May the Best Man Lose

This month's presidential election highlights an ugly truth about American politics: The most popular candidates, like John McCain, often don't get elected. The problem, mathematicians say, lies in our voting system itself.

By Dana Mackenzie

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Remember John McCain? Oh, sure you do now, but will you remember him 20 years from now? If history is any guide, McCain's 2000 campaign will end up on the curio shelf next to John Anderson's presidential run in 1980 or Paul Simon's campaign in 1988. And yet, as Al Gore and George W. Bush vie for the presidency this month, it's easy to imagine a very different election— one that might better reflect the true wishes of the people.As late as February of this year, McCain's "Straight Talk Express" was on a roll. He won the New Hampshire primary, hit a bump in South Carolina, then won a highly publicized Michigan primary, in spite of the Michigan governor's promise to deliver the state to George W. Bush. The news media loved McCain, and so did the voters. In a February 7 CNN/*USA Today/*Gallup poll, more Americans thought favorably of him than of any other candidate. When the poll was repeated on February 28, his lead had widened: 66 percent had a favorable opinion of him compared with 59 percent for Al Gore, 57 percent for George W. Bush, and 54 percent for Bill Bradley. Yet nine days later, McCain was out of the race.

What happened?

The short answer is Super Tuesday. McCain was wiped out in the delegate-rich states of California and New York, while winning only a few small states in New England. The loss in California, a winner-take-all primary, hurt most. An exit poll by the *Sacramento Bee* suggested that McCain had been too critical of right-wing religious leaders. The poll also showed that voters believed Bush was more likely to beat Gore in November than McCain was. Yet the same voters, in the same exit poll, said they would vote for Gore over Bush (51 to 43 percent), and they would vote for McCain over Gore (48 to 43 percent).

While campaign strategists and political pundits comb through the wreckage to see where McCain went wrong, ordinary voters would do well to raise a different set of questions. All the signs suggest that the most popular candidate— a candidate who could have drawn voters from both parties— is not on the ballot this November. If a candidate like McCain can't win, is something wrong with our election system?

The answer, say voting theorists Donald Saari of the University of California at Irvine and Steven Brams of New York University, is a resounding yes. America's presidential election system is fundamentally flawed, they argue, because both the primaries and the election are based on the plurality vote. McCain is just the most recent candidate done in by the paradoxes of election mathematics.

In a democratic election between two candidates, the winner is the person with the majority of the votes. But when three or more candidates run, things are seldom so simple. The winner often amasses only a plurality, not a majority, of the votes. (Bill Clinton, for example, won the presidency with 43 percent of the vote; Jesse Ventura won the Minnesota governorship with 37 percent.) The plurality winner could be everybody else's least favorite candidate and could even lose to each of the other candidates in a head-to-head battle. As Saari puts it: "The plurality vote is the only procedure that will elect someone who's despised by almost two thirds of the voters."

Voting theorists have recognized the weakness of the plurality system for centuries. "The apparent will of the plurality may in fact be the complete opposite of their true will," wrote the Marquis de Condorcet, a close friend of Thomas Jefferson and author of a proposed French constitution, in 1793. That is why runoff elections are often used in races that are expected to draw a lot of candidates.

What students don't learn in high-school civics class, and certainly won't hear from the two major parties, is that there are many voting systems besides the plurality vote and the runoff election. Some systems are even older than that of the United States. In recent years, there has been a steady increase of research on voting theory (also called "consensus theory" or "social choice theory"). For the first time ever, Saari says, "We have the tools to systematically analyze all the procedures."

So what is the best procedure?

That's the catch. Saari favors a method called the Borda count; Brams advocates a method called approval voting. Many would argue that there can be no definitive answer until other systems are tested. Alas, consensus is as elusive as ever— even on the theory of consensus.

Approval voting, the simplest of the alternative methods, dates back to at least the 13th century, when Venetians used it to help elect their magistrates. It was subsequently reinvented many times, although it didn't acquire a name until 1976. In an approval vote, a person casts one vote for every candidate he or she considers qualified for the office. It's like an opinion poll, only the results are added up to determine a winner.

Brams argues that all presidential elections should be decided this way. "With approval voting, you can eat your cake and have it, too," he says. Voters who like a dark horse don't have to feel as if they are wasting their votes: "You can vote for all the out-of-the-running candidates you want to, and a safe choice as well." Even if the safe candidate wins, the support for other candidates will be noted. At the same time, Brams believes, the quality of debate would also improve: "Campaign strategies would change. You would have to be more expansive in your appeal." Third-party candidates, like Ralph Nader or Ross Perot, would get more votes and get their ideas out to a wider audience.

Can approval voting truly change the outcome of an election? Absolutely, as the CNN/*USA Today/*Gallup poll demonstrated. If this year's election had been decided by an approval vote in February, McCain would have won. True, by the November election his rivals would have had more time to dig up dirt against him, but the polls showed that three weeks of intense campaigning by Bush didn't make a dent in McCain's approval ratings. Besides, Brams argues, negative campaigning wouldn't be as widespread if we used approval voting, because it would be more likely to backfire against its users.

The advantages of an approval vote— and the perils of plurality voting— are most apparent in contests like the Louisiana governor's race of 1991. The primary that year was dominated by three candidates: Edwin Edwards, the often-indicted former governor; David Duke, a former grand wizard of the Ku Klux Klan; and incumbent governor Buddy Roemer. Edwards won the primary with 34 percent of the vote compared with 32 percent for Duke and 27 percent for Roemer. But it was Duke's surprisingly strong showing, despite his overtly racist stance, that won national headlines. *Time* and *Newsweek* ran long articles about the politics of hate in America. Bumper stickers, anticipating an Edwards-Duke runoff election, urged Louisianans to "Vote for the crook: It's important."

In the end, Edwards walloped Duke by a 61 to 39 percent margin. But the result was hardly a triumph for the runoff system. Say what you will about Louisiana voters, it's unlikely that anyone other than Edwards's core supporters really wanted to put a "crook" in the governor's office. And the election returns from November show beyond a doubt that very few people approved of Duke, outside of the 32 percent who originally voted for him. Roemer, on the other hand, had no strikes against him except that he had recently switched parties. In an approval vote, he might well have finished first, sparing Louisianans the choice between racketeering and racism. By the same token, approval voting might have spared Minnesotans from electing a professional wrestler to the governor's seat two years ago, or New Hampshirites from handing Pat Buchanan a triumph in the 1996 presidential primary.

Some theorists object that approval voting violates the "one person, one vote" principle. But that is partly a misconception. The Supreme Court decision that enshrined that principle— *Baker v. Carr,* in 1962— was meant to ensure that each voting district in a state had the same population. Approval voting wouldn't change that. More important, Brams argues, approval voting gives each voter equal "sovereignty" over the way his or her vote is counted. "Voters are more equal if they have an equal opportunity to express themselves," he argues. "If I prefer one candidate above everyone else, I can better express myself with a bullet vote for him. Another voter who hates one candidate can express his preferences by voting for the other four."

Approval voting has some strong advocates in the voting science community, but it's hardly the clear front runner (see "Mathematicians in the Voting Booth," page 82). Its chief competitor is the Borda count, the method championed by Donald Saari. The Borda count was named after a French physicist— and later a hero in the American Revolution— named Jean-Charles de Borda, who proposed it in 1770. But it was used in the Roman senate as long ago as A.D. 105. Although it sounds obscure, sports fans should recognize it as the method used to rank college football and basketball teams.

In a Borda count election, each voter ranks all of the candidates from top to bottom. If there are, say, five candidates, then a voter's top-ranked candidate gets 5 points, his second-ranked candidate gets 4, and so on. Finally, the points from all the voters are added up to determine the winner.

Though more complicated than approval or plurality votes, Borda counts sidestep certain pitfalls. Suppose, for instance, that three voters are trying to decide whom to vote for in a primary. Alice ranks McCain first, Bush second, and Gore third. Betty ranks Bush first, Gore second, and McCain third. Cheryl ranks Gore first, McCain second, and Bush third. The situation is completely symmetrical; each candidate has one first-place, one second-place, and one third-place ranking. "I have yet to find anyone who says this should be anything else but a tie," Saari says, and in a Borda count, it is.

Consider a two-stage election, however, consisting first of a Republican primary followed by a runoff between Gore and the Republican winner. In stage one, Cheryl and Alice would vote for McCain, and so he would win the primary, 2 to 1. In stage two, Betty and Cheryl would vote for Gore over McCain, and so Gore would win the runoff, 2 to 1. In this format, the result isn't a tie: Gore wins. In fact, in this year's Michigan primary, Bush's supporters complained that some Democrats, like Cheryl, crossed party lines to help McCain beat Bush. But it wasn't their fault— the system encouraged it.

Now consider a four-person election in which two voters have exactly the opposite preferences: Andrew's preferences are, in descending order, Gore, Bradley, McCain, and Bush. Bob prefers Bush first, McCain second, Bradley third, and Gore fourth. Once again, most people would call this a four-way tie, and a Borda count would agree with them. But in a plurality vote, Gore and Bush would tie for first and Bradley and McCain would be out of luck.

"The reason voting procedures give us paradoxes and unwanted outcomes is that they do not respect the symmetries of data that give us ties," Saari says. Is there any system that would consistently declare the vote of Alice, Betty, and Cheryl a tie, and would do the same for Andrew and Bob? "Only one procedure does that," Saari says. "The Borda count." To prove his point, he published two 50-page papers on the Borda count in the January issue of the journal *Economic Theory.*

How would McCain, Bush, and Gore have fared in a general election under the Borda count? The *Sacramento Bee* poll, plus the official tallies for California's open primary, offers a clue. According to the Bee's results, McCain would have beaten Gore 48 to 43, and Gore would have beaten Bush 51 to 43. But could McCain have beaten Bush among all voters, not just Republicans? Here's one way to figure it out. In the California primary, 60 percent of Republicans voted for Bush and 35 percent voted for McCain. We can assume that those percentages would have stayed the same had they been slated to run against each other in November. On the Democratic side, nearly 800,000 voters broke party ranks in March's primary to vote for Republicans. Of these, 64 percent voted for McCain and 31 percent voted for Bush. Again, the simplest thing to do is assume that the entire Democratic party would have split the same way if they were forced to choose between those two candidates in November.

So in our hypothetical Bush-McCain race, Bush would win by 60 to 35 among the Republican voters, and McCain would win by 64 to 31 among the Democratic voters. Overall, McCain would beat Bush, 50 percent to 45 percent. (To arrive at those figures I first added together each candidate's percentage of the Democratic and Republican vote— 60 and 31 for Bush, 64 and 35 for McCain— and divided them in two. That gave 45.5 for Bush and 49.5 for McCain. I then converted those totals to percentages and rounded them off.) To see how all three candidates would do under the Borda count, we add up the results of the head-to-head matchups:

Gore 51	Bush 43	
Gore 43		McCain 48
	Bush 45	McCain 50
Gore 94	**Bush 88**	**McCain 98**

Hence, in California at least, McCain would have been the Borda count winner.

"Two cheers for the borda count," Brams laconically replies. He doesn't dislike Saari's system— it often produces a better consensus candidate than the plurality method, assuming voters rank the candidates sincerely. But it can produce head-scratching results if voters try to beat the system. For example, if all Democrats rank Gore first and Bush last, and all Republicans rank Bush first and Gore last, voters might wake up the next morning with a surprise winner— Ralph Nader, say, or Pat Buchanan— thanks to all the second-place votes. The system would clearly take some getting used to. (The problem of insincere voting was pointed out to Borda himself. His response was characteristic of a more optimistic age: "My system is only for honest men.")

Both Brams and Saari point out that their disagreement has never become personal. Yet their theories clearly live in different worlds. Brams's approach is classical, axiomatic, and deductive. Saari's is geometric and visual ("The

Geometry of Voting," page 82). Brams's theory assumes that voters will often not be able to side with one candidate over another; Saari's starts from the viewpoint that they can rank them all. Brams argues that voters should have "sovereignty"— that one voter's 1-2-3-4-5 may be different from another voter's 1-2-3-4-5. Saari laments the "indeterminacy" of the approval vote, the fact that the outcome is not fixed only by the voters' preferences but also depends on their voting strategies.

The one trait that both systems share is a need for hard choices: They both ask voters to choose whom they would pick if their favorite were eliminated. That is the information missing in our current system, Saari and Brams say, yet it's vital to reaching a consensus.

In the early 1980s, convinced of the current system's shortcomings, Brams launched a one-man voting reform movement. He wrote numerous articles about the approval voting system and took his campaign to the studios of *Good Morning, America* and to the legislatures of New Hampshire and New York, yet the crusade failed to find many converts in government. According to Brams's former student Arnold Urken, a political scientist at the Stevens Institute of Technology in New Jersey, the failure was not surprising. Before people will reform something as momentous as a political election, Urken says, they need to learn about voting methods in less formal situations.

Groups of friends, Urken suggests, could begin by using alternative voting methods to decide which restaurant or movie to go to. If the voters are well-informed and clear about their rankings, the Borda count may be best. If they are not so sure, approval voting may work better. Either way, they will probably reach a consensus faster than through a plurality vote.

The Internet is another natural place to test voting methods. Urken runs an e-business called Choice Logic Corporation out of his home in New Jersey, consulting for companies on the interpretation of customer surveys, which are a kind of vote. The companies have a big stake in getting their information right, so Urken advises them to use several different tallying methods. "I see voting as a tool for looking at data," he says.

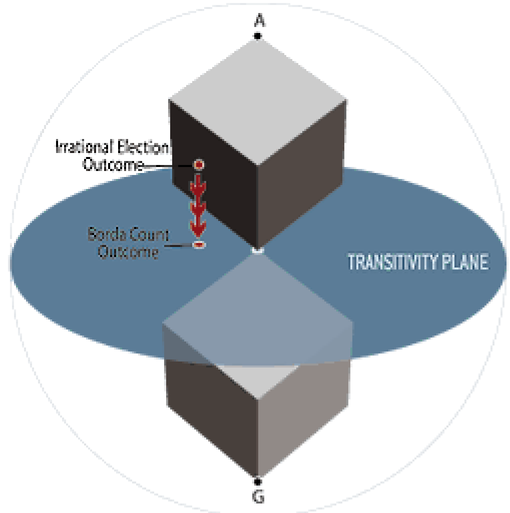
The best testing ground for alternative voting methods may be among scientists themselves. Six scientific societies now use approval voting for their elections, most notably the Institute of Electrical and Electronic Engineers, whose 350,000 members outnumber the eligible voters in Wyoming. "I think it's no accident that mathematics and engineering societies adopted it, because they can understand the logic behind it," Brams says. He also notes that none of them have gone back to other methods.

Other scientists have made good use of Saari's method. In 1973, for instance, the scientists who designed the Voyager missions to Jupiter and Saturn used a Borda count to help choose among several trajectories. Jim Parker, who works on computer vision at the University of Calgary in Alberta, uses voting methods for character recognition. One computer program might interpret a scrawled figure as a 3, while three others think it is an 8 and one thinks it is a 6. If they vote on it, they almost always get it right. With a straight majority vote, they make one mistake every 166 characters. When they use a Borda count, the error rate drops to one every 1,000.

Americans love to vote. We vote for all-star teams, music awards, soft drinks, and hamburgers. Millions of us watch two television shows that involve a vote: *Survivor* and *Who Wants to be a Millionaire?* The only time we don't like to vote, it seems, is Election Day. Some would trace this strange reluctance to the corruption and inequality in campaign financing. But the more likely reason is that people don't believe they're getting a fair choice. Sometimes the most popular candidate, like John McCain this year or Buddy Roemer in Louisiana in 1991, can't even make it onto the ballot. At other times, ballots list numerous candidates, but people hesitate to vote for their favorites because they're only allowed one vote and they don't want to "waste" it. Perhaps the best way out of our electoral malaise isn't to reform campaign finances, as politicians so often say, but to reform the voting system itself—even if it means doing so one vote at a time.

The Geometry of Voting

It's a remarkable fact, discovered by the Marquis de Condorcet, that elections with rational voters can nevertheless have irrational results. Suppose, for instance, that three candidates— X, Y, and Z— are pitted in a series of head-to-head elections. Candidate X beats Y, 60 to 40; Y beats Z, 56 to 44; and Z beats X, 52 to 48, with each number representing a percentage of the vote. If X beats Y and Y beats Z, one would think that X should beat Z. Yet the opposite occurred. Resolving such paradoxes is one of the fundamental challenges of any voting method.



In his book, *Geometry of Voting,* mathematician Donald Saari uses a "representation cube" to illustrate voting paradoxes. The large cube represents all possible election outcomes. The two smaller cubes within it represents all irrational outcomes. Point A, for instance, corresponds to an election where 100% of the voters prefer candidate X to candidate Y, 100% prefer candidate Y to candidate Z, and 100% prefer candidate Z to candidate X. The point at the center of the cube (50, 50, 50) represents the case where all the head-to-head elections result in ties. The votes described in the first paragraph would correspond to point (60, 56, 52). There's no need to plot both percentages in a given vote, Saari reasons, since they add up to 100 in any case.

Saari, along with Union College mathematician William Zwicker, discovered that there is a "transitivity plane" that slices through the cube. When the outcomes of three-party elections lie on this plane, the same candidate would win the election no matter what voting method was used. Why not simply move all irrational election results to the nearest outcome on the transitivity plane? That's exactly what the Borda count does. By subtracting six percentage points from each of the candidates' results above, one can move the outcome to the nearest point on the transitivity plane (54, 50, 46). That would mean that candidate A is preferred to B and C by equal 54 to 46 margins, while B and C are tied— a perfectly rational result, and well-deserved. In his recent work, Saari has extended this analysis to any number of candidates. He emphasizes that almost any voting method can be interpreted geometrically, and doing so often helps reveal the method's hidden assumptions.
— *D.M.*

Web Resources:

For information about how the electoral college system works, see "MathAgainst Tyranny" by Will Hively, Discover, November 1996.http://208.245.156.153/archive/output.cfm?ID=907