

Paul Johnson <pauljohn@ku.edu>

Sept 12, 2005

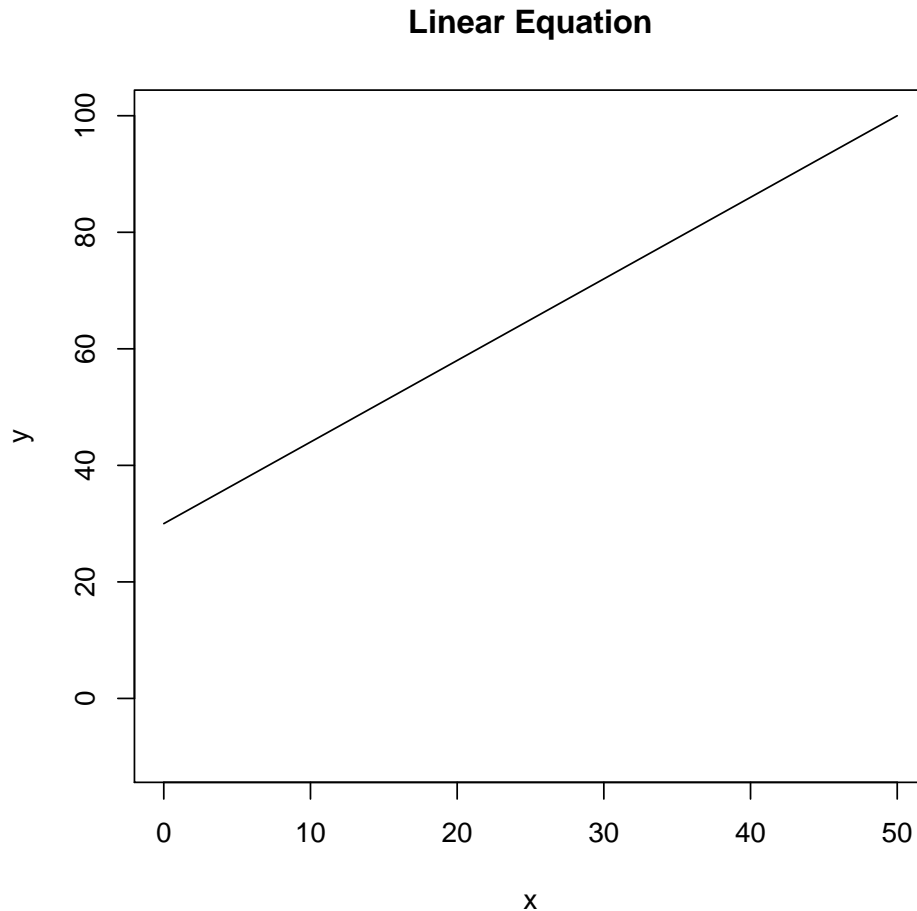
Plotting Curves

The linear equation is the basis of much social science research. In the following, the “intercept” (or “constant”) is 30 and the slope is 1.4.

$$y = 30 + 1.4 \cdot x$$

Note how the slope does not change as x moves from left to right.

```
> x <- seq(from = 0, to = 50, length.out = 200)
> y <- 30 + 1.4 * x
> plot(x, y, main = "Linear Equation", type = "l", xlim = c(0,
+ 50), ylim = c(-10, 100))
```

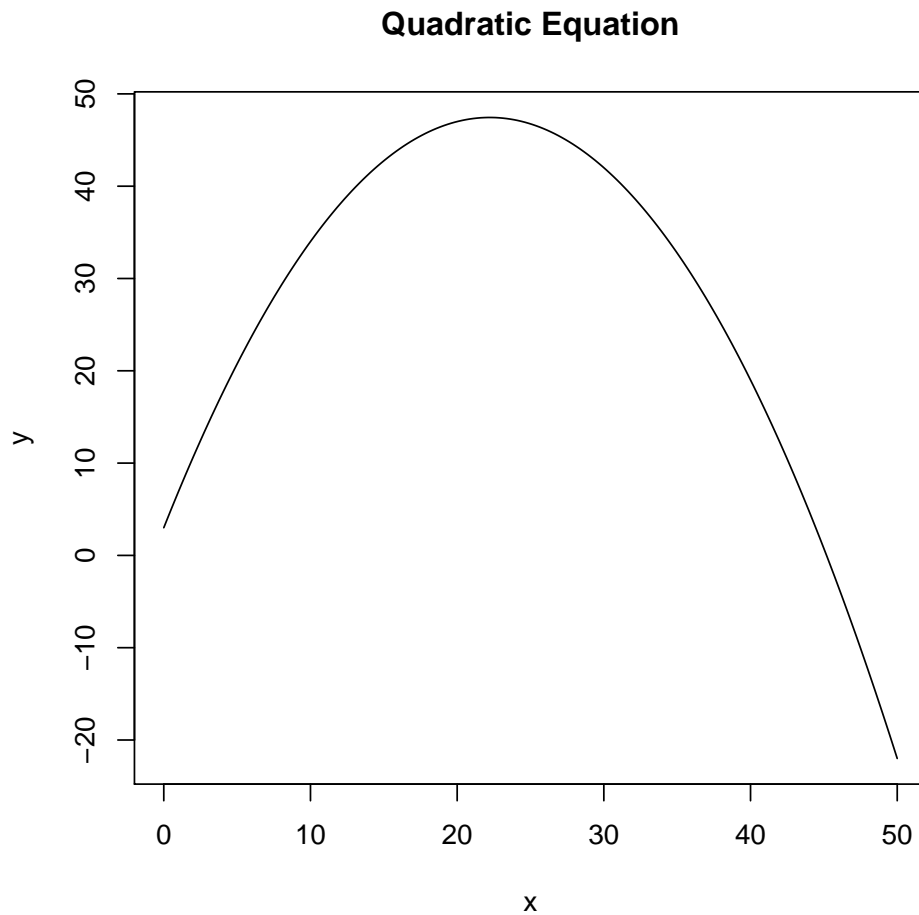


Sometimes people think there is “nonlinearity” and one of the simplest (short-sighted, silly, etc) is to throw in a squared x .

$$y = 3 + 4 \cdot x - 0.09 \cdot x^2$$

That is called a “quadratic” equation. The plot illustrating that curve is either a “hill” or a “bowl,” depending on whether the coefficient on the squared term is negative (hill) or positive (bowl). If you add in many more terms, say x^3 and x^4 , then you have a polynomial.

```
> y <- 3 + 4 * x - 0.09 * x * x
> plot(x, y, main = "Quadratic Equation", type = "l", xlim = c(0,
+ 50))
```

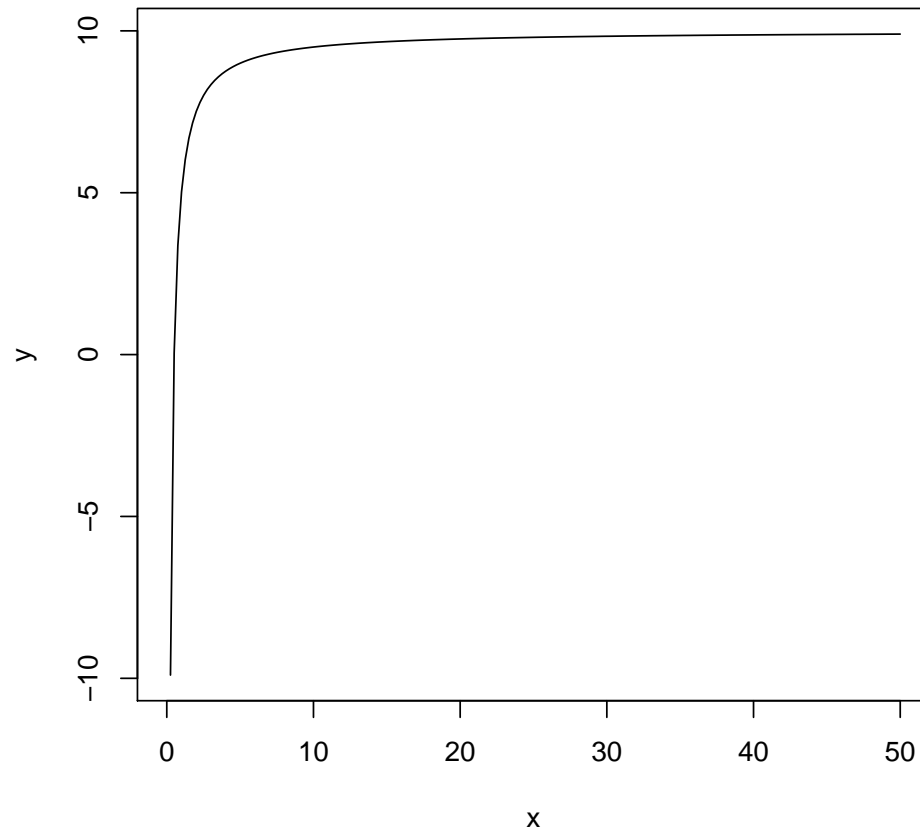


The reciprocal of x is $1/x$. Sometimes it can be used to represent an interesting relationship:

$$y = 10 + 5\frac{1}{x}$$

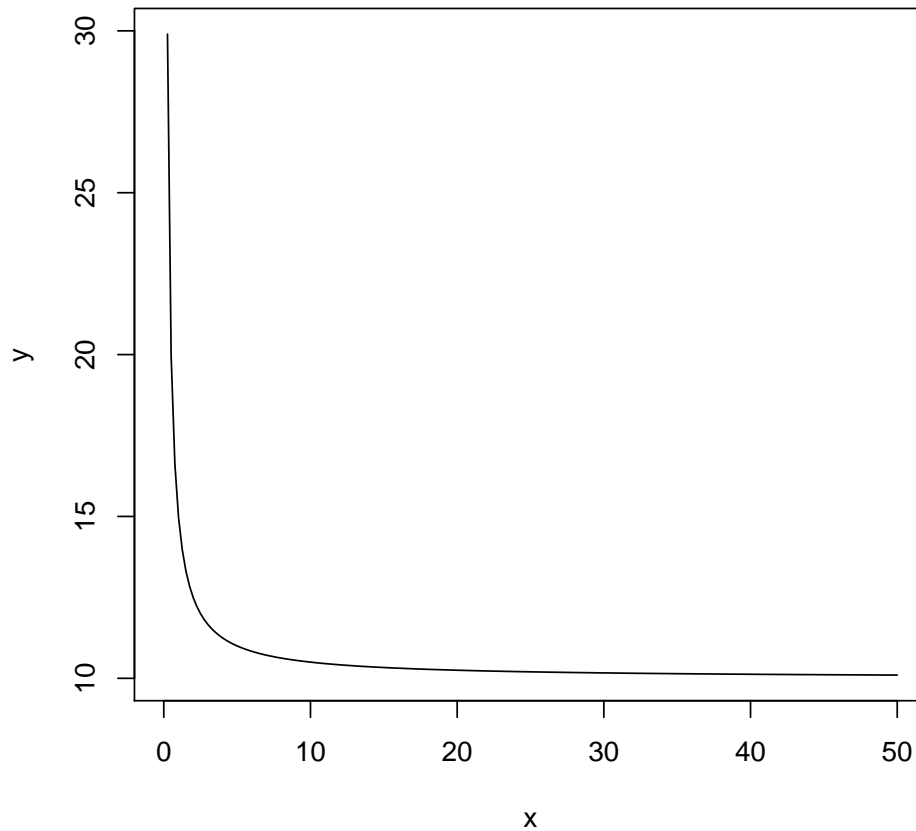
```
> y <- 10 - 5 * (1/x)
> plot(x, y, main = "Reciprocal with negative coefficient", type = "l")
```

Reciprocal with negative coefficient



```
> y <- 10 + 5 * (1/x)
> plot(x, y, main = "Reciprocal with positive coefficient", type = "l")
```

Reciprocal with positive coefficient



Please recall that

$$x^{-1} = \frac{1}{x}$$

and, more generally,

$$x^{-\alpha} = \frac{1}{x^{\alpha}}$$

so the reciprocal relationship is really just a variant on the polynomial.

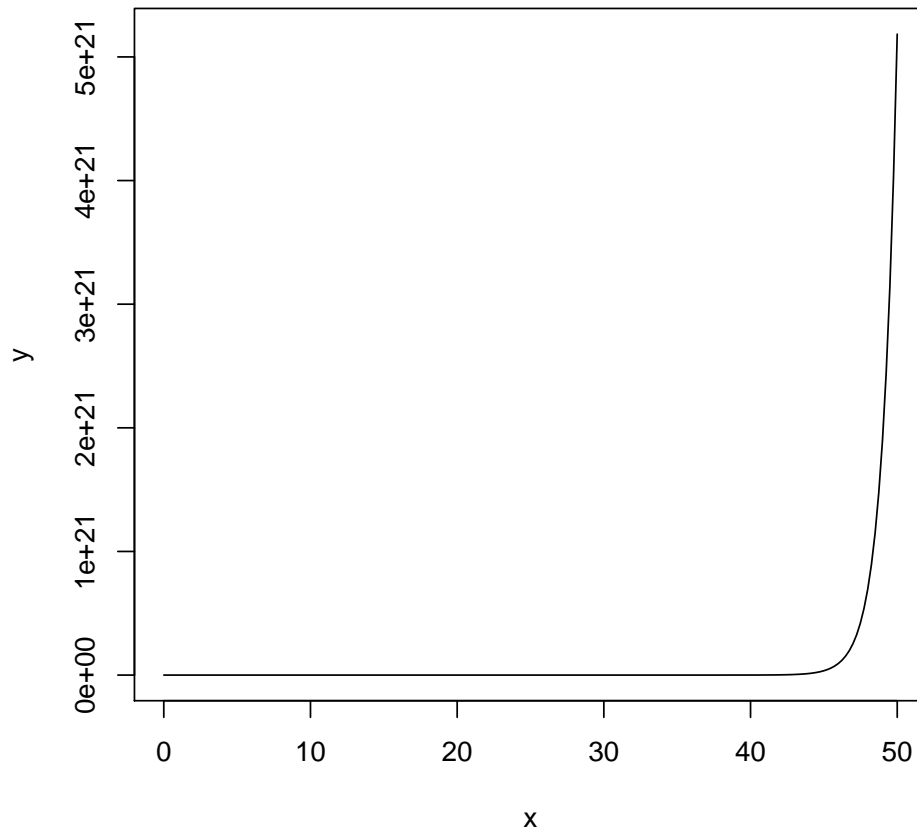
It seems to me that about one-half of the important things in statistics involve the exponential function. Euler's constant, e , is a constant similar to π (pi). The value of e is approximately 2.7182818 (but it goes on forever, like π).

The following notation is often used because it is easier to print.

$$e^x = \exp(x)$$

```
> y <- exp(x)
> plot(x, y, main = "Exponential of x", type = "l")
```

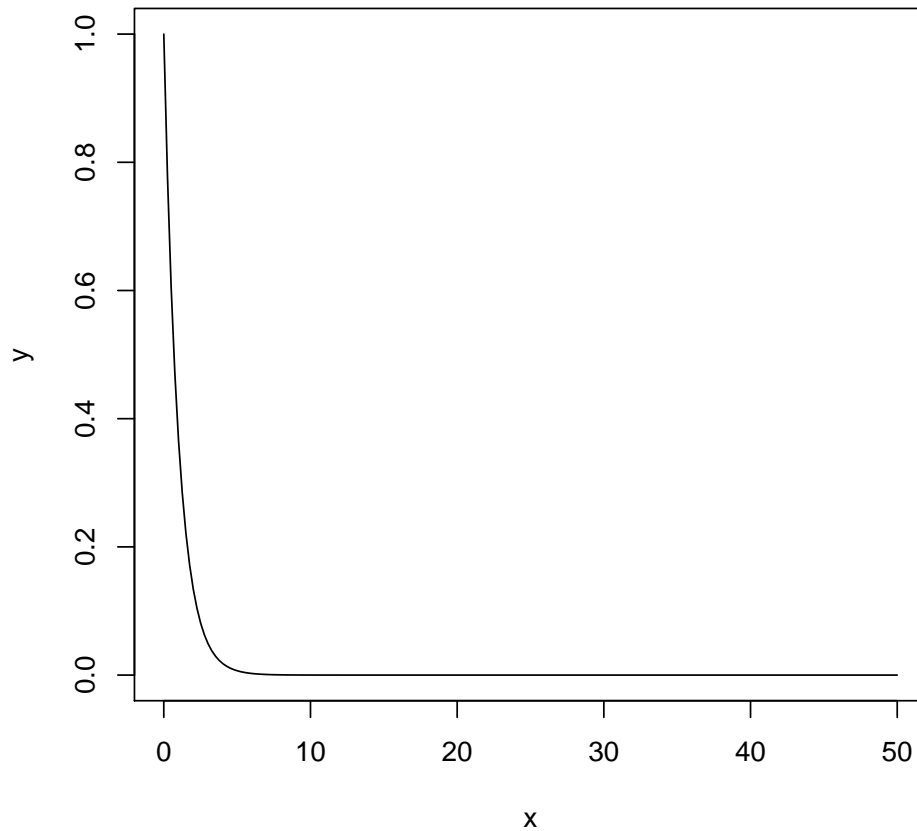
Exponential of x



Recalling the facts about negative exponents, you can figure out what it means to write $\exp(-x)$.

```
> y <- exp(-x)
> plot(x, y, main = "Exponential of -x", type = "l")
```

Exponential of $-x$



Here's an interesting thing. If you study this, you will very easily understand the Normal Distribution. Consider the squared difference between x and some "target value," which I have set at 24.

$$(x - 24)^2$$

Put that into an exponential like this

$$e^{-\frac{(x-24)^2}{100}}$$

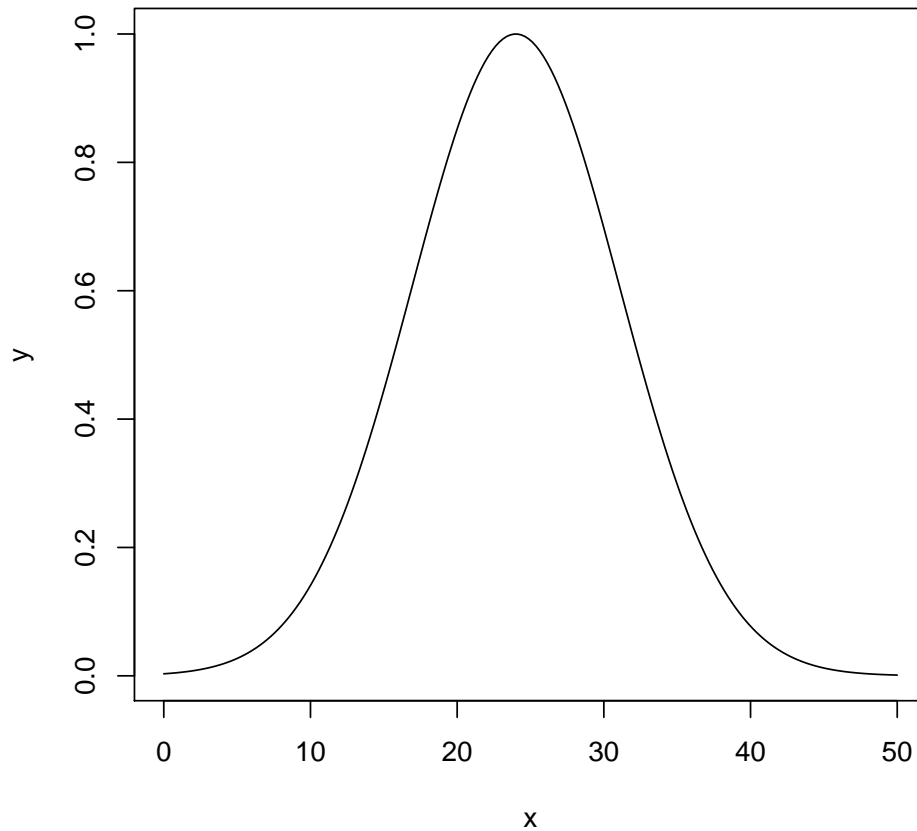
Recall, that is the same as

$$\exp(-0.01 \cdot (x - 24)^2) = \frac{1}{e^{-0.01(x-24)^2}}$$

I am using 0.01 in these expressions to "scale" the result so the values stay in a nice looking part of the figure.

```
> y <- exp(-(1/100) * (x - 24)^2)
> plot(x, y, main = "Exponential of - (0.01(x-24)^2)", type = "l")
```

Exponential of $-(0.01(x-24)^2)$



The logarithm may be familiar to you from high school. The notation

$$y = \log_b(x)$$

means that we want a number y such that $b^y = x$. Logarithms have several magical properties, such as

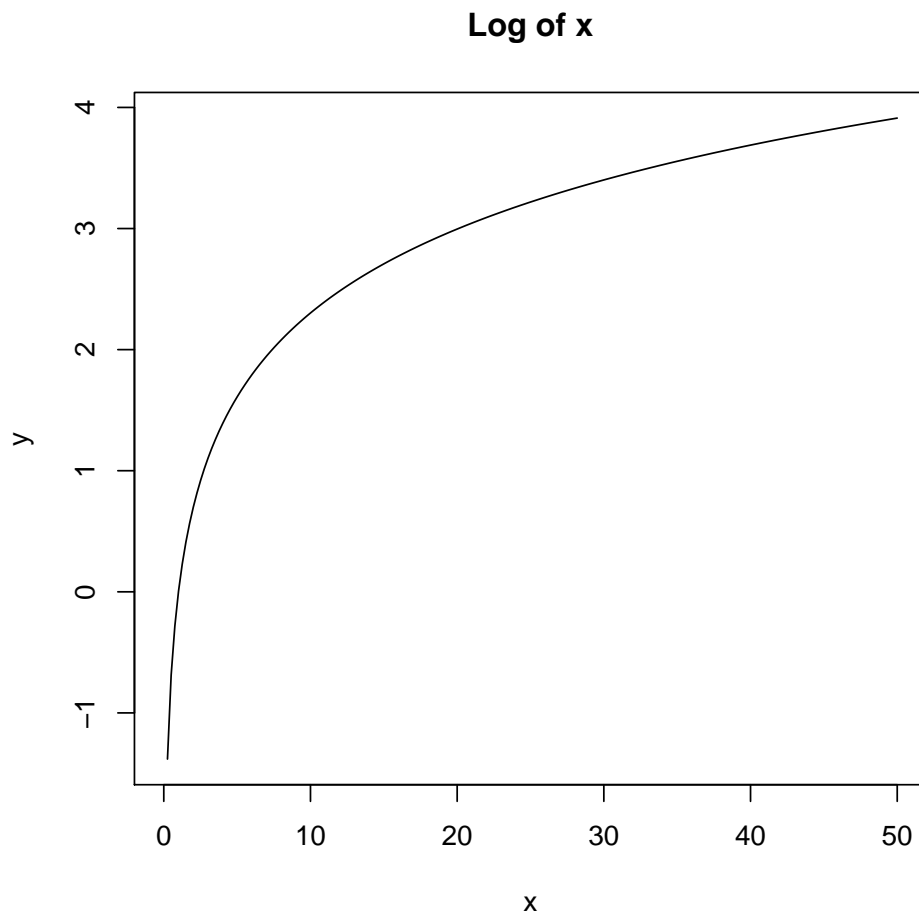
$$\log_b(z * x) = \log_b(z) + \log_b(x)$$

and

$$\log_b(x^z) = z \cdot \log_b(x)$$

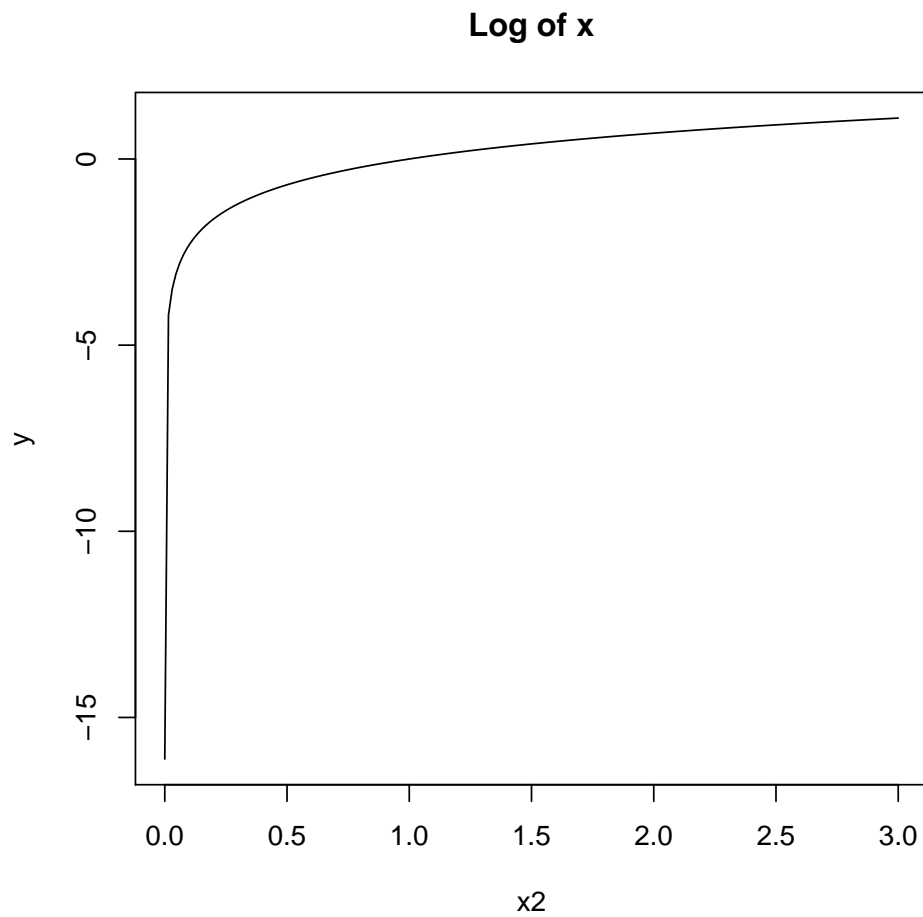
Often, e is used as the base of logarithms. It is used so often that a $\log_e(x)$ is usually represented by the simpler symbol $\ln(x)$. Euler's constant is used so often, in fact, that R's log function automatically uses Euler's constant as the base. The following illustrates that.

```
> y <- log(x)
> plot(x, y, main = "Log of x", type = "l")
```



Focus on the relationship “in the small.” Recall that $\log(0)$ is undefined, no matter what the base is.

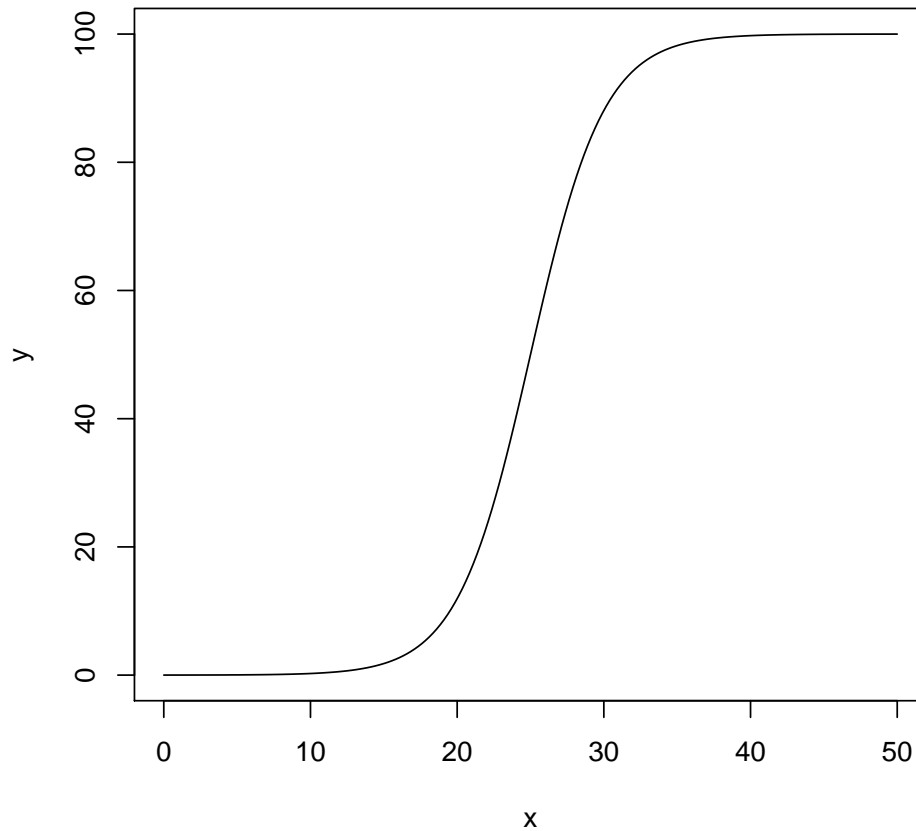
```
> x2 <- seq(1e-07, 3, length.out = 200)
> y <- log(x2)
> plot(x2, y, main = "Log of x", type = "l", xlim = c(0, 3))
```



If y has to be constrained (say, between 0 and 100 for percents) there are many possibilities .

```
> z <- -10 + 0.4 * x
> y <- 100 * exp(z)/(1 + exp(z))
> plot(x, y, main = "S-shaped curve from the logistic", type = "l")
```

S-shaped curve from the logistic



Look in R's stats package for functions that start with SS. These are standard nonlinear functions that can be put to use in making nonlinear regression models. The SSlogis function has this formula

$$y = \frac{Asym}{1 + \exp((xmid - input)/scale)}$$

$$y = Asym / (1 + \exp((xmid - input) / scal))$$

You should see my example has a simple version of this.