a given set of parameters–the model will re-run one election for each recorded "time." If the party positions are kept fixed, this amounts to replication of the same election with randomly changing voter ideal points. If party positions adapt, then it is a dynamical process and each run represents a unique trajectory.

At the current time, the model retains 3 kinds of information.

1. National level output, collects information on representation in the 2 systems. These are the number of effective parties and an index of proportionality (which we call LS, although I can't remember why!)

   run, time, effPRParties, effSMDParties prls smdls

2. Party level output, which collects these variables:

   Run Time j Party Founder Members Median PRVotes PRSeats SMDCandid SMDVotes SMDSeats

3. District level output, which collects these variables:

   Run Time District Party SMDWinner PRWinner PRVotes PrtyPol SMDVotes SMDPolRun Time District Party SMDWinner PRWinner PRVotes PrtyPol SMDVotes SMDPol

Appendix 1: Beta Distribution for ideal points.

The ideal points for voters in district k are drawn from a distribution $Beta(\alpha_k, \beta_k)$. Values from the $Beta$ distribution lie within an interval, and they depend on two parameters, α and β.

The standard $Beta$ distribution gives the probability density of a value x on the interval (0,1):

$$Beta(\alpha,\beta):\ prob(x|\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \quad (1)$$

where B is the beta function

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt$$

If $\alpha > 1$and $\beta > 1$, the mode of the $Beta$ distribution is

$$mode = \gamma = \frac{\alpha-1}{\alpha+\beta-2} \quad (2)$$

If α or $\beta < 1$, the mode may be at an edge, and we ignore those cases.

The expected value of a variable that is Beta distributed is:

$$E(x) = \mu = \frac{\alpha}{\alpha+\beta} \quad (3)$$

with variance

$$variance = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = mean \cdot \frac{\beta}{(\alpha+\beta)(\alpha+\beta+1)} \quad (4)$$

$$skewness = \frac{2(\beta-\alpha)\sqrt{1+\alpha+\beta}}{\sqrt{\alpha+\beta}(2+\alpha+\beta)} \quad (5)$$

$$kurtosis = \frac{6[\alpha^3+\alpha^2(1-2\beta)+\beta^2(1+\beta)-2\alpha\beta(2+\beta)]}{\alpha\beta(\alpha+\beta+2)(\alpha+\beta+3)}$$

Fiddle the mode

Our goal is to choose values of α_k and β_k for each district so that there is meaningful variation across districts in the types of voters. In the pictures displaying the Beta density, one's eye is drawn to the peak of the frequency distribution, which is the mode. We want it to range between $[0,1]$, but probably only in a much smaller interval, say $[.3, .7]$.

The strategy created for this project is as follows. For a given district, we set the mode, γ, and then figure out which Beta parameters α_k and β_k that correspond with that mode.

Here's a simple starting point: Suppose the mode is .50. That is the same as the mean (its symmetric), and the mode formula (2) implies:

$$.50 = \frac{\alpha-1}{\alpha+\beta-2}$$

and

$$.50\alpha + .50\beta - 1 = \alpha - 1$$

$$.5\alpha = .5\beta$$

$$\alpha = \beta$$

If one wants the mode to be in the middle, one can choose any value for α, as long as one chooses the same value for β. (Whew! What a relief. This exactly matched my intuition.)

If the mode is in the center, we know α and β are equal, but we don't know their values. The selection, it turns out, depends on how much diversity there is within a district. If one wants a distribution to have voter ideal points "tightly bunched" around the mode, then one should choose a large value for α, say 10.0,

$$variance\, of\, Beta(10, 10) = 0.01190$$

In contrast, if $\alpha = 1.5$, the variance is much greater:

$$variance\, of\, Beta(1.5, 1.5) = 0.0625$$

Seen in this light, the parameter α is a "within district homogeneity indicator." As α gets bigger, the distribution of ideal points collapses around the mode.

Although this particular calcuation works only for a mode in the center, it does outline the process that we can use to assign α and β for all other values of the mode.

Suppose the mode is .4. From equation 2

$$.40 = \frac{\alpha - 1}{\alpha + \beta - 2}$$

$$.40\alpha + .40\beta - 0.8 = \alpha - 1$$

$$.4\beta = .6\alpha - 0.2$$

$$\beta = \frac{3}{2}\alpha - \frac{1}{2}$$

or

$$.60\alpha = .2 + .40\beta$$

$$\alpha = \frac{1}{3} + \frac{2}{3}\beta$$

It is quite possible to calculate one parameter as a function of another, after specifying the mode, even if the mode is off center.

Generally speaking, for any value of the mode, $\gamma \in (0,1)$ (keeping in mind the original stipulation that $\alpha, \beta > 1$):

$$\gamma = \frac{\alpha - 1}{\alpha + \beta - 2}$$

$$\gamma\alpha + \gamma\beta - 2\gamma = \alpha - 1$$

$$(1-\gamma)\alpha = \gamma\beta - 2\gamma + 1$$

$$\alpha = \frac{\gamma\beta - 2\gamma + 1}{(1-\gamma)} = \frac{\beta - 2 + \frac{1}{\gamma}}{(\frac{1}{\gamma} - 1)} = \frac{\gamma}{1-\gamma}\beta - \frac{2\gamma+1}{1-\gamma} \tag{6}$$

So α is a linear function of β.

And

$$\gamma\beta = \alpha - 1 - \gamma\alpha + 2\gamma$$

$$\beta = \frac{\alpha - \gamma\alpha + 2\gamma - 1}{\gamma} = \frac{\alpha - \gamma(\alpha - 2) - 1}{\gamma} = \frac{(1-\gamma)}{\gamma}\alpha - \frac{1-2\gamma}{\gamma} \tag{7}$$

This indicates that, given either α or β, along with the mode, on can then calculate the missing parameter (β or α, as the case may be).

Note the Symmetry of the problem

In our modeling exercise, we want to maintain a certain amount of symmetry. It is desirable that a distribution of ideal points with a mode of .7 should be a mirror image of a distribution that has a mode of .3, assuming the within-district homogeneity parameter is kept constant. It turns out that we can easily obtain that sort of homogeneity from the $Beta(\alpha, \beta)$ distribution.

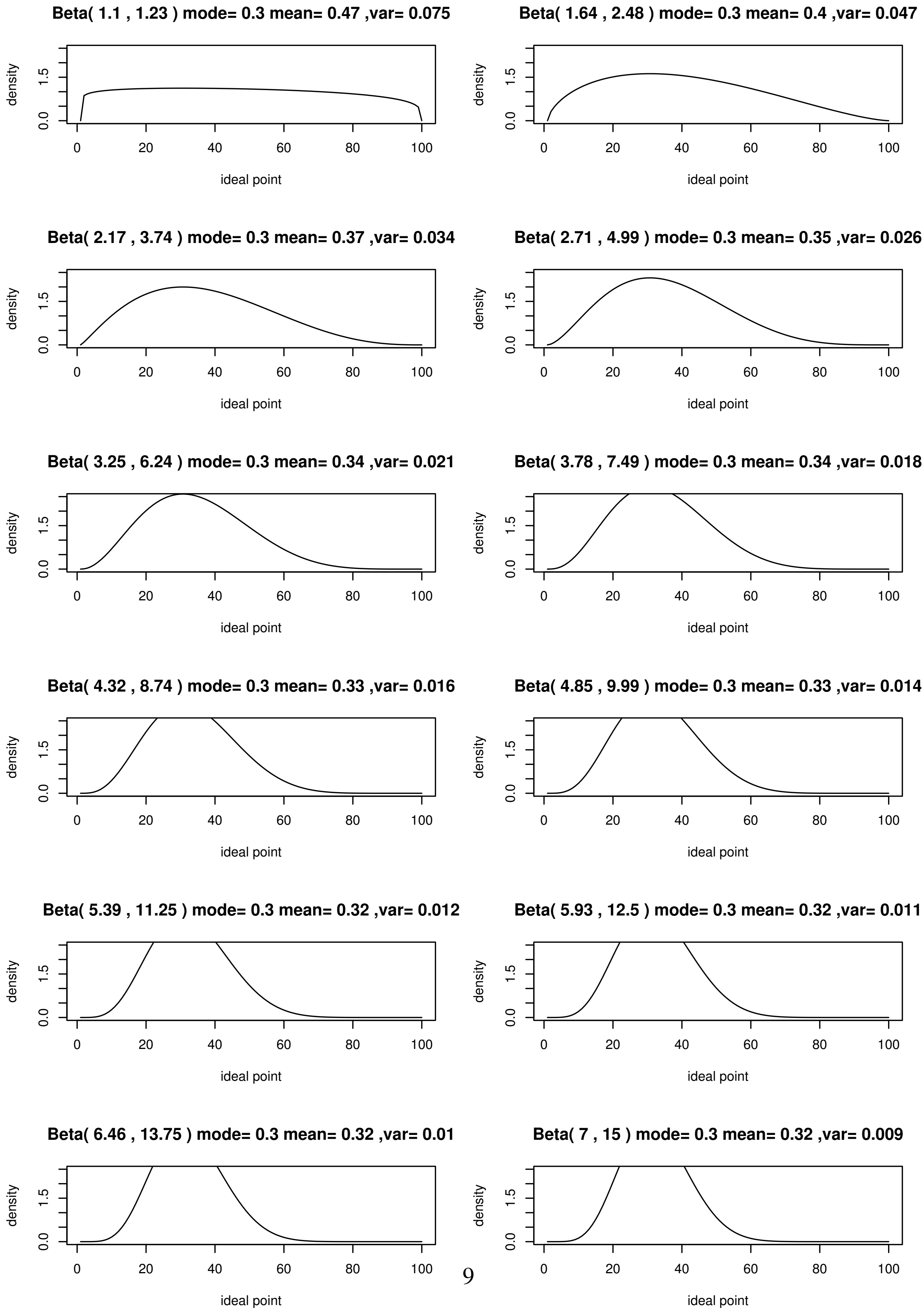
Consider the density formula 1. If two distributions, $Beta(\alpha_1, \beta_1)$ and $Beta(\alpha_2, \beta_2)$ are to be mirror images, it is necessary that $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

If the mode of $Beta(\alpha, \beta)$ is left-of-center, then that means that $\alpha < \beta$.

If the mode is right-of-center, then $\alpha > \beta$.

The smaller parameter, whether it is α or β, is the homogeneity parameter, which we can treat as an input variable in the model. The other parameter, the larger one, is given by the formula in either 6 or 7.

Modeling Strategy



The preceeding suggests a method to create groups of voters for the districts.

First, assign the district ideology for each district. The district ideology is represented by the position of the mode, the most widely held political position. The modes for the districts are,

$$\gamma = (\gamma_1, \gamma_2, ..., \gamma_K)$$

The mode for district k, γ_k , can be any value in the interval $(0, 1)$. The problem is to choose α_k and β_k for each district so that $Beta(\alpha_k, \beta_k)$ has a peak at γ_k.

Second, consider variety across districts. There should be variations across districts in ideological tendencies. The precise model that is used to set these values has not yet been decided. We can, for example, suppose that there is a uniform distribution, $\gamma_k \sim Uniform[min, max]$. Or we can suppose that γ_k follows any kind of discrete (say, multinomial) or continuous (Normal) distribution). The computer model is set up so that various conjectures can be considered. After setting the values, one can calculate diagnostics, such as $Variance(\gamma_k)$.

Through whatever mechanism, suppose district ideologies (modes) are assigned. For simplicity of graphical display, we sort the districts from "left to right" across the spectrum, so the left leaning ones are at the beginning of the array and the right leaning ones are at the end: $\gamma_k < \gamma_{k+1}$.

Third, consider variety within districts. It is necessary to select the homogeneity parameter, η. This number which will play the role of α_k in districts on the left side of the ideological spectrum or β_k in districts on the right.

After the vector γ is assigned, and the homogeneity parameter is assigned, then we proceed through the districts, setting the parameters in $Beta(\alpha_k \beta_k)$ that are consistent with $\gamma_k$and η. The graphical interface can display the distribution of voter ideal points for the entire country or one district at a time.