Exponential Distribution

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1 Introduction to the Exponential Distribution

The Exponential distribution is often used as a model for durations. It is related to the Poisson distribution in that it can be used to measure the time between successes from the Poisson process. Because the exponential represents time intervals, it is a continuous, not discrete, probability distribution. In many real life examples, the assumption that events will occur at a constant rate of time is untenable. For example, the rate of incoming phone calls differs according to the time of day. "But if we focus on a time interval during which the rate is roughly constant, such as from 2 to 4 PM during work days, the exponential distribution can be used as a good approximate model for the time until the next phone call arrives. Similar caveats apply to the following examples which yield approximately exponentially distributed variables:
* the time until you have your next car accident;
* the time until a radioactive particle decays, or the time between beeps of a Geiger counter;
* the number of dice rolls needed until you roll a six 11 times in a row;
* the time until a large meteor strike causes a mass extinction event.

Exponential variables can also be used to model situations where certain events occur with a constant probability per unit distance:
* the distance between mutations on a DNA strand;
* the distance between roadkill on a given street;"[1]

2 Mathematical Definition

The exponential density function is:

$$f(x) = \lambda e^{(-\lambda x)}$$

where $x \geq 0$. The parameter λ determines the "rate" at which events occur.

The cumulative distribution, the probability that a randomly drawn value will be smaller than k, is particularly easy to calculate in this example. The cumulative distribution is

$$F(k) = \int_0^k \lambda e^{-\lambda x}$$

$$= -e^{-\lambda x} \mid_0^k$$

$$= 1 - e^{-\lambda k}$$

[1] http://en.wikipedia.org/wiki/Exponential_distribution

3 Moments

The expected value of x for an exponential distribution is

$$E(x) = \frac{1}{\lambda}$$

The variance is

$$Var(x) = \sigma^2 = \frac{1}{\lambda^2}$$

The skewness is

$$Skewness(x) = 2$$

The kurtosis excess is

$$Kurtosis(x) = \quad 6$$

4 Illustrations

The graphs in Figure 1 show how the shape of the distribution changes as the rate of change, λ, increase.

Figure 1: Exponential Density for Various Rates